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An International Review of Spectroscopy and
Astronomical Physics

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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XLV JANUARY 1917 NUMBER 1

THE PHYSICAL INTERPRETATION OF ALBEDO. I

By LOUIS BELL

The albedo of the bodies in the solar system is one of the few clues available for the analysis of their physical characteristics. The values which have been current for albedo have unfortunately been, for various reasons, exceedingly misleading, and prior to the publication of H. N. Russell's important paper[1] there were no published values of much service in furthering physical investigation. In undertaking an examination of the subject some two years since the writer was compelled to make some drastic revisions of the current values, which Russell's contribution has made it possible to get into sufficiently satisfactory form for the purpose in hand. Primarily, the chief trouble in straightening out the matter of albedo has arisen from the fact that the term itself is used in different senses by different writers and sometimes by the same writer.

There are three quantities which have traveled under the general name of albedo as regards the heavenly bodies. First stands what may properly be called the *geometric albedo*, which is merely the proportion of the incident solar light visibly reflected from the given body at full phase. This takes no account of any fact or hypothesis regarding the relation of the reflecting power to the shape of the body, or of any law connecting the reflecting power of its surface with the angles of incidence and reflection. It depends merely on the geometrical relations of the body to the sun and earth, and on its photometric ratio to the sun. This is Russell's quantity p, which, as he indicates, is important in comparing the albedo thus defined with the reflectivities of terrestrial substances. It is also significant in giving, when compared with these reflectivities, a clue to the character of the surface.

---

[1] *Astrophysical Journal*, 43, 101, 173, 1916.

Unhappily, this quantity, which seems very direct and obvious, has often been confused with an entirely different class of quantities which may properly be called *spherical albedos* and which have to do with the proportion of visible reflected solar light, taking into account the sphericity of the body and the known or assumed relations between the reflectivity and the angles of incidence and of reflection.

Of such quantities there are as many as there may be hypotheses assumed regarding incidence and reflection. The best known of the class is what should properly be called Lambert's albedo, based upon his cosine law as applied to a perfectly diffusing sphere. It is this which has been most commonly known as "albedo" in astronomical literature. Numerically it is 3/2 the geometric albedo of such a perfectly diffusing sphere, and physically it is the reflectivity of an element of such surface at substantially normal incidence, as Lambert himself indicates in the following words: "Porro eodem hoc casu, quo nempe planetae pleno orbe lucent, claritas totius disci media est 2/3 claritatis centralis."[1]

Euler's law of reflection,[2] has occasionally been used for reckoning a value of albedo which should be carefully distinguished both from Lambert's and from others. The relation of any kind of spherical albedo to the apparent geometric albedo obviously depends on the law assumed or found for the spherical case. Numerically the spherical albedo may turn out greater than the geometric, as by the assumed laws just noticed, or less, as in some cases of Bond's albedo, *q.v.*

Another hypothetical law of reflection, the Lommel-Seeliger law, which Russell shows (*loc. cit.*), leads at times to values of the specific albedo impossibly large or even exceeding unity, was put forth by Seeliger[1] and used by Müller[2] for a tentative series of albedo values. Finally, Russell (*loc. cit.*), abandoning hypothesis in favor of experiment, has recurred to Bond's definition of albedo as the ratio of the whole amount of light reflected from the sphere to the whole amount incident on it, and has tabulated these values as A in his paper. Bond's definition stands squarely upon the experimental facts in so far as they are known, while the spherical albedos previously mentioned depend on more or less serviceable hypotheses which are actually found not to hold with precision.

---

[1] *Photometria*, § 1084, p. 484.

[2] *Astrophysical Journal*, 43, 178, 1916.

Besides, one has to deal at times with what Zöllner called true albedo, meaning thereby the specific reflectivity of an element of a diffusely reflecting surface under certain real or supposed conditions, corresponding to some definite case of incidence. This *specific albedo*, as it should be called, is important from the physical standpoint in examining reflecting surfaces where the conditions of incidence are known or can be tentatively assumed, and should properly be confined to normal incidence. Unhappily, Zöllner referred to his spherical albedo reckoned according to Lambert's law as "apparent albedo," which has been the cause of much confusion between this quantity and the geometric albedo.

The relation between these three—geometric albedo, spherical albedo, and specific albedo—may be emphasized by the following experiment of the writer, made on a precision photometer using a Lummer-Broadhun contrast prism as indicator. The apparent reflectivity of a ball 16 cm in diameter and painted with several coats of dead-white paint, having a surface as lusterless as could be obtained, showed an absolute value, as measured, of 0.48. The corresponding spherical albedo, assuming Lambert's law, was 0.72. From measurement of the phase factor the spherical albedo, according to Bond's definition (Russell's A), was 0.56, while the specific albedo of the paint used, at small angles of incidence, was 0.74. By this measurement on an artificial planet the three kinds of albedo stand out distinctly in their practical relations. In ordinary photometry the geometric albedo occasionally appears in measurements of absolute reflecting power, the spherical albedo only in rare instances of curved reflecting surfaces being used as secondary sources of light, while the specific albedo is the quantity which commonly appears in determinations of relative reflectivities which are usually compared at small angles of incidence. As Russell points out (*loc. cit.*), the spherical albedo as defined by Bond has its chief use in considering what happens to the total energy incident upon a planet; Lambert's albedo for all physical purposes is practically useless, as all but a few of the heavenly bodies depart rather widely from it, most of them by unknown amounts. It is unfortunate that it should have obtained so great currency in astronomy, for it has led to many misunderstandings.

---

[1] *Abhandlungen der K. bayer. Akad. d. Wissenschaften*, 16, 1888.

[2] *Photometrie der Gestirne*, Leipzig, 1897.

CURRENT VALUES OF ALBEDO

The first serious attempt at a comprehensive study of albedo was that due to Zöllner, who was really the founder of celestial photometry. He investigated carefully the light-ratios between Capella and the sun and between the planets and Capella, or some star with which it had been compared, thus finally obtaining the photometric ratios between the planets and the sun, from which he deducted their albedos, using the then current diameters of the planets, which were very imperfectly known. The first investigation included Mars, Jupiter, Saturn, Uranus, and Neptune. Venus and Mercury were added after later photometric measurements.[1] These are all spherical albedos, according to Lambert. In 1893, after most elaborate photometric researches on the planets lasting over some years, Müller determined their relative stellar magnitudes with a precision that, whatever it may really turn out to be, has not since been surpassed. These photometric values he connected with the best measures of diameter then available for a redetermination of the planetary albedos.[2] These albedos were expressed, not as absolute, but as relative values comparable with Zöllner's and assuming Zöllner's value for Mars as unity. These values of Müller's are therefore in terms of Zöllner's sun-Mars light-ratio, this value being probably chosen on account of the fact that from the large number of observations Mars was supposedly Zöllner's best determined planet. Unfortunately, this has turned out to be his least precise ratio, since, connecting it with Müller's magnitude of Mars, the equivalent stellar magnitude of the sun comes out −26.4. Zöllner's sun-Jupiter ratio gives −26.57, while Saturn, of which Zöllner had a valuable series of observations with the rings gone, would have given −26.9, and Uranus −26.48. Now, assuming the magnitude of the planets to be known, the assumption of too small magnitude for the sun means too high value of albedo all along the line, and when Müller's relative values were translated into absolute values on the basis of Zöllner's Mars, the result was the truly astounding tabulation shown in column 5 of Table I, beginning with Mercury at 0.17 and ending with Saturn and Venus at 0.88 and 0.92 respectively.

---

[1] *Poggendorff's Annalen, Jubelband*, p. 641, 1874.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, 8, 369.

TABLE I

Albedos of the Planets

Planet	Zöllner		Müller		Müller $\odot = -26.6$
	Relative	Absolute	Relative	Absolute	Absolute
Mercury	0.43	0.11	0.64	0.17	0.14
Venus	2.33	.62	3.44	.92	.758
Mars	1.00	.27	1.00	.27	.22
Jupiter	2.34	.63	2.79	.75	.616
Saturn	1.87	.50	3.28	.88	.721
Uranus	2.40	.64	2.73	.73	.604
Neptune	1.74	0.46	2.36	0.63	0.521

The absolute values in this table are all spherical albedos, according to Lambert. These figures have been quoted in most astronomical works from Müller's publication to the present time, in spite of the fact that they were open to suspicion on account of the very high values for Venus and Saturn. The latter, in particular, carried evidence of fatal error on its face, since the assumption of 0.88 for the albedo of the ball as a whole would necessitate considerably higher reflecting power in the bright equatorial belt, and an albedo quite certainly over unity for the bright part of the *B* ring.

Müller himself undoubtedly detected some of the sources of error shortly after his original values were published, and four years later[1] he reverted to Zöllner's sun-Capella ratio which with his own magnitude of Capella led to −26.60 as the stellar magnitude of the sun. This resulted in a general reduction of nearly 20 per cent in the albedos of the planets, which values are set out in the sixth column of Table I. For some inexplicable reason these conscientiously revised albedos attracted comparatively little attention, and the earlier series, like many things that are not so, retained world-wide currency.

ACTUAL STELLAR MAGNITUDE OF THE SUN

In view of Russell's investigation, it is unnecessary to review this matter at length. It should be pointed out, however, that the simple average of the results obtained by Zöllner, C. Fabry, W. H. Pickering, and Ceraski is of dubious value, the chances being that the first and last of these values are entitled to less weight than the others; Zöllner's on account of the widely varying results of his planetary ratios, which indicate large experimental errors, and Ceraski's[2] from the extraordinary variation between his determinations via Polaris, Procyon, and Sirius. When reduced to the Harvard scale from the Potsdam scale, the results from Polaris and Procyon are fairly concordant, but diverge widely as to Sirius, which had fewer observations, but which had the very great advantage of being a much brighter comparison object.

The determination which seems least subject to error from the photometrical standpoint appears to be that of C. Fabry,[3] who obtained −26.80 reduced to the Harvard scale of magnitudes.

Two photometric determinations by Birck[4] and by E. S. King[5] certainly cannot be taken as in any sense decisive, on account of the large and very uncertain corrections for color-index. The same doubt, of course, applies to any other photographic value. In lack of any measurements of the stellar magnitude of the sun which can properly lay claim to high exactness, Russell's tentative value of −26.72 is here assumed as correct, although in the writer's judgment the indications point to a slightly higher value.

---

[1] *Photometrie der Gestirne*, Leipzig, 1897, p. 317.

[2] *Astronomische Nachrichten*, **170**, 135, 1905.

[3] *Association française pour l'avancement des sciences*, Part 2, p. 253, 1903.

[4] *Das photographische Helligskeitsverhältnis der Sonne zu Fixsternen*, Gottingen, 1909, p. 74.

[5] *Harvard Annals*, **61**, 56.

It is certain that the quantity needs redetermination badly and by methods which have proved their precision in the ordinary work of photometry. In particular, all color-filters should have their coefficients of transmission determined by the flicker photometer instead of relying on the ordinary instruments in which simultaneous contrast throws luminous values wide from the truth.[1] Secondly, the sector disk, which is by far the most exact and reliable method of cutting down luminous intensities, should be used to the exclusion of every other device for that purpose. By these means and with the powerful electric lamps now available in photometry it should be possible easily to determine the apparent intensity of sunlight within 1 or 2 per cent, leaving the reduction for atmosphere as the only material source of error.

TABLE II

ALBEDOS

Object	m_1	r_1	Geometric Albedo	Lambert's Spherical Albedo	Bond's Albedo Russell's *A*
Jupiter I	−1.16	2.38	0.46	0.690	0.69 [.48]
Jupiter II	−1.01	2.08	.51	.765	.76 [.53]
Jupiter III	−1.62	3.62	.30	.450	.45 [.31]
Jupiter IV	−0.44	3.49	.11	.165	.16 [.11]
Titan	−1.26	2.9	.33	.495	.50
Ceres	3.70	0.53	.10	.150	.06
Pallas	4.38	0.34	.13	.195	.07
Juno	5.74	0.14	.22	.330	.12
Vesta	3.50	0.27	0.48	0.720	0.26

Given the stellar magnitude of the sun, the two quantities which appear in the determination of geometric albedo are the stellar magnitudes of the objects reduced to unit distance, and their semi-diameters at unit distance. For the small bodies the values of Russell's table, which are in close concordance with the Harvard scale, are adopted, no material discrepancy being introduced by so doing. Table II gives the albedos of the minor bodies thus found.

---

[1] Bell, "Some Factors in Heterochromatic Photometry," *Electrical World*, January 27, 1912.

ALBEDOS OF THE MINOR PLANETS AND SATELLITES

It is convenient to consider all these small bodies together, since they present in general the same kind of problems, and in particular are all, so far as theory and observation indicate, without atmosphere. Their dimensions preclude the retention of any perceptible amount of atmosphere, the value of gravity at their surfaces being far too small to prevent the escape of even the heaviest gases. If the satellites were derived from their primaries by fission, they would take very little atmosphere with them, and that probably of the lighter constituents. If otherwise acquired, there is still less likelihood of starting with an atmosphere. On any hypothesis, the loss of what atmosphere they may originally have had would be rapid.

The diameters of all these bodies are relatively small, most of them very small, so that measurement of them by ordinary astronomical means becomes exceedingly difficult except in the case of our own moon. All told, and including the moon, only ten of these minor bodies have had their diameters measured with any approach to accuracy. Of the twenty-six known satellites, only five—the original four of Jupiter, and Titan of Saturn's family—aside from our moon, have diameters as yet measured. Of more than eight hundred minor planets but four, constituting the original group, have been measured, and these only by one observer, Barnard.[1] A single, but very good check on the diameter of (4) Vesta, agreeing exactly with Barnard's figures, has been obtained by Hamy, using an interference method. The corresponding geometric albedos are given at the end of Table II. The variations between them are notable, the figures ranging from 0.10 for Ceres to 0.48 for Vesta, bespeaking a very wide difference in surface character; and the recorded observations on these minor planets check well with such variations of albedo, aside from all questions of diameter, Ceres being conspicuously dull and Vesta conspicuously bright. The albedo of the first is well within plausible limits for a dark, rocky material; that of the latter is so high as peremptorily to demand explanation.

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[1] *Monthly Notices*, 56, 55, 1895.

In reducing the geometric to the spherical albedo Russell (*loc. cit.*), has assumed a reduction factor similar to that of our moon; in fact, approximately halfway between the values taken for the moon and for Mercury. This implies, for the asteroids considered, extreme departure from Lambert's law, doubtless due to the strong shadowing, the cause which is effective in the case of the moon and Mercury. The low albedo of Ceres is easily accounted for in this way, but a geometric albedo as high as 0.48 is not consistent with such shadowing as is characteristic of the moon and Mercury, for it would imply a specific reflectivity out of all reasonable range of known massive substances. Zöllner (*Photometrische Untersuchungen*) gives for the moon, based on a study of its slopes and shadowing, a specific reflectivity, "Wahre Albedo" 0.17, while his geometric albedo must be cut down to 0.08. In other words, a body which loses brilliancy from roughness and shadowing, as in the case of the moon, may have a specific reflectivity of surface material something like double the geometric albedo. We must therefore conclude that the brilliancy of Vesta is due both to the texture and to the material of its surface. Pallas and Juno, which have intermediate albedos, may owe them to variations in surface or in shadowing or in both.

The key to this particular problem is found in the phase-factor. A smooth body which approximately obeys Lambert's law will show a phase-factor independent of its surface reflectivity. On the other hand, great roughness, with the accompanying strong shadowing, is likely to produce a large phase-factor, e.g., the moon and Mercury, and to reduce the mean geometric albedo. A rough, heavily shadowed body will therefore show lower albedo than a smooth diffusing one of the same specific reflectivity, and, therefore, other things being equal, *the geometric albedo of an airless body should vary with the phase-factor*, which is an index of the amount of shadowing. The less shadowing, the higher the albedo and the less the phase-factor; the more shadowing, the larger the phase-factor and the smaller the apparent albedo, inasmuch as the effect of shadowing is itself lessened by high surface albedo, a term in A^2 appearing in the reflection formula.

Hence it should be possible to establish an approximate experimental relation between phase-factor and albedo for bodies in which there are no complications due to the presence of an atmosphere. Such is in fact the case. When the phase-factors and geometric albedos of the four well-known asteroids, Mercury, and the moon are plotted in rectangular co-ordinates, as in Fig. 1, they fall into evident relationship. Two additional points, expressing the relation between phase-angle and albedo for a body fairly near to obeying Lambert's law, have been added from the experiments on the artificial planet already referred to, representing two different kinds of diffusing paint, one somewhat glossier than the other. The closeness with which these check the albedo and phase-factor of Vesta serve both to establish that end of the curve and to hint very strongly that the surface of Vesta is both smooth and light in color, as if owing its reflectivity to optical dust. The asteroids vary enormously in phase-factor and hence in geometric albedo. The limits set by the four directly investigated are passed in both directions, the phase-factors varying according to Müller[1] from 0.016 (Iris) to 0.053 (Frigga) magnitude per degree. Even these figures, derived from Müller's list of 34 fairly well-determined asteroids, have been somewhat extended. By graphical interpolation it therefore becomes possible to form an approximate idea of the albedo of an asteroid, provided its phase-factor is fairly well determined, and hence, having the elements of its orbit and its magnitude, to make an estimate of its probable diameter likely to be nearer the truth than any reckoning based on an assumed uniform albedo for the whole body of asteroids, as has been done by previous investigators of this question.

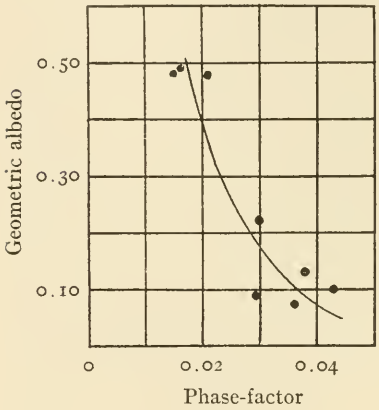

FIG. 1

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[1] *Photometric der Gestirne*, p. 378.

That these albedos are not uniform or anywhere near it, we know to a certainty, and we can establish at least an approximate relation from experimental data between the phase-factor and the probable albedo. Unfortunately, most of the asteroids are not observable through a wide range of phase-angle, and the measurements of different experimenters are in some cases rather discordant. As an example of the application of the relation between phase-factor and albedo to the determination of diameters, the writer selected from Müller's table ten asteroids, of which the phase-factors seemed likely to be the most reliable, considering the number of measurements, the concurrence of measurements by different observers, and the phase-angle covered. The approximate corresponding albedos were estimated from Fig. 1, and the corresponding diameters computed, following the data of Müller's table (*loc. cit.*). The results, including the four experimentally determined asteroids, are shown in Table III.

TABLE III

Diameters and Albedos of Asteroids

Asteroid	Diameter at Distance Unity	Geometric Albedo	Phase-Factor	Diameter	Magnitude*
				km	
(1) Ceres	1″.06	0.10	0.043	(768)	7.4
(2) Pallas	0.78	.13	.038	(565)	8.0
(3) Juno	0.28	.22	.030	(203)	8.7
(4) Vesta	0.54	.48	.021	(390)	6.5
(7) Iris	0.20	.50	.017	144	8.4
(8) Flora	0.23	.22	.028	168	3.9
(11) Parthenope	0.17	.35	.022	122	9.3
(16) Psyche	0.74	.05	.048	539	9.6
(18) Melpomene	0.31	.14	.033	224	9.3
(30) Urania	0.12	.28	.025	87	9.9
(40) Harmonia	0.11	.50	.017	78	9.2
(44) Nysa	0.17	.28	.025	121	9.8
(77) Frigga	0.31	.05 ?	.053 ?	296	11.1
(258) Tyche	0.36	0.05	0.046	259	11.1

* At mean opposition as taken from the *Astronomisches Jahrbuch*, 1916.

A glance at this table shows the great differences which probably exist between the albedos of the asteroids, since at least two of them have albedos as great or greater than that of Vesta and three others are materially less than that of Ceres. The former group, therefore, showed diameters considerably less than those previously estimated, and the latter group diameters conspicuously greater. (16) Psyche, for example, shows a diameter, both absolute and angular, nearly as great as Pallas, and hence should readily yield to micrometrical measurement, while (77) Frigga and (258) Tyche should be considerably easier to measure than Juno. The data give an interesting clue to the great variation in size of the asteroids. Bauschinger in 1900, on an estimated average albedo of 0.24, found that, of the then known asteroids, (452) Hamiltonia gave a computed diameter of only 10 km. Since his investigation, two considerably less brilliant ones, (719) Albert and 1900 GA, have been added. The latter, estimated to be of the eighteenth magnitude, would, from this figure and its distance, have on Bauschinger's assumption a diameter of only 3.4 km. At the albedo of Vesta this would shrink to less than 2.5 km, while it would be approximately doubled for an albedo like that of Psyche.

It is highly probable, therefore, that asteroids down to less than 5 km in diameter are accessible to photography if of average surface character. On the other hand, a body as dark as Psyche and of this diameter would be too near the optical limit to be detected unless under extraordinarily favorable circumstances, and anything materially smaller would be beyond reasonable chance of observation. As more phase-factors are determined with fair accuracy, asteroids of measurable diameter may be expected to turn up, and the addition of a few more to the four measured by Barnard would give valuable data concerning the physical character of the whole group.

As with increasing numbers they have been followed down to diameters of a few kilometers, there is every probability in favor of Newcomb's belief that they actually grade down to the size of meteorites and cosmic dust. Indeed, if the estimates which have been made regarding the Coon Butte meteorite have sound basis in fact,[1] a celestial body of dimensions fairly comparable with the smallest known of the minor planets actually may have fallen to earth within historic times.

---

[1] Elihu Thomson, *Proceedings of the American Academy of Arts and Sciences*, 47, 721, 1912.

As the Coon Butte crater is over a kilometer in diameter, the body whose impact caused it could hardly have been smaller than Thomson suggests (about 150 m) and may well have been double this size. As it was a meteorite, chiefly iron, its surface would have very low reflectivity, as low for instance as that of Tyche, than which at the same distance it would have been about 15 magnitudes fainter. At a million miles, however, a body of this size would acquire telescopic magnitude, so that it would comfortably bridge the gap between asteroids and the bodies which, on closer acquaintance, we know as meteors. Our own moon, were it moved to the distance of Ceres, would show a disk a couple of seconds in diameter and would outshine Ceres by more than 3 magnitudes, ranking therefore as the brightest of the minor planets. Incidentally, if the face of the moon we see is a fair sample, the moon would be slightly variable if in regular rotation, in virtue of the effect of the spots, which is found to be in evidence to the extent of nearly 0.2 between the beginning and the end of a lunation.

From the phase-factors, many minor planets have rough and irregular surfaces which bear evidence of an origin not gradual and orderly enough to permit smooth agglomeration or the grading effects of erosion. In this particular, too, they are akin to the meteorites, toward which they seem to pass in size and character. Whatever the origin of meteorites, their composition is such as to indicate little or no free oxygen and water, as was long ago pointed out by Professor H. A. Newton, and, as to the latter disqualification, evidenced by the absence of crystallized silica. There is no clue to the density of the minor planets save such as may be given by that of our own moon and meteorites, the former being about 3.3. The average of the stony meteorites runs a little higher, about 3.5, while the iron meteorites reach about 7.5. The possible asteroids of considerable mass, although of meager dimensions, lend weight to the theory of Seeliger that the aggregate mass of asteroids and of the meteor swarms to which they are allied may be great enough to show some effect in planetary perturbations.

The total light of the whole body of about 800 known asteroids is very small. Even if one took the average as approximately the twelfth magnitude, at unit distance, the whole group would not equal a fourth-magnitude star. This does not, however, prove that the enormous number of smaller bodies, individually entirely beyond telescopic reach, might not produce a visible effect, particularly if many of them should, like Eros, sweep inward to and beyond the orbit of Mars.

In recent years the average magnitude of the asteroids found steadily decreases. Bearing in mind that it would require the accession of about 10,000 asteroids of the fifteenth magnitude to double the light of those already known, or 1,000,000 of the twentieth magnitude, it is not an unsafe estimate to judge that the aggregate light of the whole swarm can scarcely exceed that of a star of the third magnitude. The question whether this light scattered in an asteroid belt can produce a visible zodiacal belt is easily answered. W. H. Pickering[1] has shown that the brightness of the full moon is equal to that due to 375 tenth-magnitude stars per square second of area, which is approximately 10^9 for the whole surface. Now the known asteroids aggregate little above 100 such stars, and it would be a most liberal reckoning which would give the equivalent of 1000 such stars simultaneously visible in the sky. Even so, the brilliancy of the whole mass concentrated in the moon's area would be barely perceptible, and scattered over the zodiacal belt known to be tenanted by the asteroids the light would be totally beyond view.

A collateral question is the visibility of single scattered bodies which may lie nearer the earth than the asteroid belt as a whole. Suppose a wanderer were to come in to a point only 0.1 unit from our planet, a location almost touched by Eros with its possible 0.12 unit. At the average albedo of the asteroids it would rise, roughly, to something like fifteenth magnitude, even if only a few hundred meters in diameter, and hence could be picked up photographically without much trouble if caught at opposition, only to be swiftly lost in the distance.

Were there many such strays it is safe to say that they would be not uncommon finds by this time; hence one must conclude that there are few brothers to Eros in our neighborhood. Still, a systematic search for small and swiftly moving bodies might well meet reward, considering the very modest dimensions here shown as necessary to visibility. There is some reason to suppose that the best field for such a search might be outside the ordinary zodiacal limits.

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[1] *Harvard Annals*, 61.

RELATION OF ALBEDO TO VARIABILITY

Any notable spotting of a celestial body, so that it presents areas of low and high albedo, will produce variations of light in revolution if the spots are unevenly distributed in longitude, or in orbital position if in latitude, the axis being inclined to the plane of the orbit. If unevenly distributed in both longitude and latitude, the rotational variation may change in magnitude and character with orbital position. Other things being equal, there would seem to be a greater chance of conspicuous spotting in small bodies than in large. Were the moon broken up into asteroids, the chance of getting large local variations of albedo in some of them would obviously be good.

H. N. Russell[1] has investigated the theory of spotted bodies and has shown that for any observed light-variation there may be an indefinitely great number of possible spot-distributions, so that to pass directly from light-curve to a detailed knowledge of the surface is out of the question. This follows, indeed, from the simple consideration that the observed light at any epoch is due to the optical integration of the light received from every element of visible surface taken with its appropriate albedo, $L=\Sigma[a\Delta s+a_1\Delta s'+\ldots a_n\Delta s_n]$, which expression tells nothing about the positions or albedos of the elements. An aggregation of dark spots interspersed with bright ones would produce exactly the effect of an area of the same mean effective albedo.

But, with this limitation, the light-curve can give some definite clues to surface characteristics. The simplest light-curve would be that of a singly spotted sphere, which corresponds in general to the case of the Algol variables, where the spot is an eclipsing body, with this difference, that the change in light due to a spot would generally be much less abrupt and also smaller. Fig. 2 may be taken as a type of curve from a dull body with a single bright area. The angular extent of the spotting in longitude determines the rate of increase and decrease of the light and the angular extent of the minimum phase. With a spotted area 180° in longitude the curve of variation would cover the entire 360°, and the light-changes would settle into a fairly smooth, regular curve like that shown by ζ Geminorum. If the body shows marked phases, particularly with a large phase-factor, asymmetry is introduced, for then the spot area will pass over only $180^\circ - \alpha$ of visible surface, coming on sharply on the preceding side and passing off slowly on the following side, producing a light-curve of the Cepheid type. Shift of phase-angle may similarly produce slight changes in the magnitude and phase of maxima and minima. Two spotted areas obviously give, if separated considerably in longitude, a double period, which may, from difference of latitude and obliquity of axis, merge into a single period. With spot-areas of different area or intensity and nearly 180° apart one could get a light-curve of the β Lyrae type.

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[1] *Astrophysical Journal*, 24, 1, 1906.

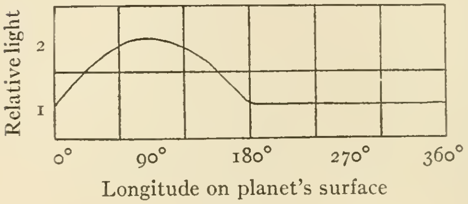

Fig. 2

The quantitative study of the light-curves can add considerably to the inferences drawn from their shape. A body shining by diffusely reflected light, and owing its variations in light to differences of albedo, has a decidedly limited range of light-change. Even were it divided cleanly in longitude into a bright and a dark hemisphere, the one having the albedo of Vesta, the other that of Ceres, the difference of magnitude at 0° inclination of axis would be only 1.7, while with chance spotting, even on a coarse and irregular scale, only small fractions of a magnitude can be expected. Even a body as conspicuously spotted as the moon shows a resulting asymmetry of scarcely $0^{M}.2$ in its light-curve, an amount which long escaped notice. Hence, it is not remarkable that light-changes due to rotation have been rarely detected among the asteroids, generally too faint for easy observation. Entirely fortuitous observational differences of at least $0^{M}.1$ are to be expected in such cases and these are quite sufficient to mask the effect of any moderate spotting.

QUASI-SPECULAR REFLECTION

When light falls upon a plane which possesses wholly or partly a polished surface, the light reflected from it, in part at least, is specularly reflected as from the surface of a mirror. The area considered may be actually a polished plane or may be made up of minute, substantially plane elements presenting slight angles to the general trend of the surface. Every elemental plane which is large compared with the wave-length of light acts as a mirror tending to form a definite image of the source of light. The summation of the rays makes up a brilliant reflected beam of total intensity proportional to the summation of the areas and their coefficients of reflection, and scattered over a solid angle of $2\ \psi$, where ψ is the maximum variation of the individual elements from the normal to the general phase. This is the condition met in a broken surface of crystalline stone which. in the sunlight, will give through a particular, and sometimes quite small, range of angles of incidence an intensely brilliant reflection, dropping to a very much lessened amount after the small angle just indicated has been passed. The familiar reflection of sunlight on the window of a distant house at perhaps many miles away is an example of the same thing, the angle ψ in this case being small compared with the variation of incidence due to the angular semi-diameter of the sun. The distribution of the light thus brilliantly reflected depends on the specific reflectivity of the material, the extent of the source which determines the angles of incidence, and the angle ψ which limits the extent of deviation from the general normal. The smaller and the more accurately plane the reflecting surface as a whole, the less the angle of spread of the resulting beam.

In practice a roughly plane broken area will have a general trend of the reflected ray, a considerable amount of miscellaneous scattering still specular in its character, and a large amount also of diffuse reflection of the ordinary kind due to irregularities of very heterogeneous character as compared with the wave-length of light, but not having their planes so directed as to form a specular beam. The distinction between pure diffusion and purely specular reflection lies chiefly in the magnitude and concordant direction of a considerable portion of the surfaces of incidence. If it is entirely heterogeneous, the light is diffuse. Possessing general community of direction, it is partly diffuse and scattered and partly of codirected specular effect. And finally, if the surface is nearly plane, the chief element of reflection will be codirected and the effect specular.

Now it is easy to compute the general effect of specular reflection in the case of one of the heavenly bodies. A really plane surface $0''.1$ in diameter would, at distance 2 from the sun and at an equal distance from the earth, reflect, crediting it with a specific reflectivity of 0.2, quite nearly $\frac{1}{5\times 10^{10}}$ the solar light; but the light of the sun equals substantially 5×10^{10} stars of 0 magnitude; hence, if the reflection from this surface were cleanly specular it would shine with something like the brilliancy of Arcturus, and on a shift of phase equal to the solar semi-diameter as seen from it this light would be practically extinguished. A cleanly specular area even $0''.0001$ in diameter would still shine as a star of about the fourteenth magnitude and would be quite obliterated on a similar change of phase-angle. Were the reflecting surface only approximately plane, the resulting beam would be lessened by its spread, but in any case there would be a probability of a very sharp increase in light at one particular angle followed by rapid decrease, leaving, after a swing of a few minutes of arc at the most, only a feeble reflection to take its place. The sharper the specular element of the reflection, and the smaller the element in which it takes place, the more abrupt will be the changes of light ensuing. There is little doubt that we actually get such reflection from some of the wall surfaces of the moon, which contains a very prominent specular element. The variations in brilliancy from point to point as found by W. H. Pickering[1] are so enormous as to offer no other explanation, and the variations of brilliancy with phase of various of these points is so notable as to enforce the same conclusion.

THE LIGHT-ENVELOPE OF A RADIANT

It is customary in photometry to express the intensity of light about a radiant (considered as a luminous point) by a polar curve from this point as origin. For every angle θ reckoned from the appropriate axis, a radius r is drawn of length proportional to the total luminous intensity in this direction and its end thus determined defines a point p on the curve. The locus of all points p from $\theta=0$ to $\theta=360^\circ$ defines the light-curve in a plane.

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[1] *Selenographical Journal*, 1882.

Similarly in three dimensions one may express the intensity in any direction in polar co-ordinates, θ, ϕ, for the angles from the two axes of origin, and radii proportional to the corresponding intensities. Then the loci of all points p' determined in varying θ, ϕ, from 0° to 360° with the corresponding values of r, determine a light-surface of which every radius drawn from the origin represents the total intensity along that angular direction.

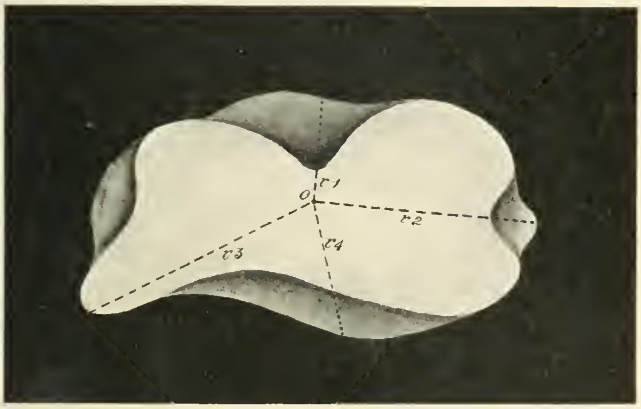

Fig. 3

In general, this surface is not a figure of rotation about any axis whatever. It must be considered as revolving synchronously with the source if this revolves; while if this be subject to phase-shifting, as in the case of a planet, each shift of phase produces a new set of values for r drawn to the observer and a consequent modification of the light-surface. As a simple example, Fig. 3 shows the light-surface about an electric arc lamp as instantaneous source.

If the source shines by reflected light, a regularly diffusing body will give a relatively symmetrical light-surface, while an area of quasi-specular reflection produces a protuberance due to the increased values of r, of form depending, for any given phase-angle, upon the area and constants of the specular surface. Except in the case of a perfectly diffusing body of regular shape the form of the light-envelope gives little, and often misleading, information as to the real contour of the body itself.

VARIABILITY OF THE ASTEROIDS AND SATELLITES

Aside from the variations of brilliancy due to phase-angle few periodic changes have been found in the small bodies. In some of the satellites, e.g., the old moons of Jupiter and Titan and Iapetus, there has been found periodic variation of light with orbital position indicating irrotational relation to the primary, as in our moon. Of these, all save the last show variations hardly exceeding half a magnitude, as has been shown by Guthnick.[1] Iapetus presents a different condition, since its range of variation[2] is approximately $1^{\mathrm{M}}.4$ (3.6 to 1). This is so great as to be highly improbable as a result of mere spotting. Guthnick[3] shows the range as reaching $1^{\mathrm{M}}.8$.

The smooth form of the light-curve found by E. C. Pickering (*loc. cit.*), precludes any great irregularity of form, while the ratio of nearly 4:1 required in the mean albedos of the two hemispheres is too great to be plausible if due to fortuitous spotting alone. A moderate degree of elongation, even less than the 9:11 ratio suggested by E. C. Pickering, with aggregation of material of low albedo toward one hemispheroid, suitably explains the phenomena.

The variability of the asteroids discloses more striking conditions. The phase-factor accounts for most or all of the once surmised long-period variations. Beyond these there appear in a few asteroids well-marked short-period variations clearly rotational in character. Of these, two, (7) Iris, *c*, Fig. 4, and (15) Eunomia, *d*, Fig. 4, investigated by Wendell[4] show variations of small amplitude and simple character taking place in about $3^{\mathrm{h}}6^{\mathrm{m}}$ and $3^{\mathrm{h}}2^{\mathrm{m}}$ respectively. Doubtless a great many similar cases will be found when a search is made with instruments of light-gathering power sufficient to make the photometry of these faint objects easy.

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[1] *Astronomische Nachrichten*, 198, 230, 1914.

[2] *Harvard Annals*, 11, 67.

[3] *Astronomische Nachrichten*, 198, 252, 1914.

[4] *Harvard Circular*, Nos. 75 and 94.

A radically different type of variability appears in the cases of (433) Eros, and (345) Tercidina, introducing factors of great interest to the photometrist. The first named, carefully studied during the opposition of 1901 by numerous astronomers, disclosed a variation of nearly $2^{\mathrm{M}}.0$ with a period of about $0^{\mathrm{d}}.11$, or probably double this, if the successive maxima are unsymmetrical. The great variation of light makes simple spotting an improbable cause; the form of the light-curve makes it practically impossible. Moreover, the variation, nearly 2^{M} early in February 1901, had sunk early in April to less than $0^{\mathrm{M}}.5$, and in May it became imperceptible.

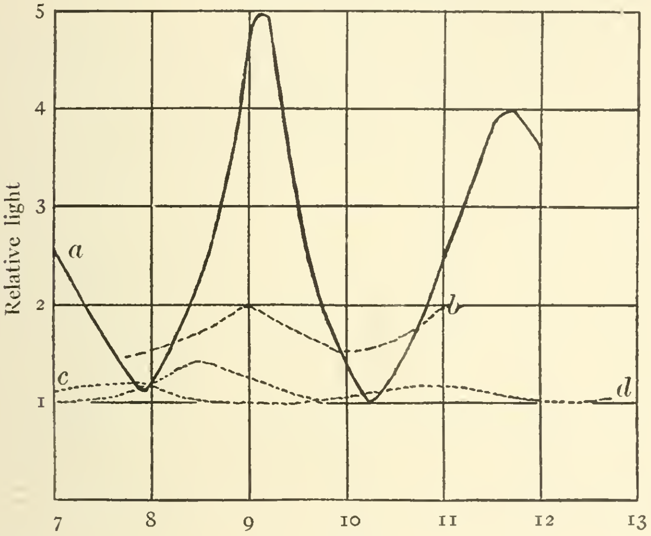

FIG. 4.—Light-ratio curve of Eros, 1901

Fig. 4 shows the actual light-curve of Eros as deduced from the observations of Guillaume, Lecadet, and Lnizet.[1] Curve *a* gives the variations on February 21, 1901, and curve *b* those on April 19 following. Now the amplitudes, and particularly the sharpness of the principal maximum, at once eliminate spotting as a possible cause, since no plausible difference of albedo accounts for the amplitude, with a spot small enough to account for the sharp rise. The case is all the worse when the double spotting required to account for the secondary maximum is considered. Likewise eclipsing, as in the Algol class, seems to be barred by the continuous variation and by the same difficulties as to amplitude and rate of variation, taking into account the limited range of albedo to be expected. E. C. Pickering[1] has gone over these hypotheses and has very properly rejected them. The kindred hypothesis of greatly elongated form requires a high average light at times of little or no variability, contrary to the observed facts.

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[1] André, *Les Planètes et leur origine*, p. 91.

The curve *b* adds to the complications, for it shows complete regularity within the experimental errors combined with a very rapid change in amplitude for a small change in relative positions of the earth and Eros. In case of eclipsing bodies moving about an inclined axis, failure of eclipse through shift of aspect should generally raise the average light instead of decreasing it.

Curve *a*, however, though extraordinary from the astronomical standpoint, concerned chiefly with self-luminous bodies, is of a form perfectly well known in technical photometry. It is an entirely typical case of quasi-specular reflection, and not a very extreme one at that. Curves at varying incidence from surfaces showing mixed specular and diffuse reflection always present these phenomena of sharp rise, a maximum which narrows with the angle subtended by the specular area and a fall which may or may not be symmetrical with the rise, according to the structure of the surface. An admixture of a few per cent of specular in the general diffuse reflection would give a curve like *a*.

The light-surface resulting from such a surface takes the form of a protruding horn of moderate angular extent, the point of which may be shifted out of the line of sight by a very small apparent angular displacement of the axis of revolution due to change of orbital position. With increasing displacement the reflection becomes in effect less and less specular, the amplitude of light-change greatly decreases, and may quite disappear, together wth any asymmetry due to the specular surface.

From this viewpoint we need consider Eros only as a somewhat irregularly flattened mass revolving on an inclined axis lying between the flattened surfaces. At a single point during opposition the maxima of its light-surfaces may sweep into the line of sight and then pass away from it rather rapidly, leaving only the trivial variations due to a diffusing body somewhat irregular in form and texture. The great eccentricity of the orbit of Eros and its inclination of 10°50′ produce so great variation in the geometric conditions of successive oppositions that many years may elapse before the photometric variations of 1901 are reproduced. It is not surprising, therefore, to find that at the prior dates cited by E. C. Pickering (*loc. cit.*) and at the opposition of 1907, carefully observed by Guthnick,[1] varying results are shown, in the latter case no definite short-period variation, save a suspected one of $0^{M}.25$, with double maxima and minima in a scant 5 hours, derived from the observations in the middle third of October. Guthnick did find, however, a much smaller phase-coefficient and also an apparent magnitude less by at least $0^{M}.4$ than were found in 1901, which is quite what one would expect on the present theory.

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[1] *Harvard Circular*, No. 58.

The Harvard observations of 1896[2] are in fair agreement with the figures obtained two oppositions later.

At the opposition of 1903 a long series of measurements of Eros was obtained by Bailey at Arequipa, together with others on several asteroids.[3] These showed a double period of $0^{d}.2196$ for Eros, with an apparent range of $0^{M}.5$ to $0^{M}.75$. The photometric and photographic results agreed closely. The periods deduced from the several groups of observations concurred well, but the curves show strong indications of some variation in amplitude and perhaps in form, although the general type was sinusoidal, as in the later stages of the previous opposition. The time of revolution found is substantially identical with that already noted during the preceding opposition after the amplitude had fallen into a regular sinusoid.

Two oppositions later Guthnick found, as already cited, no variation save during a brief season, when the period appeared as somewhere near $0^{d}.2$, as before. During the opposition of 1914 Miss Margaret Harwood[4] investigated the variations photographically and found an amplitude of approximately $0^{M}.5$, but with the startling period of $0^{d}.3064$, some $2^{h}5^{m}$ longer than before. It seems clear from her data that the former period could not possibly meet her observations, while the longer one did. Incidentally her light-curve shows departure from the previous forms, in having a relatively short minimum.

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[1] *Astronomische Nachrichten*, 178, 1, 1908.

[2] *Harvard Circular*, No. 58.

[3] *Harvard Annals*, 72, 165.

[4] *Ibid.*, 76, 161.

The facts, however, do not demand a real change of rotation period on the part of Eros, the inherent improbability of which is enforced by the uniform regularity found up to the opposition of 1907. Any cause ultimately producing so great a change can hardly be expected to be inactive for so long and then suddenly potent without any observed effect on the orbital motion of the planet. Considering the photometric history of Eros an explanation at once suggests itself. If the light-envelope of a body in any aspect is approximately a surface of revolution, the visible intensity is substantially constant so far as rotation is concerned. If this surface is other than a spherical zone, by change of angle between the axis of revolution and the line of sight, the radius of the envelope will change from r to r' and the light will vary in the ratio $\frac{r}{r'}$. Hence any periodic shift of axis will produce variability not dependent on the period of rotation and generally non-sinusoidal.

Now, Eros, from its erratic light-curves, appears to be a sort of Maxwell's top, spinning in space, and subject to the characteristic wabbling precession of an unbalanced rotation, unchecked by the internal viscosity that acts as stabilizer in the major planets. From this viewpoint, the apparently lengthened period of 1914 is not a rotation period at all, but the effect of a sort of nutation period, or subperiod, on the light-envelope, resulting in the somewhat unsymmetrical period shown. The effect of such a motion would be to impress irregularities on ordinary rotational light-changes in certain positions of the axis with reference to the line of sight, and such seem to be in evidence in the observations of 1901 and 1903. The light-changes of Eros are quite inexplicable as due to spotting, but are what may reasonably be expected of a somewhat irregularly flattened body revolving about an axis considerably inclined to the plane of its orbit, with the rapid and wabbling precession due to the relations between the axes of its ellipsoid of inertia. In such case the light-variations in the next few oppositions may follow those of the last, recur to former habits, or start on a fresh line, since the gravitational forces cannot be lost sight of at opposition. The condition of staggering in rotation was foreseen by Seeliger,[1] and it now seems probable that the corresponding light-variation has been observed.

(345) Tercidina seems to present conditions similar to those of Eros in changes of variability, probably owing to a like cause, though the apparent staggering in path shown on one of Wolf's plates[2] can hardly be a genuine phenomenon. Its only dynamical analogue is the spiraling flight sometimes found in rifled projectiles, but due to the resisting medium, inapplicable to the astronomical case.

One other instance, that of (77) Frigga, which became entirely invisible for six consecutive oppositions, may fall into the same category. It has a very large variation with phase-angle, showing marked irregularity of some sort, and if this is associated with a large proportion of specular reflection, it is by no means unlikely that precessional staggering may from time to time reduce its light enough to make it difficult of observation. It certainly deserves to be watched. In fact, any asteroid with a large phase-angle is worth examination for variability. The short periods indicated in the few cases investigated, and the appearance of new types of variability, give hope of disclosures of importance regarding the origin of the group. One must, however, point out that the irregular form of the light-envelope, which bears no definite relation to the form of the generating body, makes it necessary to exercise extreme caution in drawing dynamical conclusions from the photometric facts.

SOME PECULIARITIES OF THE SATELLITES

The common variability of satellites with orbital position, beginning with Cassini's study of Iapetus, is familiar. It holds true for the four old satellites of Jupiter, for Titan, and, with two or three possible exceptions, for the rest of Saturn's tribe. Of this list Titan presents no remarkable features save the high geometric albedo of 0.33. It has, approximately, the diameter of our moon, and the same density, near to that of a stony meteorite, and about the same degree of variability as that indicated for our lunar surface, suggesting miscellaneous spotting on a scale telescopically irresolvable, as would probably be the case were the moon at a similar distance. But its albedo is nearly four times as great as the moon's. The difference is rather too large easily to account for in bodies of similar constitution, and the natural inference is that Titan suffers less from shadowing and very likely carries considerable optical dust, which is much in evidence in Saturn's neighborhood, perhaps in part owing to impact with meteorites known to have raided Saturn's system on one,[1] and probably on many, occasions. A beaten surface of rock may rise to Titan's albedo, while a solid surface is extremely unlikely to rise so high, judging from terrestrial materials.

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[1] *Astronomische Nachrichten*, 155, 78, 1901.

[2] *Ibid.*, 156, 87, 1901.

The four old satellites of Jupiter present an entirely different case. They are all characterized by very low density: I, not much in excess of that of water; II, barely twice that figure, and the others between. All have at times been noticed as bearing visible shaded areas, clear variations from circular contour have been noted,[2] and all exhibit light-variations of an interesting character—regular with orbital position, apparently permanent, and surprisingly sudden, especially in the cases of I and II, which also have the highest albedos. The curves given by Guthnick (*loc. cit.*) for III and IV, while varying systematically, are smoother and cover less range. The first obvious deduction is that one must put aside W. H. Pickering's interesting hypothesis, to account for low density and apparently variable markings, that these bodies are loose aggregations of matter. Aside from the necessary stringing out of such a mass by collisions, the persistence of sharp changes of brilliancy in the same reasonably restricted longitudes for more than a quarter-century clearly marks the satellites as definite solids. Especially in II, the very abrupt light-changes are characteristic of the shifting of a region of quasi-specular reflection. No mere spot of different albedo can well shift the total light back and forth

[1] *Monthly Notices*, 74, 101, 1913.

[2] *Annals of the Lowell Observatory*, 2, 30.

by 20 per cent in as many degrees of longitude. Satellites III and IV, too, show sudden, though less marked, peaks of brilliancy, and it is not impossible that Barnard[1] may have actually glimpsed one of the specular regions during a transit of III.

The abrupt light-changes, too, make it clear that these satellites cannot at all follow Lambert's law, and consequently Russell's values for A, quoted in Table II, must be radically revised. Assuming Russell's reduction factor q as for our moon, one reaches the values in brackets. From the high geometrical albedos of I and II a very high reflectivity would follow if one were dealing with pure diffuse reflection, hardly to be accounted for save by the presence of light optical dust as in the case of Titan, but there must be at least small quasi-specular effects from the shape of the light-curve.

.This shape also makes it clear that these satellites can have no great amount of atmosphere, of which the diffusion in transmission and reflection must inevitably produce fairly regular diffusion. The low densities point to constitution very different from our moon, to material, in fact, of exceptionally low specific gravity as a mass. Perhaps pumice comes as near to the physical requirements as any terrestrial material, having a specific gravity of from 0.7 upward, and specific reflectivity from 0.25 when dense, up to at least 0.70 when reduced to optical dust, with frequent examples of quasi-specular effects.

Concerning the outer satellites of Jupiter and Saturn and the few others of the solar system it can only be said that most give indications of variable light with orbital position, but that of their albedos there is no definite indication, even the phase-angle relation being denied us. Hence estimates of diameter from the light reflected are futile, save as the limits of albedo confine them. As between the highest and lowest albedo of the small bodies there is a ratio of about 9:1; hence it may fairly be assumed that a diameter may be certain to a 3:1 ratio, or if the assumed albedo be taken at a mean ratio for the geometric albedos ascertained for the small bodies (very nearly 0.20), then the diameter of the unknown is quite certainly determined within 1.5:1, and probably still closer.

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[1] *Astronomische Nachrichten*, 144, 325, 1897; and quoted in André's *Les Planètes et leur origine*, p. 192.

SUMMARY

The subject of albedo, forming an important clue to the physical characteristics of the heavenly bodies, has been greatly befogged by misunderstanding as to the definition of the term. Three quantities have been so called: (1) the geometric albedo, which is merely the apparent reflectivity of the body at full phase, as determined from its photometric magnitude and position in space; (2) the spherical albedo, in which the shape of the reflecting body and the relation of incident and reflected rays, according to some known or assumed laws, are taken into account; the commonest form of this assumes the application of Lambert's law to a perfectly diffusing sphere, which gives a numerical value 3/2 the geometric albedo of the same body, and equal to the specific reflectivity of an element of surface at normal incidence; (3) the specific reflectivity of an element as found or deduced from a definite assumption as to incidence, usually taken as normal.

Albedo in astronomical literature has most often meant Lambert's albedo. The commonly current values of this quantity due to Müller[1] are not only much in error, but, in spite of the fact that Müller himself partially corrected them twenty years ago, have often led to fantastic efforts to explain, in defiance of physical facts, figures without substantial basis.

The chief sources of error were a value too low for the stellar magnitude of the sun and some loose data regarding planetary diameters. These have been corrected by Russell, whose values for the geometric albedo p have been assumed in this investigation with the mental reservation that they may be a slight percentage high. Russell's A is spherical albedo on Bond's definition and is not here employed.

There exists among the small bodies having no atmosphere a relation between the variation of light with phase-angle and the geometric albedo, both in substance and in theory, owing to the fact that both quantities depend in considerable measure on the heavy shadowing of a rough surface. This relation leads to a graphical method of estimating albedo from phase-factor, serviceable in approximating the diameters of small bodies. In default of knowledge of phase-factor a geometric albedo of 0.2 is likely to give the minimum percentage of error in diameter.

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[1] *Publikationen*, Potsdam, 8.

From these data the asteroids grade down steadily to diameters of a few kilometers and probably to meteoritic dimensions. Their total luminosity is, however, insufficient to account for visible glow, which must be attributed to matter lying within the asteroid belt.

The variability of the small bodies cannot satisfactorily be accounted for by local differences of albedo, since the range is often too great, that of so conspicuously spotted a body as our moon being hardly $0\overset{M}{.}2$, and in some cases (e.g., Eros) the form of the light-curve is typically that due to another and well-known cause. This is quasi-specular or "mixed" reflection due to a specular component, as from definitely aligned faces, superimposed on diffuse reflection. It produces more or less definitely directed beams of relatively high intensity, and gives the reflecting body a light-envelope of very unsymmetrical form. The variations in the light of Eros at its various oppositions are such as would be produced by a somewhat flattened body revolving on an axis considerably inclined to the orbital plane and passing between the flattened areas, which show specular components. Change of angle between axis and line of sight leads to small and regular variations like those late in 1901, which may finally vanish. The apparent change of period in 1914 can hardly be due to real slowing of the rotation, considering the steadiness from 1896 to 1907, but may readily be accounted for by the staggering precessional motion of an irregular body, a cause of variation suggested by Seeliger in 1901. Tercidina and Frigga furnish possible cases of the same sort.

The satellites, like the asteroids, give large variations of apparent albedo, which show in the different aspects presented around their orbits. These may be complicated by elongated form, but are equally well accounted for by slight specular factors, plainly in evidence in Jupiter's I and II. These light-curves confirm the lack of atmosphere, and their permanence shows the Jovian moons to be solid in spite of their low density.

BOSTON, MASS.
November 1916

A QUALITATIVE DETERMINATION OF THE REFLECTION COEFFICIENTS OF SOME METALS IN THE SCHUMANN REGION

By I. C. GARDNER

The great majority of the investigations of the optical properties of metals have stopped short at λ 1850. This wave-length constitutes a natural boundary in the spectrum, as the absorption of air makes necessary, for work with light of shorter wave-length, the use of a vacuum spectroscope and Schumann plates with accompanying complications. If we proceed farther into the region of extremely short wave-lengths, we soon reach a point (λ 1250) beyond which light is transmitted by no solid substance, so far as is known. We are therefore soon forced to use in any optical system, not refracting, but reflecting members exclusively. It is then evident that a knowledge of the reflection coefficients of various substances is necessary, before apparatus is designed for work in the region of extremely short wave-lengths.

The work which has been done upon the reflection coefficients of metals for light of wave-lengths lying between, say, λ 2800 and λ 1850 may be briefly reviewed. Schumann,[1] in collaboration with L. Mach, published an account of an investigation he made on alloys in different proportions of aluminium and magnesium. The work is entirely qualitative and extends as far as λ 1850. Two alloys, No. IV (Al 1.25, Mg 1) and No. V (Al 27, Mg 24.3), reflected the ultra-violet better than anything else he tried. Hagen and Rubens[2] have done a great deal of work upon metallic reflection, but they did not include the region shorter than λ 2500. They studied eleven metals and with one exception found that they were characterized by low reflection coefficients (less than 50 per cent) for λ 2500. At this point they found magnalium to have a reflection coefficient of approximately 67 per cent. Glatzel[3] measured the reflecting power of silver, steel, nickel, and copper as compared with Brandes-Schünemann alloy. He does not state explicitly the shortest wave-length with which he worked, but it seems probable that it was about λ 2380. Beyond λ 2500 none of the metals tried was superior to the alloy. Nutting[1] investigated the reflection coefficients of ten metals by a photographic method and gives graphs showing variation of reflection coefficient for the region extending from λ 4000 to λ 1850. At λ 1850 he ascribes to aluminium and brass a reflection coefficient of 65 per cent. The other metals fall below this, with 35 per cent for speculum metal. At approximately λ 2200 the absorption of the gelatine of the photographic plate begins to play an important rôle, and it seems improbable that quantitative results of much accuracy can be obtained beyond this point without the use of Schumann plates.

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[1] *Sitzungsberichte der K. Akad. der Wiss. in Wien*, 108, II*a*, 154, 1899.

[2] *Zeitschrift für Instrumentenkunde*, 22, 42, 1902.

[3] *Physikalische Zeitschrift*, 2, 176, 1900.

In all of the work described above the reflection coefficient has been determined by the "direct" method. There is a second method, the "indirect," in which the reflection coefficient is derived from a measurement of the constants of the elliptical polarization produced when a beam of plane-polarized light is reflected from the substance in question. A very convenient method of doing this has been devised by Voigt[2] and applied by Minor,[3] Meier,[4] Fréedericksz,[5] Erochin,[6] and Foersterling and Fréedericksz.[7] These five papers provide data obtained by the same method for some twenty different metals extending in some few cases as far as λ 2260 and in nearly all cases to λ 2573. In general, the reflection coefficient for the shortest wave-length measured lies between 20 and 40 per cent. The following values are perhaps worthy of mention: silver, λ 2263, 18.4 per cent; platinum, λ 2573, 37.1 per cent; mercury, λ 2570, 57.9 per cent; Wood's metal (composition not given), λ 2573, 52.7 per cent; chromium, λ 2570, 69.8 per cent; and tin, λ 2570, 71.4 per cent.

The most recent and by far the most extensive work upon the reflection coefficients of metals which extends to λ 1850 has been carried on by Hulburt.[1] He used the direct method and measured the intensity of light by means of a photo-electric cell. Twenty-eight metallic mirrors were tried, namely aluminium, antimony, bismuth, cadmium, carbon, carborundum, chromium, cobalt, copper, gold, lead, magnalium, magnesium, molybdenum, nickel, palladium, platinum, selenium, silicon, silver, speculum metal, steel, stellite, tantalum, tellurium, tin, tungsten, and zinc. For each of the metals tried there is published a graph showing reflection coefficient as a function of wave-length between λ 1850 and λ 3800. With one exception, carborundum, the trend of the curve as we pass to the region of shorter wave-length is downward. In the case of carborundum the reflection coefficient is 15 per cent at λ 3800 and rises to 20 per cent at λ 1850. Furthermore, with one exception, silicon, 60 per cent, the reflection coefficient is 30 per cent or less at λ 1850. Magnalium, for which, in view of the work of Schumann and Mach, and of Hagen and Rubens, referred to above, we should expect a large reflection coefficient, reflects less than 10 per cent of the light at λ 1850.

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[1] *Physical Review*, **13**, 193, 1901.

[2] *Physikalische Zeitschrift*, **2**, 303, 1901.

[3] *Annalen der Physik*, **10**, 581, 1903.

[4] *Ibid.*, **31**, 1017, 1910.

[5] *Ibid.*, **34**, 780, 1911.

[6] *Ibid.*, **39**, 213, 1912.

[7] *Ibid.*, **43**, 1227, 1914.

There are no data dealing with the optical properties of metals for wave-lengths less than λ 1850. Lyman[2] has recently extended the spectrum to λ 600, and it therefore seems desirable to continue the investigation of reflection coefficients into the region beyond λ 1850. The minimum wave-length dealt with in this paper is λ 1030, at which point we come to a second natural boundary in the spectrum. By the use of extremely pure hydrogen and an oscillatory discharge, the spectrum may be extended to λ 900, and the extension to λ 600 was achieved by substituting an atmosphere of helium for hydrogen in the spectroscope.

Since the difficulties of measurement of energy in the Schumann region have proved prohibitive up to the present time, and since the nature of the Schumann plate renders the results of photographic photometry of very questionable value, it has not seemed advisable, in the present work, to attempt to obtain quantitative determinations of the reflection coefficients of various metals such as is desirable for the testing of theoretical treatments of the optical properties of substances. The purpose has therefore been restricted to the investigation, in a qualitative manner, of the reflection coefficients of several metals in order to determine what ones are best suited for use in the construction of apparatus for work in the region of extremely short wave-lengths.

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[1] *Astrophysical Journal*, **42**, 205, 1915.

[2] *Ibid.*, **43**, 89, 1916.

As speculum metal has in the past been used for the construction of gratings for the resolution of Schumann light, it was adopted arbitrarily as the standard reflecting metal with which the other metals were compared. We obtain then, not the reflection coefficient directly, but determine at once what metals are superior to speculum metal for the making of gratings and other optical parts.

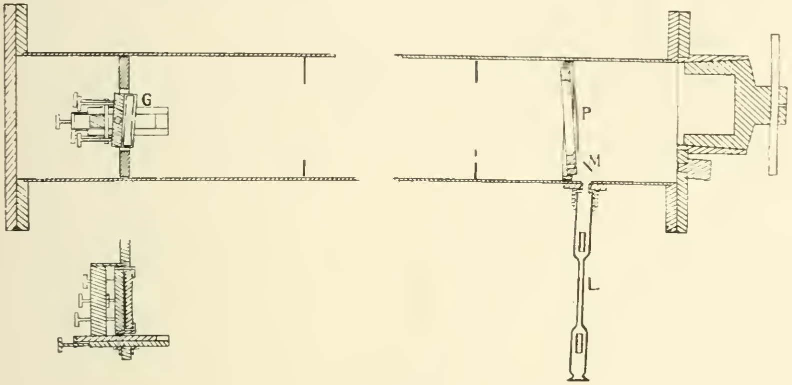

Fig. 1

A bit of speculum metal and the metal to be tested were so mounted that their polished faces were adjacent and in the same plane. They were then placed in such a position that the light from the quartz discharge tube was reflected (angle of incidence approximately 45°) through the slit and upon the grating. One half of the slit was illuminated by light reflected from speculum metal, the other by light reflected from the metal to be tested. For light of the wave-length to be studied, the astigmatism of the grating in the first-order spectrum is so slight that we obtain simultaneously two spectra side by side, the relative strengths of which enable us to determine by inspection which is the better reflector. The method is illustrated in detail in Fig. 1, which is a horizontal section taken through the vacuum spectroscope and discharge tube. *L* is the discharge tube which serves as source of light, M is the composite mirror, and G is the grating. The spectrum is received on the photographic plate P. The grating used has a radius of curvature of 102 cm, and is ruled with approximately 6000 lines to the centimeter. The ruled portion is 2.8×5.3 cm. By photographing the two spectra at the same time we eliminate any effect due to variation of intensity of the source. There remains an outstanding error due to possible unsymmetrical placing of the source in such a manner that the two mirrors are unequally illuminated. This may be largely eliminated by interchanging the two mirrors and taking a second exposure. This method, while simple, is sufficiently accurate for the purpose of the work. For if the reflecting powers of two metals are so nearly identical that the eye detects no differences in the two images on the Schumann plate, obviously one metal is, so far as the reflection coefficient is concerned, as suitable for use in the construction of apparatus as the other.

The vacuum grating spectroscope used was patterned very closely in its essentials after the one used by Lyman.[1] It differed in that it used a much larger plate (2.5×8.5 cm) which in turn made necessary the very large hole for the removal of the plate-holder. This hole was closed by a conical plug 9.6 cm in diameter. The reflection of the light by the mirror necessitates the placing of the discharge tube on the side of the spectroscope instead of on the end plate. A novel feature, perhaps worthy of mention, is the mounting of a miniature incandescent lamp on an arm so that it may be swung down immediately in front of the center of the grating. If the mirrors are in correct adjustment the light from this lamp passes through the slit of the spectroscope, is reflected by the mirrors down the capillary of the discharge tube, and the image of the slit may be seen by looking through a telescope pointed along the axis of the capillary. It would have been extremely difficult to test the adjustment of the mirrors without some such arrangement.

The quartz discharge tube which served as source of light was of the external capillary type. It is shown in section at L, Fig. 1. The electrodes were of aluminium, and the internal diameter of the capillary was about 0.6 cm. There is no window at the inner end separating the interior of the discharge tube from the interior of the spectroscope. The spectroscope is commonly filled with hydrogen at a pressure of approximately 1.8 mm and the spectrum obtained is that of hydrogen. The spectrum usually extends from λ 1640 to λ 1030 and is filled with lines very close together.

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[1] *The Spectroscopy of the Extreme Ultra-Violet*, Longmans, Green, & Co., 1914, p. 34.

The directions for making plates given by Schumann[1] were followed quite closely. A device for dropping the plate-holder controlled by a magnet on the outside and similar to that used by Lyman[2] was provided so that four exposures might be taken on one plate without opening the spectroscope. The lengths of the four exposures were usually 30, 60, 120, and 240 seconds.

The method has been applied to nine substances, namely, silicon, gold, stellite, platinum, copper, nickel, aluminium, silver, and steel. The majority of the metals were selected because the ease of working and permanence of surface seem to make them adaptable for the construction of mirrors, if they should prove to be good reflectors. Silicon was chosen as a direct result of the work of Hulburt, who attributed to it a reflection coefficient of 60 per cent at λ 1850. The aluminium was very difficult to polish. I was led to test it by the fact that Nutting (*loc. cit.*) gives it a reflection coefficient of 65 per cent at λ 1850. Following will be found a complete account of the preparation of the mirrors and the results obtained with them.

The speculum metal mirrors used in this work were sawed from a broken grating and polished by Alvan Clark & Sons, Corporation. They were repolished from time to time as conditions demanded.

Silicon.—The specimen of silicon was obtained from the Carborundum Works at Niagara Falls and was sawed into slabs and polished by means of rouge on a beeswax lap at the laboratory. Silicon polishes easily and the surface is permanent and does not tarnish. Silicon is much the best reflector of all the substances that I have tried. Throughout the entire range (λ 1640–λ 1030) it is a better reflector than speculum metal, although the difference is less marked for wave-lengths shorter than λ 1300. Between λ 1600 and λ 1300 I estimate that the reflection coefficient of silicon is twice as large as that of speculum metal. This estimate is onl approximate and indicates that, to the eye, the spectrum reflected from silicon with a two-minute exposure is fully as strong as that reflected from speculum metal with a four-minute exposure.

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[1] *Annalen der Physik*, 5, 349, 1901.

[2] *Op. cit.*, p. 35.

Gold.—The gold mirror was prepared by deposition from a cathode. Gold spatters very readily, and from six to ten minutes are sufficient to produce an opaque film. The spectra reflected from gold and from the speculum metal are very nearly identical in strength. Between λ 1300 and λ 1030 gold is perhaps slightly superior, but the difference is not sufficiently marked to encourage the use of gold for mirrors.

Stellite.—The stellite mirrors were cut from a block of stellite obtained from the Stellite Works, Kokomo, Indiana. It is an alloy, chiefly of chromium and cobalt. The exact composition is a commercial secret. The mirrors were polished by means of diamantine on a beeswax lap. Hulburt found stellite superior to speculum metal between λ 2000 and λ 3000. His diagram indicates that its reflecting power falls off as we approach λ 1850. The spectra reflected from the speculum metal and stellite are identical in strength. In the Schumann region there is no advantage in substituting stellite for speculum metal.

Platinum.—The platinum mirrors were deposited on glass from a cathode. About thirty minutes were required to deposit an opaque film. Between λ 1600 and λ 1300 platinum is not superior to speculum metal, in fact one or two of my negatives indicate that it is slightly inferior. But beyond λ 1300 the spectrum reflected from the platinum is distinctly the stronger. It would be very interesting to investigate the reflecting power of platinum throughout the region discovered by Lyman (λ 1250–λ 600). As noted above, silicon is a much better reflector than speculum metal in the portion of the spectrum with which this article deals, but, as compared with speculum metal, its reflecting power is growing less as we pass to the shorter wave-lengths, whereas in the case of platinum the reflecting power is increasing.

Copper.—Copper spattered readily on glass from a cathode. The mirrors tested were quite opaque. The copper mirrors which

I made were markedly inferior to those of speculum metal through the entire region. Hulburt finds that at λ 1850 the copper was slightly better as a reflector than the best speculum metal mirror which he tested.

Nickel.—About three hours were required to produce a brilliant opaque film on glass by cathode deposition. Between λ 1600 and λ 1300 the reflection from nickel and from speculum metal is the same. Beyond λ 1300 the nickel is very slightly superior to the speculum metal.

Aluminium.—Two pieces of aluminium cut from commercial sheet stock were polished by Alvan Clark & Sons, Corporation. It was very difficult to polish, but a surface was finally obtained which was quite good. Aluminium reflects very poorly in the Schumann region. Only a few of the stronger groups of lines show in the four-minute exposure.

Silver.—The silver mirror was cut from a coin and polished. Speculum metal is much superior to silver over the whole range upon the plate. At the end of longer wave-length the speculum metal reflects twice as well as does the silver.

Steel.—The steel mirror was hardened and polished by Alvan Clark & Sons, Corporation. Steel reflects markedly better than speculum metal from λ 1600 to approximately λ 1300. At λ 1200 the speculum metal reflects slightly better than the steel.

Conclusion.—The reflecting power of nine metals, silicon, stellite, copper, nickel, gold, platinum, aluminium, silver, and steel, has been investigated in the portion of the spectrum lying between λ 1600 and λ 1030. The purpose has not been to determine the reflection coefficients on an absolute scale, but rather to determine directly what metals are superior to speculum metal for the construction of reflecting surfaces to be used for light in this region. The results indicate that:

1. Silicon is much the best reflector of all the metals tried, and over a portion of the region it reflects the light approximately twice as well as speculum metal.

2. The reflecting power of platinum as compared with that of speculum metal is increasing as we pass to shorter wave-lengths.

It may possess advantages over other metals for work beyond λ 1030.

3. Stellite, copper, nickel, gold, and steel show only minor variations of reflection when compared with speculum metal.

4. Silver and aluminium are very much poorer than speculum metal, with silver ranking above the aluminium.

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November 1916

THE UNITS AND NOMENCLATURE OF RADIATION AND ILLUMINATION

By HERBERT E. IVES

Illumination and its measurements (photometry) are, strictly speaking, merely departments or special cases of radiation and radiometry. In a comprehensive treatise on radiation, photometry could be discussed as such a special case with considerable resultant economy both of space and of the student's mental effort. Thus the various quantities dealt with in radiation become parallel illuminational quantities when passed through a template or filter yielding a spectral luminosity-curve. The operation of measureing light with a photometer is merely an example of the use of a null method of radiometry with the eye as the sensitive radiometer. The greater part of practical photometry, i.e., photometry with no difference of color, could thus be dismissed. The complications introduced when the radiations are of different composition then take their place as disturbances due to the inability of the eye simply and definitely to evaluate as light radiations of various wavelengths. Unlike other selective radiometers, which are selective merely in responding in different *degree* to various radiations of the same intensity, the eye recognizes or creates a difference of *quality* as well. The problem of photometry with a difference of color is that of eliminating the confusing effect of this difference of quality.

However, if the attempt is made to handle photometry in this way, a difficulty is met at the outset in the fact that the derived science, photometry, has been worked out in much greater detail as to instruments, methods, and terminology than has the parent science. On going back from things photometric to things radiometric many of the latter are found to be missing or else in confusion. This remark applies with particular emphasis to the terms used to describe the various radiational quantities. The nomenclature of illumination, on the other hand, has been put into businesslike shape by technical usage and by painstaking committees of the technical societies, with a resultant economy of words and clearness of meaning that are sadly lacking in the more fundamental science.

To indicate briefly the existing confusion, which later will be dwelt on more in detail, it may be said that the word "radiation" is used as the title, not only for a branch of physical science, but for the "stuff" radiated; for the process of ejecting energy from a body without regard to where it goes; for the process of throwing the energy upon a body without regarding whence it comes; and for either the quantity of energy or the rate of transfer of energy. When this stage is passed and an attempt is made to differentiate between various manifestations of radiant energy each author seems to make it a point of honor to use such terms as "emission," "emissive power," and "intensity" in a sense quite different from anyone else. I have yet to find an author who has conscientiously selected and arranged in tabular form a set of terms for radiational quantities in a manner comparable to that found quite indispensable when a branch of science reaches the stage of application.

A very considerable part of the problem of treating illumination as a special case of radiation therefore appears to consist in the standardization of the terminology of radiation, and this note is concerned largely with suggestions to that end.

The accompanying table (I) of radiational and illuminational quantities is an attempt to arrange these in parallel columns in the manner suggested at the outset. Logically, the illumination columns should have been derived from those of radiation. Actually, the greater completeness and orderliness of that technically applied division of radiation fits it to act as a guide to the standardization of the terminology of the more fundamental quantities. The table thus consists of practically the accepted illuminational quantities with a set of suggested terms for the radiational quantities, the latter culled from a variety of sources and chosen with regard to freedom from ambiguity and to similarity to the parallel illuminational quantities. Justification for the selection of names will be ventured below.

In the first column are the radiational quantities; in the second are these same quantities after the radiation has been passed

TABLE I

Relations between Radiation and Illumination

Radiation	Illumination			
1. Radiant energy (joules)	After passage through a luminosity-curve template become	Luminous energy (light-watt seconds)	After division by the mechanical equivalent of light become	Luminous energy (lumen seconds)
2. Radiant flux (watts)		Luminous flux (light-watts)		Luminous flux (lumens)
3. Radiant emission (watts)		Luminous emission (light-watts)		Luminous emission (lumens)
4. Specific radiant emission or radiant emissivity (watts per square centimeter)		Specific luminous emission or luminous emissivity (light-watts per square centimeter)		Specific luminous emission or luminous emissivity (lumens per square centimeter)
5. Radiant intensity in a given direction (watts per unit solid angle)		Luminous intensity (light-watts per unit solid angle [pyrs])		Luminous intensity (lumens per unit solid angle [candles])
6. Specific radiant intensity in a given direction, or radiance (watts per projected square centimeter per unit solid angle)		Brightness (light-watts per projected square centimeter per unit solid angle [pyrs per square centimeter])		Brightness (lumens per projected square centimeter per unit solid angle [candles per square centimeter])
7. Irradiation (watts [incident])		Illumination (light-watts)		Illumination (lumens)
8. Specific irradiation (watts [incident] per square centimeter)		Specific illumination (light-watts per square centimeter)		Specific illumination (lumens per square centimeter [phots])

9. Reflecting power = ratio of reflected to incident energy
10. Absorbing power = ratio of absorbed to incident energy
11. Transmitting power = ratio of transmitted to incident energy
12. Emitting power* = ratio of emission to that of black body at the same temperature

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* The alternative equivalent endings in "ive," such as "emissive," are sometimes preferable from the standpoint of euphony.

through a template giving a spectral luminosity-curve,[1] that is, they are now illuminational quantities in the same units as used in measurements of radiation, e.g., watts; in the third column are the quantities of the second column after division by the mechanical equivalent of light;[2] that is, they are illuminational quantities expressed, as a rule, in the everyday units of illumination based on the candle.[3]

In order to illustrate clearly the pressing need for standardizing the nomenclature of radiation, I append the accompanying list (Table II)—undoubtedly far from complete—of terms which have been used in various recent textbooks on radiation and general physics.

TABLE II

Proposed in Table I	Various Textbooks
Specific radiant emission or radiant emissivity	Emissive power Specific emissivity Coefficient of radiation Emission Total energy stream
Radiant intensity	Radiation intensity Coefficient of emission Normal energy stream
Radiance	Specific intensity Brightness Radiation intensity of a surface Emissive power of a surface
Specific irradiation	Intensity
Absorbing power	Absorption Coefficient of absorption
Emitting power	Emissivity

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[1] A full discussion of the relation between radiant flux and luminous flux will be found in the *Journal of the Franklin Institute*, 180, 409, 1915, in the article "The Establishment of Photometry on a Physical Basis."

[2] 1 lumen = 0.00159 light-watt.

[3] A full discussion of the illuminational units will be found in the standard lists of the Illuminating Engineering Society and the Institute of Electrical Engineers. In details where these lists have not been followed the present table is in substantial agreement with an excellent paper by E. B. Rosa, *Bulletin of the Bureau of Standards*, 6, 543, 1910.

For some quantities it appears from my search that most authors have no terms at all, and it is evident that the same terms are used by different authors with quite different meanings. It is only just to state that the better writers, having once defined their quantities, are likely to use them consistently, however confusing the chosen names may be. When, however, it comes to the discussion of Kirchhoff's law, which must figure in every treatment of radiation, and the statement of which is more or less stereotyped, the confusion in nomenclature has resulted in really serious confusion of ideas.

Before taking up the reasons for the choice of the various radiational terms it is perhaps advisable to make clear their use by an example which includes all except the various "powers" as follows:

Let us take a *black body* with an opening of area *s;* from this proceeds *radiant energy;* the amount of this given out in unit time, or the rate of emitting energy, is the *radiant emission* of this black body; this divided by *s*, or the rate of energy emission per unit area, is the *specific radiant emission or radiant emissivity*, which is equal to $\sigma(T^4-T_0^4)$, where σ is approximately 5.7×10^{-12} watts per degree^4, T being the temperature of the black body and T_0 that of the body which it irradiates. If we now place a radiometer of area a at such a distance ~~x~~ from the black-body opening that the latter is, to within the errors of measurement, a point source, and that the radiometer subtends from the black-body opening the solid angle $\Omega\left(=\frac{a}{r^2}\right)$. we can measure the *radiant intensity* of the black body *in that direction* by the quotient of the rate of reception of energy divided by Ω. This will vary from a maximum, normal to the opening, to zero when the radiometer lies in the plane of the opening. If now we divide this radiant intensity in any direction by the projected area of the opening in that direction, or by $s\cos\theta$, where θ is the angle giving the direction measured from the normal, we obtain the specific radiant intensity or *radiance* in that direction. For the black body, which obeys Lambert's law, the radiance is a constant in all directions outside the body, and is equal to $\frac{\sigma}{\pi}(T^4-T_0^4)$.

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See p. 44

Leaving now the radiating body, consider the space between it and any body on which the radiant energy falls. Through this space radiant energy is passing at a certain rate. This rate constitutes the *radiant flux* through the space. Thus the radiant flux through the space surrounding a single radiator is equal to the radiant emission of the radiator, but radiant flux in space can be measured irrespective of whether we know what the radiator is, or in case the latter is multiple or otherwise complex in a manner to interfere with its handling by summational processes.

Somewhere in the path of the radiant flux from our black body let there be a body of area a (e.g., the radiometer assumed above). Radiant energy is incident upon this at a certain rate; this rate of reception of energy constitutes the *irradiation* of the body. The *specific irradiation* is the irradiation divided by a, the area. At any finite distance from the radiator the specific irradiation of a body will be different from point to point, and we can measure only the mean irradiation or the irradiation of an element small enough to be considered as uniformly irradiated. If the radiant energy is incident at an angle ϕ to the normal, the specific irradiation is $\cos\phi$ times the value for normal incidence. The irradiation of the body considered, owing to the assumed black-body opening, is therefore

$$\frac{s\sigma(T^4-T_0^4)}{\pi}\cdot\frac{a}{r^2}\cos\theta\cos\phi,$$

r where r is the distance between the two bodies; the specific irradiation is

$$\frac{s\sigma}{\pi r^2}(T^4-T_0^4)\cos\theta\cos\phi.$$

Let us now consider this black body as a light-source. This merely means that our radiometer is to be selective, its distribution of sensibility according to wave-length being that of the eye. The light-evaluated radiant energy proceeding from the opening of the black body we call *luminous energy;* the rate at which it is given out is the *luminous emission* of the body. It is the integral

$$s\int_0^\infty J_\lambda L_\lambda d\lambda,$$

where J_λ is the specific radiant emission or emissivity for the wave-length interval $d\lambda$; L_λ is the luminosity of the normal equal-energy spectrum, in terms of the maximum as unity, or the *luminous efficiency*. This divided by s is the *specific luminous emission*, or *luminous emissivity*. *Luminous flux* and *luminous intensity* are similarly derived directly from the radiational quantities, and so on down the line. It is only when we take the further step of expressing our illuminational quantities in the ordinary units referred to the candle by dividing by the mechanical equivalent of light that we must learn new terms, namely, those for the unit of luminous flux, the *lumen*,[1] for the unit of luminous intensity, the *candle*, and for specific illumination, the *phot*. The latter, the lumen per square centimeter, proposed by M. Blondel, is the exact parallel of the watt per square centimeter by which specific irradiation is measured, and (at least in the form of its subdivision, the milliphot) should take the place of the anomalous meter-candle or lumen per square meter. ("Illumination," it may be noted, is rather commonly used where "specific illumination" is meant. The illumination of a body, on all parts of which luminous energy is incident, is greater the larger the body, while the specific illumination, whether we measure it in phots or meter-candles, is not.) In the case of the black body, which has been taken as our illustration, the illumination of the surface a at the distance r is

$$\frac{L}{m}\frac{s\sigma(T^4-T_0^4)}{\pi}\frac{a}{r^2}\cos\theta\cos\phi,$$

and its specific illumination is

$$\frac{L}{m}\frac{s\sigma(T^4-T_0^4)}{\pi r^2}\cos\theta\cos\phi,$$

where L is the luminous efficiency of the black body at the temperature at which the observation is made (assumed to be calculated from the spectral distribution of emission, e.g., the Planck equation, and the spectral distribution of luminosity, e.g., the equation derived by Kingsbury[1]), and *m* is the mechanical equivalent of light.

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[1] It would be highly desirable to use the term "lumen" for the watt of luminous flux, calling the present "lumen" the "candle-lumen," in the same manner that we designate the lumen derived from the Hefner as the "Hefner-lumen." Some of the other quantities in the second column of the table might appropriately receive names recalling worthies in photometric science, in the manner that "volt," "ampere," and "farad" do in the electrical units. The "rumford" as the rational unit of specific illumination (corresponding to the phot) would be particularly fitting.

Taking up now the justification for the selection of names used in the radiation table, I assume that there will be little exception taken to the first three terms—"radiant energy," "radiant flux," and "radiant emission." All three of these are commonly spoken of as "radiation," but they must be differentiated as soon as any detailed discussion of the processes of radiation is undertaken. Nor should there be opposition to the last four terms, the "reflecting," "absorbing," "transmitting," and "emitting" powers, when it is pointed out that two other common ways of denominating these properties, shown in Table II, conflict with other branches of physics and with well-established usage in nomenclature. Thus the use of the word "coefficient," as in "absorption coefficient," immediately encounters the difficulty that the coefficient of absorption is used in physical optics as the fraction absorbed (and sometimes curiously enough, as the fraction transmitted) by unit thickness, which is an altogether different meaning from the one associated with absorbing power in Kirchhoff's law, for instance. The termination "ity," found in the rather frequently used "emissivity" (emitting power), is quite generally in use to indicate a property referred to *unit* mass, thickness, area, etc., as in "resistivity." "Emissivity," therefore, does not belong among the list of properties, but finds its proper place as the equivalent of "specific emission."

The greatest novelty in these suggested terms lies in the choice of the words "radiance" and "irradiation," for the radiational quantities corresponding to "brightness" and "illumination." These have not been selected from terms in use, because, speaking broadly, no one appears to have attempted to give these quantities names. In the case of "irradiation" the vast majority of writers have not been interested in what becomes of the radiant energy after it leaves the radiator. In the case of illumination the *raison*

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[1] See Ives and Kingsbury, "The Mechanical Equivalent of Light as Determined from the Brightness of the Black Body," *Physical Review*, 8, 177, 1916.

d'être of the whole science is putting the luminous energy upon something, so that not only have we a term for the rate of doing this, but this term becomes the name of the whole science. By analogy one might be led to call the rate of putting radiant energy upon something "radiation," and this usage of the word is, in fact, found in connection with the solar constant. This analogy is wrong, however, for etymologically the radiational equivalent of "illuminate" (in-luminate) is "irradiate." Strict etymological consistency would suggest that the science of illumination might more properly be denominated "lumination." It is interesting to note that all the standard dictionaries give as the first definition of "irradiate" "to throw light upon," and give as their authority the etymology together with well-supported literary usage. A dictionary is of course not the place to go for definitions of scientific terms, since the dictionary merely records the usage of scientific writers, but this appears to be a case where scientific writers have not taken advantage of a word lying ready to hand. The problem is handled by the German language in a similar manner, by the use of the prefixes *zu*, *aus*, *be*, *-strahlung*.

An objection to the employment of the word "irradiation" in this sense will occur to everyone, namely, that this word is already used for quite a different thing, the apparently larger area of a bright as compared with a dark object of the same actual dimensions. But this objection is not serious; first, because there is no great likelihood of confusion between phenomena in such different fields of optics, and second, because if the point is pressed it will be found that "irradiation" as used in physiological optics is clearly the wrong word. Etymologically it should be "eradiation," a word which, on reference to standard dictionaries (not knowing whether such a word was recognized), I find to be "a shooting out as of rays," an exact description of which is often called "irradiation."[1]

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[1] "Eradiation" for radiant emission, and "elumination" for luminous emission, with the corresponding verbs "to eradiate" and "to eluminate," might claim recognition, but they are obviously impracticable because of the danger of confusion with the words having the prefix *i*. "Radiate" is used pretty generally in the sense of the suggested "eradiate," and, as noted, "radiation" for radiant emission, with probably little danger of ambiguity. For "eluminate" we already have the good word "shine." Why not call the luminous emission of a light-source its "*shine*"?

It appears, therefore, that however unusual "irradiation" may seem in the sense used in the table, it is soundly based. It will probably be granted that, in describing the condition holding in a "black" inclosure, there is conciseness and non-ambiguity in the statement that, if we are dealing with luminous flux, the *specific illumination* of all bodies therein is equal, and if we are dealing with radiant flux, the *specific irradiation* of all bodies therein is equal.

"Radiance," as a time-saving word for "specific radiant intensity," is frankly taken from the dictionary, as the appropriate synonym for "brightness."

The question may be raised in some quarters whether there is any real need for names for these quantities. Why should we not simply speak of "incident energy" and "incident energy per unit area," and so on? Apart from the service which is rendered to any science by a well-considered terminology, which should be sufficient excuse for undertaking such an arrangement as is here attempted, there is the fact to be considered that the problem of heating a body by the incidence of radiant energy is taking on technical importance. "Irradiation" may in the future figure in a manner comparable with "illumination." The measurement of radiant energy, both at the point of emission and at the point of incidence, may assume an importance comparable with that of photometry. If so, an arrangement and simplification of the terminology of radiation to fit it for technical application is inevitable.

Returning now to Kirchhoff's law, this is most frequently given, following Kirchhoff's own terminology, as a statement about the ratio of the emissive *power* of a body to its absorptive *power*. The use of the same word *power* in these two compound terms is highly objectionable, because the impression is inevitable that these two "powers" are similar properties of a body, just as are absorbing power and reflecting power. In fact, I find one writer specifically stating that they *are* similar, *since they are both functions of* λ *and* T. Actually, one is a simple dimensionless numerical coefficient, the other a rate of emitting radiant energy. The absorbing power may or may not vary with temperature. In the black body it does not. In many metals it is constant over a very large range of temperature. The radiant emissivity (the "emissive power" of Kirchhoff) practically always varies continuously and rapidly with temperature. The two things are fundamentally different, and should not be described by similar names. The use of letters from the same case to symbolize them, as is common in the practice of representing the one by capital A and the other by capital E, is objectionable.

In accordance with the terms listed above the correct statement of Kirchhoff's law becomes: *The radiant emission of any body, due to temperature alone, is equal to the product of the absorbing power of the body, by the radiant emission of a black body, of the same dimensions, at the same temperature.*

Thus stated, Kirchhoff's law is not a relationship between two apparently similar *properties* of the *same* body (as would be a relationship between reflecting power and absorbing power), as Kirchhoff's terminology appears to indicate, but a relationship between the *behaviors* of two *different* bodies, under the same condition of temperature. A very considerable part of the service rendered by the discovery of this law is the fact that it makes possible the prediction of behavior under extreme conditions of temperature from an optical property—this property, as it happens, being one that as often as not is not variable with temperature over the region in which we are interested.

The desire, which seems to be general, to derive from Kirchhoff's law some property which is common to all bodies may be satisfied, using the terminology here suggested, by stating the simple relationship that, for any body whose radiant emission is due to temperature alone, *the emitting power is equal to the absorbing power.*

Physical Laboratory of the United Gas Improvement Co.
Philadelphia
November 18, 1916

THE ECLIPSING VARIABLE STAR SS CAMELOPARDALIS[1]

By R. J. McDIARMID

The variation of this star was discovered by Miss A. J. Cannon, in 1908, and announced in *Harvard Circulars No. 140* as a variable of the Algol type with a photographic range of over one magnitude, from 9.8 to 11.0.

The position of the star, $\alpha = 7^h 4^m$, $\delta = +73° 30'$ (1900), makes it a very suitable object for observation throughout the entire year, at the latitude of Princeton. In March 1913 the writer added this star to the program of variables under observation with the sliding-prism polarizing photometer attached to the 23-inch equatorial. The period of the star was unknown at this time, as well as the nature of the variation in light. The first night's observations, March 31, 1913, gave a variation of $0^M\!.50$, as the star was then in primary eclipse. During the two years' observation the variation never exceeded $0^M\!.60$ and this was very puzzling, as it was announced with a variation of over one magnitude. This discrepancy was not cleared up until after the true period was determined, and it was found that the difference in photographic and visual range was due to the color-index of the star.

The period of this star was very difficult to determine owing to the length of the eclipses, which last over 21 hours. In the autumn of 1914 Professor Pickering of Harvard very kindly sent me 30 photographic observations taken at and near primary minimum, extending as far back as 1896. These observations were quite well satisfied with a period of 4.825 days, and a range of $1^M\!.2$; a note was added that in some cases the measures were difficult to make and were perhaps in doubt, owing to the adjacent bright star B.D.$+73°366$. The variable is referred to in *Harvard Circular No. 140* in the following terms: "This star follows $+73°366$, magn. 7.5, 24^s, north $0'\!.5$". The period $4^d\!.825$ applied to the visual observations was found to be too long. When the data furnished by the visual and photographic observations were combined, more weight being assigned to the visual, the period $4^{d}19^{h}47^{m}6^{s}.4$ was determined, which satisfied both series of observations very well. The photographic observations used in this case, as well as in that of TX Cassiopeiae[1] and T Leonis Minoris, proved to be of extreme value.

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[1] Paper read at nineteenth meeting of the American Astronomical Society, *Popular Astronomy*, 24, 668, 1916.

The observations made with the Princeton photometer extend over a considerable period, from March 31, 1913, to December 10, 1915. In all, 10,992 measures were made by comparing SS Camelopardalis with the star whose co-ordinates[2] are $\alpha = 7^{h}2^{m}21^{s}.8$, $\delta = 73^{\circ}\,12'27''$ (1900), and visual magnitude 11.14. The magnitude of the comparison star was found by comparing it with B.D.$+73^{\circ}366$, whose magnitude is given in *Harvard Annals*, **45**, as 7.70.

After correction for light-equation, the 687 observations (of 16 settings each) were grouped into normals according to phase, with the finally adopted period; each normal was the weighted mean of five observations, except in three or four cases; the weights were assigned in accordance with notes taken at the time of observation (maximum weight being 5).

The observed curve thus formed presented some unusual characteristics for an eclipsing variable. The depth of the primary eclipse is $0^{M}.57$, and that of the secondary is $0^{M}.15$. The primary eclipse lasts 21 hours, and there is a period of constant brightness of 7 hours at minimum phase. The secondary is of the same duration, but the interval from primary to secondary is shorter than that from secondary to the following primary, showing that the orbit is eccentric. The uneclipsed portions of the light-curve show a slight ellipticity of the stars, but, in addition, the portion of the curve between primary and secondary is about $0^{M}.05$ lower than that between secondary and primary. The observations of these two branches were taken at different seasons, and it might be possible that the difference in "level" is due to the difference in sensitiveness of the observer's eye; this, however, was never found to be the case in the four[1] other systems previously worked up. The comparison star might itself be a variable, as in the case of TX Cassiopeiae;[1] this did not seem to be the case, as the comparison star was compared with B.D.$+73°366$ on several different nights; the measures thus made showed no greater range than that corresponding to the probable error usually found. It would, therefore, appear that this difference is real, and that SS Camelopardalis is fainter after the primary eclipse than before it. This, however, is the opposite of what might be expected, since after primary minimum the components are near periastron, which is associated with unusual brightness in other systems; but the observational evidence appears to be conclusive. To eliminate this influence, the ellipticity-constant was determined from the mean of the branches.

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[1] *Astrophysical Journal*, **42**, 412, 1915.

[2] The co-ordinates of the comparison star were computed for 1900 from the data furnished in the *Astrographic Catalogue* of the Greenwich Observatory, **2**, 126. Plate 3008; number of star on plate, 2989.

The "rectified" depths of the two minima, after correction for ellipticity, are $0^{\mathrm{M}}.486$ and $0^{\mathrm{M}}.082$, and the loss of light in the two cases $1-\lambda_p$ (primary) $=0.3610$ and $1-\lambda_s$ (secondary) $=0.0727$. The primary eclipse is obviously total, since the intensity remains constant at minimum for over 7 hours. Hence from the relation for uniform disks, $k^2=\frac{1-\lambda_s}{\lambda_p}$, the value of k, the ratio of the radii of the stars, is 0.335. Since the branches of the observed curve at primary minimum seem to show a slight asymmetry, the two branches were combined in one, and this curve was used in the solution, taking $k=0.335$ and proceeding in the usual way.[2]

The elements resulting from the solution were computed in the manner outlined in *Astrophysical Journal*, **36**, 404, 1912, and are given in Table III. The residuals for primary and secondary were read off from the theoretical curve, and are given in the table of observations under $(O-C_u)$. The probable error of a normal point for primary is $\pm 0^{\mathrm{M}}.016$ and for secondary, $\pm 0^{\mathrm{M}}.018$.

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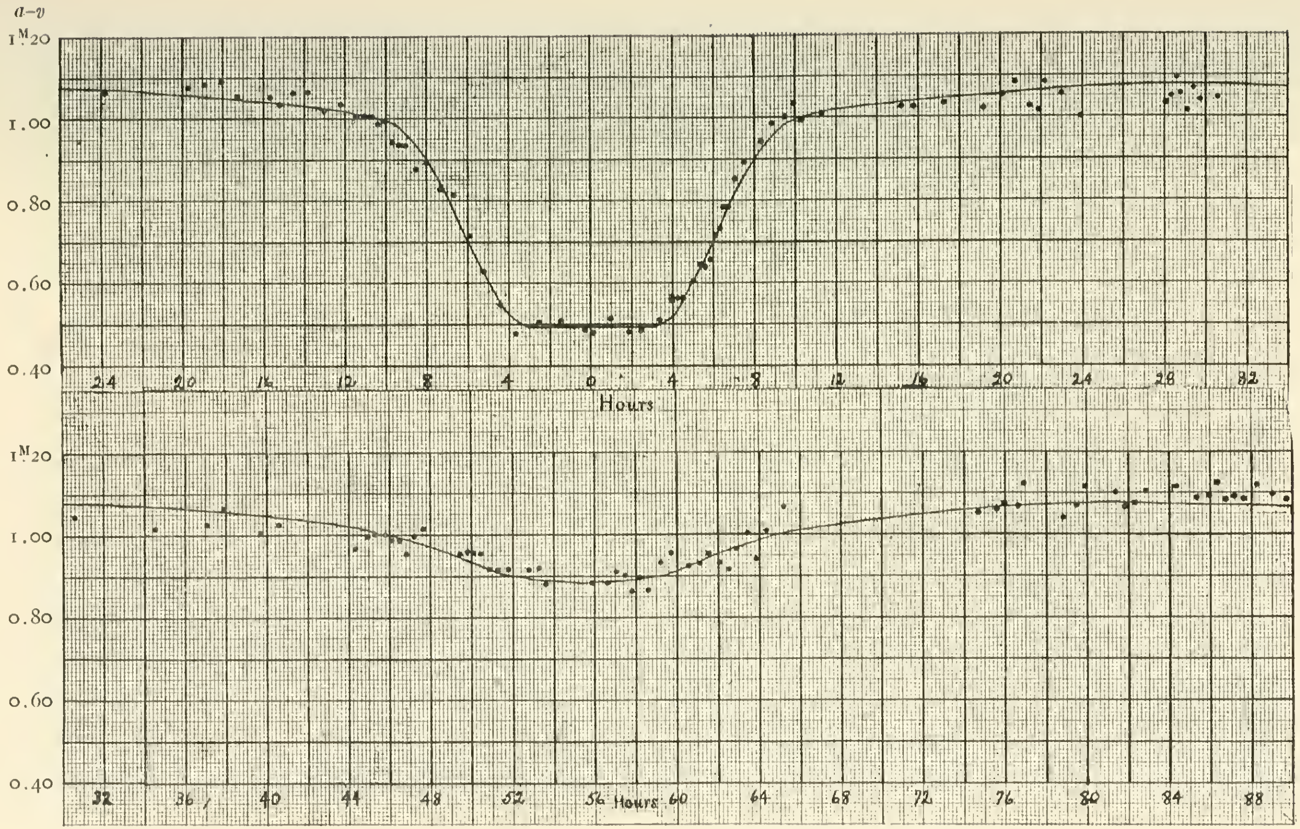
[1] *Astrophysical Journal*, **42**, 412, 1915.

[2] *Ibid.*, **35**, 315, 1912; **36**, 54 1912.



Mean light-curve of SS Camelopardalis

FIG. 1

In the case of the solution which considers the stars darkened to zero toward the edge, the primary eclipse was total and the secondary annular, as in the previous solution. The secondary was assumed slightly deeper, viz., $1-\lambda_s=0.0850$. Then from the relation

$$Q(k\alpha_0)=\frac{1-\lambda_2}{\alpha_0-(1-\lambda_1)}$$

where

$$1-\lambda_2=0.0850$$
$$1-\lambda_1=0.3610$$
$$\alpha_0=1.00 \text{ for total eclipse}$$

we obtain

$$Q(k\alpha_0)=0.133.$$

From Table V,[1] for central annular eclipse $\alpha_0''=1+x$, $k=0.307$; for grazing annular eclipse $\alpha_0''=1.00$, $k=0.358$. After several trials were made the best "fit" was found by adopting

$$\alpha_0''=1+0.25x \text{ and } k=0.345.$$

The primary curve was now computed for $k=0.345$ by means of Table IIx,[2] and the curve for secondary eclipse, for $k=0.345$ and $\alpha_0''=1+0.25x$, by use of Table Iy.[3] The corresponding "darkened" elements are given in Table III. The residuals for primary and secondary eclipse are given in Table I in the column headed $O-C_d$. The probable error of a normal point is $\pm 0^{M}.016$; and for secondary, $\pm 0^{M}.018$.

The tabulated densities in both cases are corrected for polar flattening as well as for probable difference of the masses of the brighter and fainter components. The hypothetical parallax was computed, and also the galactic co-ordinates.

The plot of the photographic observations on the new period gives the brightness as $9^{M}.8$ at constant light and $10^{M}.86$ at primary minimum, giving a loss of light of $1^{M}.06$. Adopting the elements previously obtained, which after rectification would probably be about $0^{M}.95$ and $1-\lambda_p$ (photographic) $=0.583$, the light of the bright star is 0.583 of the total, that of the faint one being 0.417;

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[1] *Astrophysical Journal*, **36**, 394, 1912.

[2] *Ibid.*, 245, 1912.

[3] *Ibid.*, 390, 1912.

hence J_b/J_f (ratio of surface brightness) = 12.2 for the uniform solution, and for the darkened solution, 11.7.

TABLE I

Observations SS Camelopardalis

PRIMARY MINIMUM

Normal No.	Phase	Mag. Diff. *v-a*	$O-C_u$	$O-C_d$	Normal No.	Phase	Mag. Diff. *v-a*	$O-C_u$	$O-C_d$
1........	$-12^{h}11\overset{m}{.}3$	1.035	+0.015	+0.015	24......	$+1^{h}51\overset{m}{.}1$	0.480	−0.012	−0.012
2........	11 26.2	1.004	− .010	− .007	25......	2 26.5	0.481	− .011	− .011
3........	11 2.4	1.008	− .005	− .003	26......	3 21.6	0.510	+ .028	+ .012
4........	10 39.6	1.007	+ .004	.000	27......	3 57.7	0.554	+ .036	+ .037
5........	10 22.5	0.987	− .018	− .018	28......	4 13.9	0.563	+ .028	+ .033
6........	10 3.9	0.996	− .005	− .002	29......	4 32.7	0.564	+ .005	.000
7........	9 40.6	0.940	− .050	− .050	30......	4.59.9	0.605	.000	.000
8........	9 21.3	0.936	− .045	− .045	31......	5 22.7	0.644	+ .010	+ .006
9........	8 58.8	0.932	− .035	− .028	32......	5 37.2	0.639	− .018	− .020
10........	8 32.1	0.878	− .052	− .052	33......	5 50.9	0.655	− .022	− .025
11........	8 00.1	0.892	.000	− .010	34......	6 06.0	0.716	+ .010	.000
12........	7 21.6	0.829	− .005	− .008	35......	6 18.0	0.730	.000	− .004
13........	6 40.3	0.815	+ .035	+ .035	36......	6 30.2	0.784	+ .034	+ .034
14........	5 53.7	0.714	+ .014	+ .024	37......	6 43.6	0.784	+ .014	.000
15........	5 12.3	0.629	+ .005	+ .003	38......	7 03.6	0.851	+ .050	+ .030
16........	4 24.9	0.548	.000	− .005	39......	7 31.4	0.892	+ .040	+ .042
17........	3 39.5	0.477	− .015	− .033	40......	8 21.2	0.943	+ .018	+ .016
18........	2 31.7	0.504	+ .012	+ .012	41......	8 51.0	0.987	+ .037	+ .030
19........	1 26.4	0.509	+ .017	+ .017	42......	9 32.0	1.003	+ .018	+ .018
20........	0 48.9	0.495	+ .003	+ .003	43......	9 57.7	1.034	+ .036	+ .040
21........	−0 17.0	0.485	− .007	− .007	44......	10 17.6	0.992	− .010	− .008
22........	+0 6.0	0.480	− .012	− .012	45......	11 14.1	1.008	−0.005	−0.007
23........	0 59.5	0.512	+0.030	+0.030	....	....	....	....	....

SECONDARY MINIMUM

Normal No.	Phase	Mag. Diff. *v-a*	$O-C_u$	$O-C_d$	Normal No.	Phase	Mag. Diff. *v-a*	$O-C_u$	$O-C_d$
70........	+44 17.2	0.966	−0.046	−0.050	89*.....	+55 52.4	0.882	−0.014	−0.003
71........	44 56.6	0.998	− .010	− .014	90......	56 38.3	0.884	− .012	− .005
72........	45 19.8	1.001	− .006	− .005	91......	57 3.5	0.910	+ .014	+ .022
73........	45 47.2	1.006	.000	.000	92......	57 30.7	0.901	+ .005	+ .012
74........	46 9.4	0.986	− .008	− .010	93......	57 50.2	0.861	− .035	− .035
75........	46 28.3	0.984	− .005	− .008	94......	58 11.1	0.899	+ .003	+ .004
76........	46 50.0	0.952	− .032	− .036	95......	58 38.9	0.866	− .030	− .040
77........	47 10.1	0.997	+ .015	+ .015	96......	59 12.6	0.932	+ .030	+ .030
78........	47 38.3	1.014	+ .041	+ .036	97......	59 41.3	0.958	+ .048	+ .050
79........	49 26.1	0.952	+ .012	+ .007	98......	60 34.6	0.923	.000	+ .002
80........	49 49.6	0.960	+ .030	+ .020	99......	61 6.9	0.930	− .005	− .004
81........	50 7.9	0.958	+ .032	+ .028	100......	61 35.4	0.953	+ .012	+ .010
82........	50 27.6	0.957	+ .039	+ .030	101......	62 3.3	0.931	− .016	− .024
83........	50 52.8	0.918	+ .005	+ .000	102......	62 31.8	0.916	+ .042	+ .048
84........	51 15.1	0.915	+ .010	+ .005	103......	62 57.1	0.967	.000	− .005
85........	51 49.1	0.913	+ .015	+ .013	104......	63 26.8	1.002	+ .030	+ .024
86........	52 48.7	0.916	+ .020	+ .022	105......	63 55.5	0.939	− .042	− .043
87........	53 16.4	0.920	+ .026	+ .028	106......	64 23.1	1.009	+ .017	+ .017
88*.......	53 35.3	0.881	−0.015	−0.009	107......	65 12.2	1.068	+0.068	+0.065

*Zero phase of secondary is $55^{h}24\overset{m}{.}0$.

The primary eclipse is total; this is certain, since the light remains constant for 7 hours. Hence, knowing the magnitude of the system, we can readily determine the magnitude of each star. The visual and photographic magnitudes resulting are given in

Table III. In this system we meet again the combination, already noticed in other eclipsing binaries,[1] of a small, relatively dense star, of greater surface brightness, and presumably hotter, with a large star of low density and surface brightness, redder color,

TABLE II

UNECLIPSED OBSERVATIONS

PRIMARY TO SECONDARY				SECONDARY TO PRIMARY			
Normal No.	Phase	Mag. Diff. (a–v)	O−C	Normal No.	Phase	Mag. Diff. (a–v)	O−C
46	$+15^{h}12^{m}.4$	1.027	−0.012	108	$+74^{h}40^{m}.2$	1.052	−0.010
47	15 49.3	1.024	− .015	109	75 38.5	1.060	− .008
48	17 15.0	1.032	− .012	110	76 00.0	1.077	+ .008
49	19 12.3	1.020	− .028	111	76 35.1	1.070	.000
50	20 6.3	1.052	− .005	112	76 54.5	1.022	+ .052
51	20 44.0	1.084	+ .022	113	78 53.8	1.041	− .035
52	21 26.2	1.026	− .035	114	79 29.9	1.069	− .005
53	21 52.8	1.013	− .046	115	79 56.2	1.115	+ .040
54	22 12.6	1.083	+ .018	116	81 23.3	1.102	+ .025
55	23 0.1	1.056	− .015	117	81 50.4	1.067	− .010
56	23 56.4	1.000	− .070	118	82 20.5	1.077	+ .000
57	28 4.7	1.032	− .045	119	82 52.7	1.104	+ .026
58	28 21.4	1.049	− .026	120	84 20.6	1.115	+ .041
59	28 35.7	1.095	+ .015	121	85 20.6	1.089	+ .013
60	28 48.7	1.055	− .023	122	85 59.4	1.092	+ .022
61	29 8.5	1.012	− .066	123	86 22.9	1.126	+ .056
62	29 26.5	1.069	− .010	124	86 46.8	1.084	+ .014
63	29 44.7	1.040	− .035	125	87 12.7	1.091	+ .022
64	30 36.2	1.046	− .032	126	+87 37.6	1.086	+ .016
65	34 32.8	1.016	− .054	127	−27 28.8	1.120	+ .051
66	37 6.0	1.024	− .036	128	26 44.5	1.098	+ .030
67	37 55.1	1.065	+ .010	129	26 2.5	1.084	+ .016
68	39 43.8	1.007	− .043	130	23 48.6	1.065	− .005
69	40 36.9	1.023	−0.020	131	19 44.1	1.078	+ .020
				132	18 53.8	1.085	+ .030
				133	18 5.0	1.091	+ .042
				134	17 17.1	1.056	+ .008
				135	15 41.5	1.052	+ .010
				136	15 11.1	1.035	− .005
				137	14 32.4	1.062	+ .027
				138	13 46.2	1.065	+ .035
				139	−12 58.3	1.020	+0.005

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[1] *Contributions from the Mt. Wilson Solar Observatory*, Nos. 64 and 68; *Astrophysical Journal*, **36**, 368, 1912; **37**, 204, 1913.

and doubtless lower temperature. The unusual feature is that, visually, the larger star, although fainter per unit area in a ratio of 1:5, gives the greater amount of light. Photographically the reverse is true; the photographic plate is affected chiefly by the light of the smaller star, which is 12.2 times brighter per unit area than the larger star. The spectral type of the system is given as F ?; if this is the case, the smaller star may well be of Class A and the larger of Class G. On this assumption the hypothetical parallax comes out $0''.0007$ from the data for the small bright star, and $0''.0011$ from those for the large faint star.

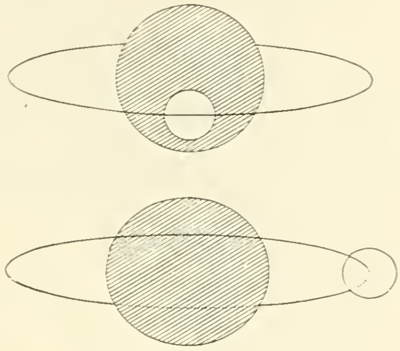

FIG. 2

After the completion of the discussion of the observations, a paper by R. S. Dugan on the variable RV Ophiuchi was published.[1] The light-curve of this star has characteristics similar to those of SS Camelopardalis. The difference in brightness between the uneclipsed portions of the curve is almost exactly represented by a sine term. Professor Dugan explains it as due to a difference of brightness between the advancing and following sides of the brighter star. This assumption was applied to SS Camelopardalis, and it was found that by using a term 0.020 sin θ (θ being the orbital longitude) the residuals throughout the curve were greatly reduced. In this case the system is brightest before principal minimum, but, since the star then eclipsed gives out less light than the other, this is actually the case when the brighter of the two components (though not the one of greater surface brightness) is approaching us.

SUMMARY

1. SS Camelopardalis is an eclipsing variable. The elements for the principal minimum are: J.D. $2,420,842.594+4^{d}.82438=$ 1915 December 10, $14^{h}21^{m}$ G.H.M.T.$+4^{d}19^{h}47^{m}6^{s}.4$.

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[1] *Astrophysical Journal*, 43, 130, 1916.

2. The range from maximum to principal minimum is $0^{\mathrm{M}}.57$, and to the secondary minimum is $0^{\mathrm{M}}.15$. The stars are distinctly ellipsoidal, and when correction is made for this the depths of the minima become $0^{\mathrm{M}}.485$ and $0^{\mathrm{M}}.082$ respectively. The brightness is greater by about $0^{\mathrm{M}}.05$ halfway between secondary and primary than midway between primary and secondary. The principal eclipse lasts 21 hours and is total for 7 hours; the secondary is annular. The photographic range at principal minimum is about $0^{\mathrm{M}}.95$.

TABLE III

RESULTS

Elements of System	Uniform	Darkened
Maximum radius of small star . . . . a_2	0.150	0.153
Minimum radius of small star . . . . b_2	0.136	0.144
Maximum radius of large star . . . . a_1	0.446	0.444
Minimum radius of large star . . . . b_1	0.406	0.417
Ratio of the radii of the two stars . . . . k	0.335	0.345
Ratio of the axes of the spheroidal stars . . . . $1+\frac{1}{2}z$	1.088	1.054
The least apparent distance of centers . . . . $\cos i$	0.2024	0.2305
Inclination of orbit i	78°.3	76°.0
Eccentricity of orbit . . . . $e \cos \omega$	−0.03	. . . .
Maximum percentage loss of light at primary minimum . . . . $\alpha_0 p$	1.00	1.00
Maximum percentage loss of light at secondary minimum . . . . $\alpha_0 s$	1.00	*1.096
The light of the bright star (visual) . . . . L_b	0.6397	0.6397
The light of the faint star (visual) . . . . L_f	0.3603	0.3603
Ratio of the surface brightness of the two stars (visual) . . . . J_b/J_f	5.02	4.74
The light of bright star (photographic) . . . . L_b	0.583	. . . .
The light of faint star (photographic) . . . . L_f	0.417	. . . .
Ratio of surface brightness of the two stars (photographic) . . . . J_b/J_f	12.2	11.7
Most probable density of smaller star . . . . ρ_b	0.094	0.078
Most probable density of larger star . . . . ρ_f	0.0050	0.0046
Hypothetical radius of brighter star in terms of solar radius	2.32	2.32
Hypothetical parallax		0″.0009
Distance in light-years from the galactic plane		1700
Distance in light-years projected on galactic plane		3200
Visual magnitude of system		10.15
Visual magnitude of big star (end on)		10.65
Visual magnitude of small star (end on)		11.26
Photographic magnitude of system		9.9
Photographic magnitude of big star (end on)		10.86
Photographic magnitude of small star (end on)		10.48

*α_{0s} Expressed in terms of the light lost at internal tangency.

3. The system consists of a large red star of low surface brightness and a smaller white star of a little more than one-third of the diameter of the other, but of five times the surface brightness of the large star visually and 12 times its surface brightness photographically, so that the smaller star is visually the fainter and photograpically the brighter of the two. Both stars are ellipsoidal, the ratio of the longer to the shorter axis in the equatorial plane being about 1.06.

4. The density of the large red star is about 1/200, and that of the smaller white star about 1/12 that of the sun. The combined spectrum is recorded as F ?. It is not improbable that the small star is of spectral type A and the larger of type G or redder.

OTTAWA, CANADA
September 1916

THE MINIMUM RADIATION VISUALLY PERCEPTIBLE

By HENRY NORRIS RUSSELL

In a recent paper[1] Ives has shown that the least quantity of radiant energy capable of producing the sensation of light is much smaller than had previously been supposed, the error of the earlier estimates arising from the treatment of all radiations within the visible spectrum as if they were equally efficient in producing the sensation of light, which is extremely far from the truth. The numerical results given in his paper depend, however, upon four factors, two of which were well determined, while the other two could be but roughly estimated in the absence of sufficient data. It is the purpose of the present communication to supplement Dr. Ives's work by the introduction of improved values for these latter factors, derived partly from material inaccessible to him and partly from unpublished observations.

The well-determined factors are the mechanical equivalent of light and the stellar magnitude of the standard candle. The former may be taken as 1.59 ergs per second per square centimeter,[2] for light of intensity one meter-candle, and wave-length 0.55 μ (corresponding to the maximum visual sensation for equal energy). The meter-candle, according to the mean of several fairly accordant determinations,[3] is of stellar magnitude -14.18. It follows that a source emitting light of wave-length 0.55 μ, and appearing like a star of magnitude 6.0, would furnish energy at the rate of 1.35×10^{-8} ergs per second per centimeter. These data are taken unchanged from Ives's discussion.

The uncertain factors are the diameter of the pupil of the eye when accommodated to complete darkness, and the stellar magnitude of the faintest visible objects.

Upon the first of these points the only really reliable information appears to be contained in a brief paper by W. H. Steavenson[4] which reached this country just after Dr. Ives's paper was written. Measurements were made by photographing the eyes, accommodated to complete darkness, by flashlight, since the presence of light strong enough to permit of direct measurement caused a decided contraction of the pupil. The mean diameter for five healthy men was found, under these conditions, to be 0.336 inch, or 8.5 mm, with an average deviation from the mean of only 3 per cent. This makes the area of the pupil 0.57 sq. cm—just double the value resulting from Ives's estimate of diameter 6 mm, and nearly three times that derived from the conventional figure of "one-fifth of an inch."

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[1] *Astrophysical Journal*, 44, 124, 1916.

[2] Ives, Coblentz, and Kingsbury, *Physical Review*, **5**, 269, and **6**, 319, 1915.

[3] Russell, *Astrophysical Journal*, **43**, 129, 1916.

[4] *Journal of the British Astronomical Association*, **26**, 303, 1916.

More important is the second question. There is abundant evidence to show that the visibility of faint stars is greatly affected by the diffused light of the sky. How great this influence is, even for the brighter stars, is often accidentally illustrated when a star happens to be seen through a gap in the foliage of a tree, upon a relatively dark background. To the writer, at least, it appears so much brighter than usual under these circumstances as to make immediate identification difficult.

A very valuable series of observations of the visibility of stars upon a dark background has been made by H. D. Curtis.[1] Faint stars were observed through a small hole in a large screen attached to the remoter end of the twelve-inch telescope, and the reality of the observation was controlled with the telescope. Though the faintest star seen directly upon the background of the sky was of magnitude 6.53, stars of magnitude 7.2–7.4 were easily seen upon the dark background, four others of magnitudes between 7.9 and 8.1 were usually seen with little difficulty, two of magnitude 8.3 were "seen with difficulty," and a star of magnitude 8.9 was "glimpsed at intervals, very doubtful." It appears, therefore, that if seen upon a perfectly dark background the limit of visibility for a stellar point must be not far from the magnitude 8.5—especially when it is remembered that the dome of an observatory, under ordinary working conditions, is by no means perfectly dark, even in the absence of artificial lights.

[1] *Lick Observatory Bulletin*, **2**, 67, 1901. The magnitudes of some of the stars have been replaced by more recent Harvard photometric determinations.

Further evidence bearing upon this matter is found in certain observations by the writer, part of which have been published.[1] If a room is completely darkened except for a small part of one window, less than a square foot in area, and the light of a star, shining through this aperture, is received on a white screen on the opposite side of the room the faint luminous patch may be easily observed if the eye has become thoroughly accommodated to darkness. The light of the first-magnitude stars is unmistakable, and that of Sirius really conspicuous; but in this connection the following transcript from the notebook is of interest. "Sky near Sirius much brighter than paper illuminated by Sirius-light, so much so that it dazzles the eye, appearing as a dark spot on the Sirius-lit paper, after a glance at the window. To see the fainter lights to advantage, it is necessary to remain in the dark for at least half an hour, and to avoid looking at anything as bright as Sirius-light for a few minutes before observing." Under favorable conditions, the illumination of the paper by stars fainter than the second magnitude was recognized, the faintest stars whose light was certainly identified being of magnitudes 2.8 and 2.9. As these stars were at a moderate altitude, it is probable that the illumination of a white surface by a star of the third magnitude in the zenith would be distinctly perceptible.

The following observations, which have not previously been published, are copied from the notes written at the time:

> February 8, 1902. Investigations on *minimum visibile*. Two approximately circular disks of white cardboard were pinned on black cloth, and the greatest distances at which they could be seen, when illuminated by starlight, marked with pins on a tape measure. Disk A (larger) maximum distance, light from Sirius, 39 inches. Disk B (small) maximum distance, light from Sirius, 20½ inches. Disk A, light from Rigel, maximum distance 16½ inches. The disks lost all definite outline at less than half these distances and appeared as faint, ill-defined luminous spots, like the old luminous match-safes. Diameter of disk A, 0.45 inch, of disk B, 0.80 inch.

From these data the *minimum visibile* may be calculated in terms of stellar magnitude as follows. The disks were illuminated and viewed nearly at right angles to their own plane. If the radius of the disk is r, the distance of the observer's eye R, and if it is assumed that the disk reflects diffusely the fraction A of the incident light, in accordance with Lambert's law, the light received by the eye from the disk should be to that received directly from the star in the ratio $A\frac{r^2}{R^2}:1$, to which corresponds a difference of magnitude amounting to $5 \log R/r - 2.5 \log A$. If the value of A for the white cardboard is taken roughly as 0.8, the last term is $+0.25$, and the three observations give (correcting the obvious interchange of A and B at the end of the original note):

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[1] "Shadows Cast by Starlight," *Popular Astronomy*, 10, 242, 1902.

Star	Disk	Mag. of Star	R	r	5 log R/r	*Minimum Visibile*
Sirius.............	A	-1.58	39	0.40	9.95	$8\overset{\mathrm{M}}{.}6$
Sirius.............	B	-1.58	20.5	0.225	9.80	8.5
Rigel.............	A	$+0.34$	16.5	0.40	7.95	8.5

The observations, which from the nature of the measures must have been quite independent, agree better than might have been expected. The resulting *minimum visibile*, $8\overset{\mathrm{M}}{.}5$, is in entire agreement with that found by the Lick observers, and is ten times fainter than the ordinarily assumed limit for the visibility of a star against the relatively brilliant background of the sky.

The amount of energy which would enter the eye from a light-source of maximum efficiency, and of magnitude 8.5 is, according to the foregoing, $1.35\times10^{-8}\times0.57\times0.10$, or 7.7×10^{-10} ergs per second—which may be taken as the best available approximation to the true *minimum visibile*. This is just one-fifth of the value found by Dr. Ives, as follows at once from the facts that the values here assumed for the area of the pupil and the faintest visible light are respectively twice and one-tenth of those adopted by him.

The faintest visible illumination of a surface (by a third-magnitude star) corresponds to 1.3×10^{-7} meter-candles, and, for light of the maximum efficiency, to an energy-density of 2.1×10^{-7} ergs per square centimeter per second. Since there must be some losses, both on reflection at the surface and in passing through the eye, it follows that a sensible stimulation of the retina can be produced by an energy-density less than 2×10^{-7} ergs per square centimeter per second at the surface of the retina itself, provided that this stimulus is applied over a sufficiently large area (which, to make the incident energy equal to the minimum previously given, must have an apparent angular diameter of at least 10°). Ten times this illumination, corresponding to the light of a star of magnitude 0.5, is conspicuous to the fully rested eye, so that the "shadow bands" caused by the irregularities of density in the earth's atmosphere can be seen on the screen under favorable conditions.

The results here given are subject to possible corrections for the difference in area of the luminosity-curve for high and low intensity, but Dr. Ives has shown that these are probably small. The greatest outstanding uncertainty is that of the observational determinations of the *minimum visibile.* According to the present estimate, the minimum perceptible radiation corresponds to the reception by the eye of about 200 elementary quanta of radiation per second, or, in ordinary units, of *one erg in forty years*, or one gram-calorie in 1700 million years.

Further investigations of this question by laboratory methods would be of great interest, especially if an attempt was made to determine how long a time was required for such very faint light to produce a perceptible sensation.

Princeton University Observatory
December 11, 1916

MINOR CONTRIBUTIONS AND NOTES

ON DE LURY'S HYPOTHESIS AS TO EFFECT OF HAZE ON MEASURES OF SOLAR ROTATION[1]

R. E. De Lury suggests[2] that certain variations of results occurring in spectroscopic measurements of solar rotation are probably due to the superposition, on the spectrum of a definite point of the sun's image, of the spectrum of scattered light from the sun as a whole. He mentions various sources of the scattered light, including atmospheric haze, instrumental stray light, diffraction-effects, bad seeing, and possibly dust particles in space. He computes that certain results published by St. John, Adams, and Ware indicate an overlapping spectrum of the haze of about 9 ± per cent at Mount Wilson for observations of 1914–1915 in the region λ 5200.

The work of the Smithsonian Astrophysical Observatory has yielded measurements bearing on this point. In the years 1904–1907 at Washington, and 1913–1916 on Mount Wilson we observed at various wave-lengths the distribution of light along the diameter of the solar image.[3] If De Lury is right about the amount of the stray light, our work on the distribution of solar radiation would be worthless, as well as that of the observers of the solar rotation. It is worth noting that our Washington results agree closely with our Mount Wilson work and differ from it in the sense contrary to what would be expected if both were vitiated by stray light, and Washington work most affected owing to less favorable location.

In this work, having set the spectrobolometer at the desired wave-length, we set the solar image a little beyond the slit and inserted the shutter. Then we stopped the coelostat clock, and as the image drifted toward the slit we opened the shutter.

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[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, 44, 177, 1916.

[3] See *Annals*, 2 and 3, and *Smithsonian Misc. Coll.*, 65, No. 4, 1915.

At the central position we closed the shutter for 10 seconds, and after the image had drifted entirely off the following limb we inserted the shutter again. Thus the record runs: shutter, sky, sun, shutter, sun, sky, shutter. If now on the automatic bolographic record a straight edge is laid on the trace of the three shutter positions, the relative intensities of sky and sun are measureable. It is to be noted that "sky" means all stray light from all the sources enumerated above. As our telescope is nearly similar to that used by St. John, Adams, and Ware, excepting as their silvered mirrors may have had better or worse polish than ours, the results we obtained should give a fair test of the matter in question.

As to the results obtained in Washington work I quote: "From the examination of numerous records it appears that the total deflection produced by the sky radiation and all other causes combined, at a point 1.5 per cent of the radius outside the computed width of the disk, is certainly less than 1 per cent of the deflection at the centre of the disk."[1] (At λ 5200 the edge has about 50 per cent as great intensity as the center of the disk.)

As to the results obtained on Mount Wilson, on account of the better definition and less intensity of the sky-light there, the inspection of numerous plates does not indicate appreciable intensity at a point 1.5 per cent of the radius outside the limb. So far as can be measured, there is no difference between the shutter and the sky. I feel safe in saying that the stray light there is at any rate much less than 1 per cent of the sunlight at the limb for λ 5200.

From these results I conclude that, with the long-focus telescopes used, and for good observing days, neither at Washington nor (much more) at Mount Wilson does the spectrum of a point of the sun's image at λ 5200 derive more than 1 per cent of its intensity from all the light scattered from positions distant more than 1.5 per cent of the solar radius. Experts will draw their own conclusions, as to what influence scattered light of such intensity may produce on measurements of solar rotation.

Our work has been done at high sun, i.e., zenith distances almost always less than 50°. We have not observed on very hazy days,

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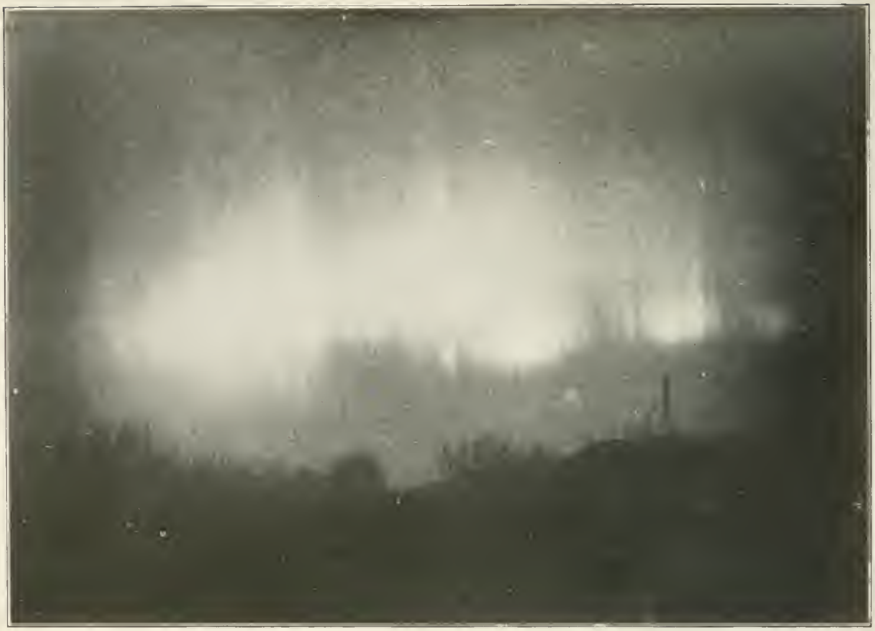
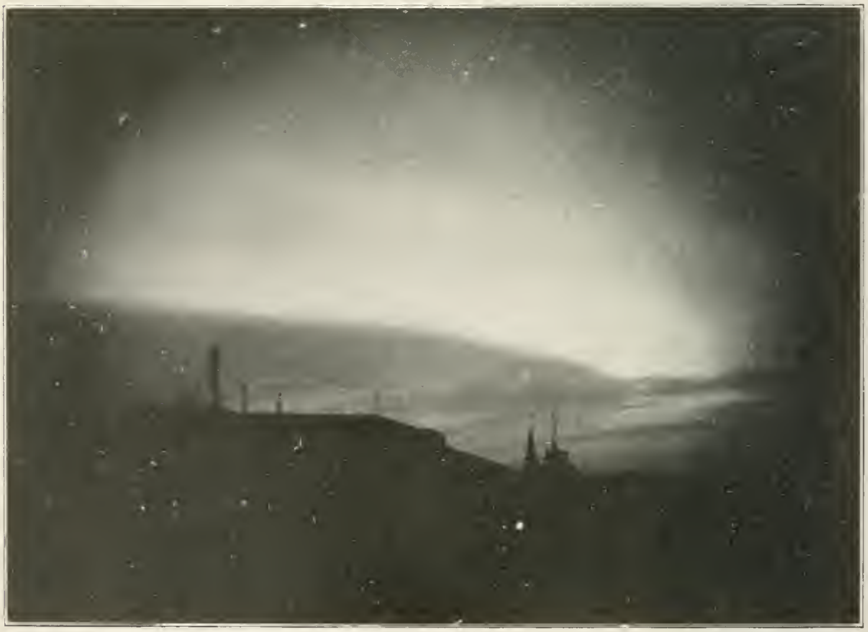
[1] *Annals*, 2, 223.

PLATE I



Auroral Curtains. 1916 October 13. 9^h34^m G.M.T.
Exposure 3 seconds



Auroral Arch. 1916 October 13. 9^h57^m G.M.T.
Exposure 10 seconds

such as would be obviously unfit for our investigations of the solar constant. It is possible that if observations were made soon after sunrise, or through clouds, or on extremely hazy days, the superposed spectrum would reach such values as are suggested by De Lury. Perhaps observers of solar rotation may find it a useful precaution to expose a plate to light from just within and just without the solar limb, in such a manner as to get a rough indication of the intensity of stray light on each day when the solar rotation is observed.

C. G. Abbot

Smithsonian Astrophysical Observatory
December 18, 1916

PHOTOGRAPHS OF THE AURORA BOREALIS

A very fine aurora was seen in the northern sky at Christiania on October 13, 1916. The photographs reproduced here (Plate I) were taken with the camera used during my expedition to Bossekop in 1913 for observing the aurora. It had a kinematograph lens of 25 mm aperture and 50 mm focal length. "Ultra rapid" plates from Hauff of Feuerbach were used. Enlarged positives were first made from the small original negative; new negatives were made from these, and the illustrations are reproduced from prints taken from these enlarged negatives.

Some of the stars in Ursa Major may be seen above the aurora in the lower picture, and in the foreground of both pictures the trees and buildings of the town may be discerned. The photographs were taken by myself from the roof of the Astronomical Observatory.

Carl Störmer

Christiania
November 1916

NOTICE TO SUBSCRIBERS AND CONTRIBUTORS

The war has greatly increased the difficulties of the publication of scientific journals. This is peculiarly true of the *Astrophysical Journal,* inasmuch as three-fifths of its subscribers before the war were in Europe or in countries other than the United States. The difficulty in renewing subscriptions from abroad in these troublous times has diminished the income, while the greatly increased cost of paper has unduly enhanced the expense of manufacture of the *Journal.* Even in time of peace the income has not been over one-half of the expense of production of the *Journal* in the style that has been followed for the last forty-four volumes, but the necessary subsidy has been generously maintained by the Trustees of the University of Chicago.

Hoping that the unfavorable conditions are temporary, the Editors seek to avoid the necessity of an increase in the subscription price; but we shall be obliged for the present to restrict the size of the numbers to about 64 pages and to limit the illustrations to those actually necessary. (It may be appropriate to mention that authors frequently furnish the blocks for their illustrations.) We ask our contributors to express themselves briefly and to avoid expensive tabular composition where possible. We are further obliged, for the present, to suspend the privilege of free reprints for articles received after this notice is issued. Reprints will be supplied at cost, which may be estimated as follows:

	25	50	100	150	200	250	300	350	400	500	Additional 100
4 pages.......	$1.30	$1.40	$1.60	$1.80	$2.00	$2.20	$2.40	$2.60	$2.80	$3.20	$0.40
8 pages.......	1.75	1.90	2.20	2.50	2.80	3.10	3.40	3.70	4.00	4.60	0.60
16 pages.......	2.80	3.20	4.00	4.60	5.20	5.80	6.40	7.00	7.60	8.80	1.20
Plates*........	0.80	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30	2.80	0.60
Covers........	1.00	1.20	1.60	2.00	2.40	2.80	3.20	3.60	4.00	4.80	0.80

*One double plate equals two single.

THE EDITORS

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THE LAW OF PHOTO-ELECTRIC PHOTOMETRY

By JAKOB KUNZ

In the field of present photo-electric investigations four main groups may be distinguished. The first group is concerned with the fundamental problem of the velocity of electrons emitted, and the frequency of the incident light. After a long series of experiments of gradually increasing accuracy the equation $\frac{1}{2}mv^2=hn-P_0$ has finally been established by the measurements of R. A. Millikan. The second problem is that of the relation between the number of electrons emitted and the intensity of the incident light. Other problems arise in connection with the so-called normal and selective effects, and with the influence of gases. The present investigation deals with the second problem. Several physicists have attacked this question with widely discordant results or conclusions. Lenard, Elster and Geitel, and Richtmyer concluded that a linear relation exists between the photo-electric current and the intensity of light. Griffith and H. E. Ives, on the other hand, do not find a linear relation. Ives especially[1] gives a large variety of curves and suggests a cell which will yield a straight line, without giving data for this statement.

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[1] *Astrophysical Journal*, **39**, 428, 1914; **43**, 9, 1916.

A closer investigation of the literature, however, shows that there is very little if any evidence in favor of the simple, straight-line relation. Ives has already remarked that Lenard was not justified in his conclusion, as his figures really show a current increasing more rapidly than the illumination. Richtmyer also concluded from his data that the action of light is strictly proportional to the light-intensity over very wide ranges. If I divide, however, the relative light-intensities I by the deflections d, then the ratio I/d varies widely, as will be seen from Table I.

TABLE I

I	d	I/d	I	i	I/i
0.0100	0.318	314	19.0	5.9	322
0.0236	0.646	370	30.0	8.7	345
0.0400	1.033	389	45.0	12.5	360
0.0625	1.573	398	63.0	16.4	385
0.0816	2.055	397	84.0	22.6	372
0.111	2.682	414	118.0	31.4	376
0.131	3.20	409	155.0	38.8	398
0.160	3.93	407	210.0	52.6	400
0.197	4.80	410	304.0	79.5	383
0.297	7.51	396	375.0	97.5	385
0.387	9.79	396	475.0	123.0	386
0.760	18.53	411	620.0	160.0	387

The ratios I/i and I/d vary so much that the conclusion seems hardly justified that the photo-electric current is proportional to the intensity of illumination. Richtmyer has given further evidence in favor of his conclusion with experiments involving a photo-electric cell itself as a resistance. I have repeated these measurements with the same result, using cells, however, which do not give a straight-line relation when tested with different methods. Elster and Geitel have made photometric measurements also over a very wide range of intensities, using the polarization of light by a Nicol prism for the variation of the illumination. They also conclude that there is a linear relation between the two variables. In each set of experiments, however, they vary the intensity only in the ratio of 1 to 4, by means of the angles 30°, 45°, 60°, 90°, the deviations from the straight line varying only between 0.1 and 3 per cent. I was able to test an Elster and Geitel cell over a wider range of intensities, keeping the potential-difference constant. Table II contains the results, *d* being the corrected deflections of the electrometer needle, *I* the relative intensity, *I/d* their ratio, which ought to be constant.

TABLE II

d	*I*	*I/d*	*d*	*I*	*I/d*
2.0	4	200	115.3	178	155
4.55	9	198	152.6	224	147
8.1	16	198	208.4	294	141
19.35	36	186	261.5	352	135
58.25	100	162			
90.25	144	160	310.85	400	130

The source of light was a tungsten lamp of 42 candles, which was moved over an interval between 30 and 300 cm from the sensitive surface. If the relative intensity of illumination varies from 4 to 16, then the ratio *I/d* varies, indeed, only 1 per cent, but if the intensity varies more, then the deviations from the straight line become conspicuous. Similar tests on other cells show clearly that the statements made by Ives are justified. Ives suggested, in his last contribution made in this journal, a cell which would give a straight line. Such a cell has been constructed according to the specifications given by Ives. Table III does show a very nearly constant ratio between the light-intensity and the photo-electric current.

TABLE III

d	*I*	*I/d*	*d*	*I*	*I/d*
3.45	4	116.0	128.1	144	112.5
8.0	9	112.5	159.4	178	112.0
14.1	16	114.0	202.6	224	111.6
31.6	36	114.0	257.6	294	114.0
86.4	100	116.0	306.7	352	114.5

This cell indeed satisfies very nearly the theoretical requirements of photometry. However, the light-intensities which act on the sensitive potassium surface are comparatively small and other cells of the older type also give a nearly linear relation for smaller light-intensities. A milk glass has almost necessarily to be used in connection with this cell, and thereby the intensities acting on the surface are rather small. The electrostatic field in this cell is not uniformly radial, because it is well-nigh impossible to distil the alkali metal away from the glass stem which carries the central bulb covered with the potassium. This potassium cell contained argon. With the exception of this cell of Ives, it seems to me that there is no satisfactory evidence in favor of the single law of direct proportionality between the intensity of illumination and the photo-electric current. This situation gave rise to the following investigation.

During our study of the best conditions of the sensitiveness of photo-electric cells for the use of stellar photometry, Professor J. Stebbins and I found that in order to eliminate the dark current it was necessary to cover the larger part of the photo-electric cell with alkali metal, leaving only a small hole free for the entrance of light, and to use, not a single ring electrode, but a ring electrode with fine wires crossing each other and forming a wire net. By this means there is established a more or less uniform field between the sensitive surface and the anode, and only few electrons will pass through it and accumulate on the opposite glass surface where the light enters. A description of the construction of these cells has been given in the *Physical Review*, 7, 62, 1916.

The preliminary experiments have been made with these cells; their diameter is 3.2 cm, the anode loop 2 cm in diameter, and the opening for the beam of light 1.5 cm. The cells contained alkali metals, partly sensitized and evacuated, partly sensitized and filled with argon.

In the first set of experiments a galvanometer of high sensitiveness $2.4 \cdot 10^{-10}$ was used for the measurement of the current, a constant potential was applied to the cell varying between 4 and 120 volts, the intensity of illumination was changed by moving a tungsten lamp, nearly a point-source, on a photometer bench. Some of the curves obtained are given in Fig. 1, which shows the deviations from straight lines for different conditions. It was soon observed that the curves approached a straight line when the source of light was rather weak, 6.5 candles for instance, and when the cross-section of the entering beam of light was sufficiently decreased. In the last case, even when a tungsten lamp of 388 candles was moved from 300 to 40 cm away from the sensitive surface, and with an opening of 7 mm, I found a straight line on

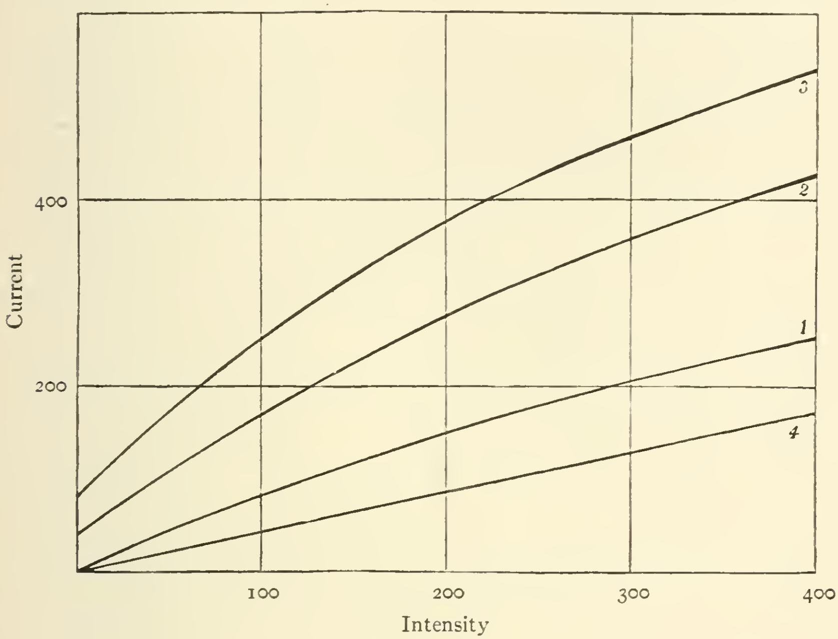

Fig. 1

plotting the data obtained; but calculating the ratio of illumination to the current I found a small variation in that ratio, as will be seen from Table IV.

TABLE IV

d	*I*	*I/d*	*d*	*I*	*I/d*
5.05	11.1	220	68.6	156.2	228
11.25	25.0	222	89.5	204.0	228
20.1	44.5	222	122.3	278.0	227
44.2	100.0	226	175.6	400.0	228

These data were obtained by means of an evacuated potassium cell without the treatment with hydrogen, under the influence of strong light. This result pointed strongly toward the final result of a straight line, especially because it was found later that cells partly filled with argon or helium give more satisfactory results than those without the inert gas. The results obtained are independent of the alkali metal; sodium, potassium, and rubidium under otherwise the same conditions gave exactly the same results. In curve 4 of Fig. 1, 215 volts indicate that the potential-difference between the electrodes of the cell and the current through the tungsten lamp are kept constant. As long as the galvanometer was used, rather high light-intensities were required in order to get a sufficient current. In the next set of measurements the current was measured by means of an electrometer and a high graphite resistance, as recommended by Ives. These resistances are very satisfactory when properly treated and kept in a dry atmosphere. They can be changed readily from 10^7 to 10^{11} ohms. They are constant as long as the potential-difference applied is less than 1 volt, but when the tension increases, deviations from Ohm's law will be observed. The openings for the beam of light were 0.5 cm in diameter. Some of the results are represented by Fig. 2, which seems to consist of a series of straight lines. But it was observed in these series that the last point for the highest intensity was a little too high without exception, while one should have expected that point to be a little too low, because the potential-difference between the electrodes of the cell must have been decreased sometimes as much as 0.7 per cent, but usually less. Table V shows the data, which are represented by curves 1 and 5 of Fig. 2.

TABLE V

d	I	I/d	d	I	I/d
7.15	11.1	155	5.55	11.1	200
15.9	25.0	157	12.4	25.0	202
28.5	44.5	156	22.05	44.5	202
63.6	100.0	157	50.65	100.0	198
98.6	156.2	158	80.15	156.2	194
128.2	204.0	159	105.1	204.0	195
175.7	278.0	158	145.0	278.0	192
212.0	331.0	157	173.8	331.0	191
262.2	400.0	154	213.3	400.0	190

Though the curves are nearly straight, there is a slight variation in I/d in these experiments, always in the same sense, namely, that with increasing intensity I, the current d becomes too large. It will be seen in addition that as long as the intensity of illumination varies only from 11 to 100 the current remains within 1 per cent proportional to the illumination. Now, in stellar photometry, the brightness of the stars varies hardly more than 10 per cent, and the illumination remains 10,000 to 100,000 times smaller than in these experiments, and it has always been found that the current becomes more nearly proportional to the intensity as the light decreases. Hence these cells in the form prescribed are sufficiently accurate for stellar photometry and for many other purposes; for instance, in plant physiology. On the other hand, if these cells are used for wider variations of the illumination a calibration is to be recommended. So, for instance, J. B. Nathanson, in the reflecting power of the alkali metal,[1] took the deviation from the straight-line relation into account. The intensity has been varied from 11.1 to 400, or from from 1 to 36. To the potassium-hydrogen-argon cell, 120 volts were applied; the tungsten lamp

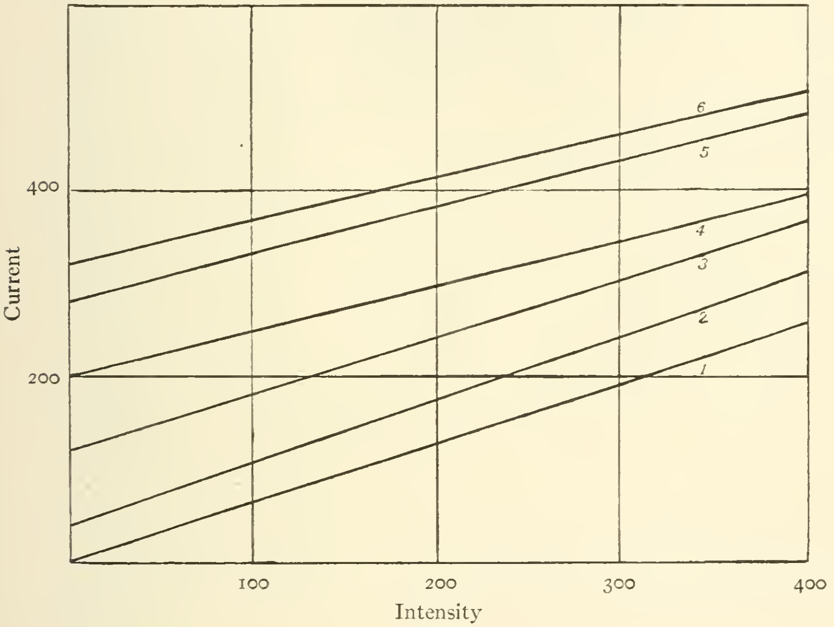

FIG. 2

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[1] *Astrophysical Journal*, 44, 137, 1916.

moving between 50 and 300 cm from the sensitive surface had a brightness of 40 candles in curves 1 and 2 of Fig. 2; 83 candles in curves 5 and 6; of 0.06 in curve 4, and of 1.02 candles in curve 3.

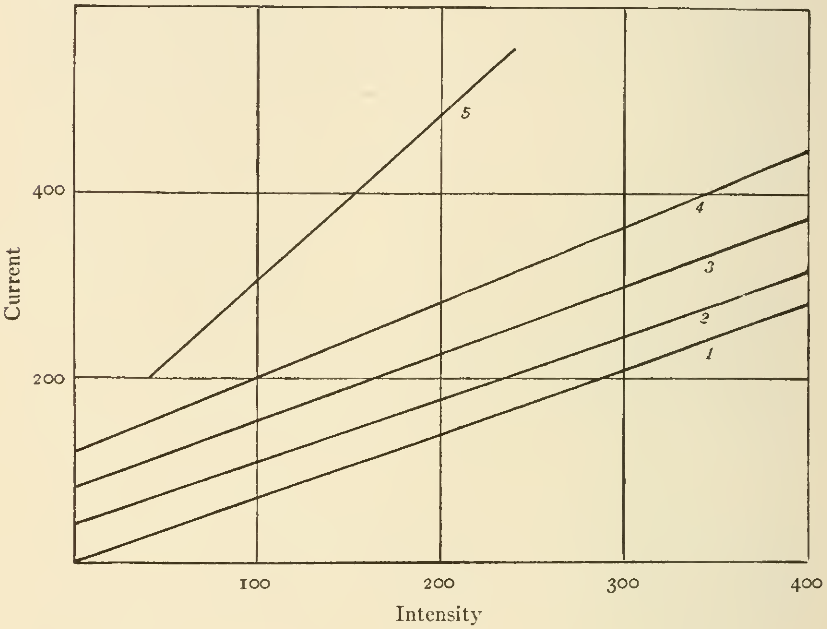
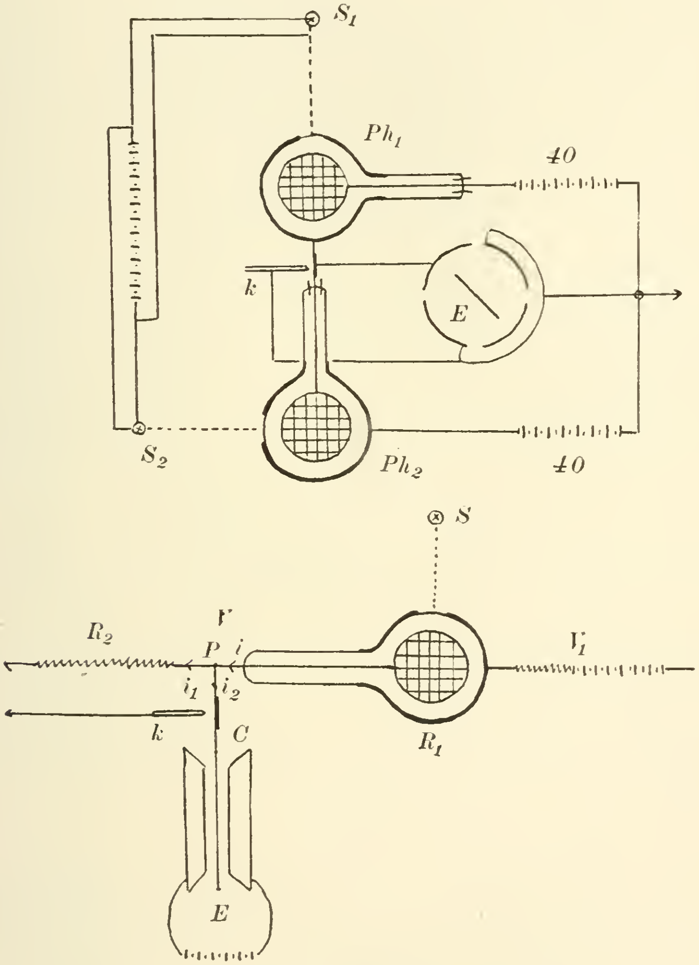


Fig. 3

TABLE VI

d	I	I/d	d	I	I/d
9.0	12.8	142	9.63	12.8	134
34.5	48.8	141	35.2	46.8	133
70.9	101.3	142	75.2	100.0	134
106.5	151.5	142	112.8	152.0	136
139.6	200.0	144	147.7	200.0	136
186.6	268.0	144	196.6	269.0	137
251.8	360.0	144	244.0	336.0	138
276.7	397.0	144	264.0	362.0	138
....	....	....	289.2	400.0	138

The light-intensity was also varied by means of the polarization by two Nicol prisms and by means of the rotating sector disk. Some of the corrected data are given in Fig. 3 and in Table VI, curves 1 and 2 corresponding to the variation of light by means of Nicol prisms, curves 3 and 4 to the variation by revolving sector disks. The points of curve 4 marked with crosses were obtained by the motion of the source, the points marked with circles, by means of the rotating sector disk. The two curves, respectively straight lines, coincide exactly. This fact can be stated in the form: Talbot's law holds for the photo-electric cell. The ratio between the illumination and the current deviates again slightly from the constant value for the higher intensities, as will be seen from Table VI, which corresponds to curves 1 and 3 of Fig. 3.



FIGS. 4 AND 4*a*

Instead of a graphite resistance, an electrolytic resistance of dilute cadmium sulphate and cadmium-electrode, and, finally, a photo-electric cell itself, were used as a resistance. In the latter case, the arrangement of the experiment is given by Fig. 4. The photo-electric cell Ph_2, illuminated by the source S_2, serves as resistance; k represents a key; the source S_1 was given various distances from the photo-electric photometer. Over a short interval of intensities the required relation is a straight line, as indicated by curve 5 of Fig. 3. This system is exceedingly sensitive to small variations of illuminations, but on this account it requires great care; there is probably no other photometer so sensitive as is this system.

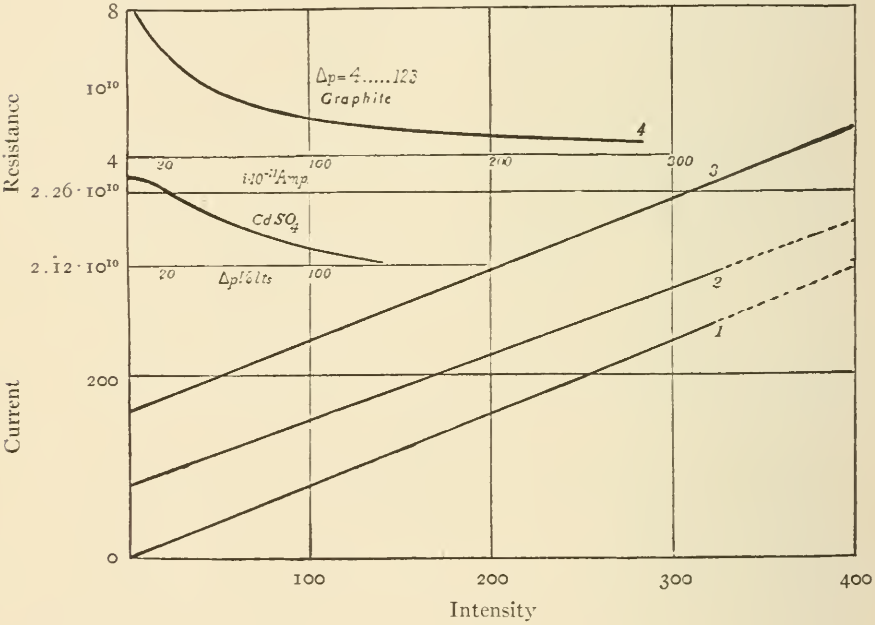


FIG. 5

Two other methods have been used for the measurement of the current, namely, that of the accumulated charge and that of the rate of drift of the electrometer thread. The results coincide with those obtained by means of the resistances. Some of the data are plotted in curves 1, 2, and 3 of Fig. 5. Curves 1 and 3 refer to the accumulated charge, and 2 to the drift. In these data a slight deviation from the straight line can still be observed. The method of accumulated charge is very convenient; if the leakage is well eliminated, then the photo-electric cell has the same accumulating properties as the photographic plate. The results obtained with this method show that the quantity of electricity generated is directly proportional to the quantity of incident light; this latter is proportional to the quantity of light absorbed, and therefore the quantity of electricity evolved is proportional to the quantity of light absorbed, a photo-electric law analogous to the electrochemical law of Faraday.

For the method of the rate of drift the quadrant electrometer was replaced by a string electrometer of high sensitiveness, in order to eliminate the inertia of the needle. In Fig. 4*a* let *C* represent the capacity of the photo-electric cell, *E* the string electrometer and the connecting wires. We shall choose the following initial conditions:

$$\text{for } t=0\text{:}\quad v=0;\quad i=0$$

$$\text{at } t=t\text{:}\quad V=V;\quad i=i$$

$$i=C\frac{dV}{dt}=\frac{V_1-V}{R_1}$$

Where R_1 is the resistance of the system, essentially the resistance of the photo-electric cell, V the potential of the string in the moment t, i the current.

The solution is as follows:

$$v=V_1\left(1-e^{-\frac{t}{CR_1}}\right).$$

For small values of t, we have:

$$V=V_1\left\{1-\left(1-\frac{t}{CR_1}+\tfrac{1}{2}\frac{t^2}{C^2R_1^2}\;.\;.\;.\;.\;\right)\right\}.$$

$$V=V_1\frac{t}{CR_1}\;;$$

thus the potential of the string rises proportional to the time for small values of t and large values of C and R_1. In order to get a uniform motion of the string across the field of the microscope, one should choose large capacities and large resistances; however, if these quantities are large the sensitiveness decreases, because the current

$$i=\frac{V_1-V}{R_1}=\frac{V_1}{R_1}\left(1-\frac{t}{CR_1}\right)=\frac{V_1}{CR_1^2}(CR_1-t)$$

becomes small.

We might also attempt the following arrangement: Let us attach another high resistance R_2 (for instance, another photo-electric cell) in the point P connected to the ground. The current i from the photo-electric cell splits into two parts, i_1 and i_2, i_2 being the charging current of the string between the knife-edges. We should find the following relations:

at $t=0$, we shall assume $V=0$; $i=0$

at $t=t$ $V=V$; $i=i$

$$i=\frac{V_1-V}{R_1};\quad i_1+i_2=i;\quad i_1=\frac{V}{R_2};\quad i_2=\frac{dq}{dt}=C\frac{dV}{dt}$$

$$C\frac{dV}{dt}=\frac{V_1}{R_1}-V\frac{R_1+R_2}{R_1R_2}$$

$$V=\frac{V_1R_2}{R_1+R_2}\left(1-e^{-\frac{R_1+R_2}{CR_1R_2}t}\right);$$

finally we get

$$V=\frac{V_1R_2}{R_1+R_2};\qquad i=\frac{V_1}{R_1+R_2}=i_1\qquad i_2=0.$$

The first arrangement, especially, is much used in stellar photometry by J. Stebbins and other astronomers.

Reviewing the results so far reported, we see that the curves obtained with the galvanometer show a tendency toward saturation; that is, the current increases more slowly than the illumination. The curves obtained with the electrometer show a slight deviation from the straight line in the opposite direction. In these measurements smaller intensities and a smaller opening for the beam of light have been used than in the previous experiments. The former effect, occurring with higher intensities of illumination, may be explained by negative charges being accumulated on the free glass surface; by volume charges between the anode and the glass wall of the cell, and, perhaps, by reflection of electrons. The opposite effect, where the current increases a little more quickly than the illumination, is not so easily explained, provided one assumes a linear relation for the primary phenomenon. Of course, surface charges and reflection of electrons might bring about very complicated effects.

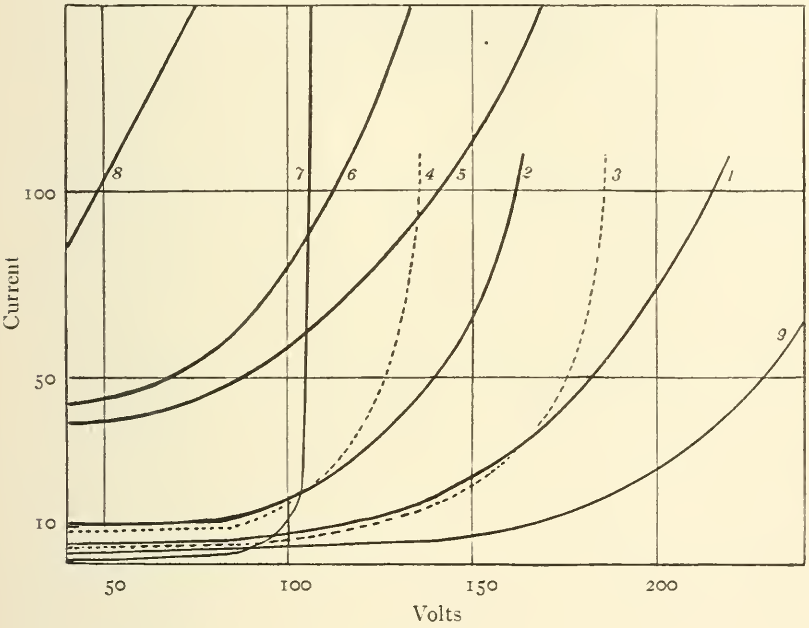


Fig. 6

Before entering upon the final arrangement of the experiment it is useful to study the characteristic curves of the cells, that is, the connection between the current and the varying potential-difference applied for a constant illumination. A very large number of such curves have been plotted. They show always the same characteristic features as will be seen from Fig. 6. The illumination is that of 2 candles at a distance of 50 cm. The curves 1 and 3 correspond to rubidium cells of the type so far used, filled at first with hydrogen. After the formation the hydrogen was pumped out and replaced by argon. Then the curves 2 instead of 1, and 4 instead of 3, are obtained. Argon in each case raises the sensitiveness and decreases the critical value of the potential, at which the glow sets in. The curves 5 and 6 correspond to a potassium-argon cell of the same type, but of larger size, 5.8 cm in diameter. Curve 9 represents a quartz cell of the same type, in which the dark current is vanishingly small, even in comparison with the small currents which occur in stellar photometry. Curve 7 is obtained by means of the Ives cell, which gave a straight line; and curve 8 is the characteristic curve of the very sensitive cell which was used for the final measurements. In this case the curve was nearly straight.

All of these cells show practically the same features, yet they are of very different value for accurate photometric measurements. When the illumination changes over a wide interval, the potential-difference across the cell has to be kept constant. Now, it is very easy to keep the tension constant if the characteristic curve is nearly horizontal, but very hard if the slope becomes considerable. At the same time high sensitiveness is desirable in order to work with as small potential-difference as possible. For these reasons the cell of curve 5 satisfies the theoretical requirements better than the other curves. The Ives cell has a rather extreme characteristic curve. Not more than 100 volts could safely be applied. When the cells are newly formed and sealed off, the color of the sensitive layer changes more or less, and the sensitiveness falls off a little, but in the course of time the cells become constant and have been in use over six months without a noticeable change.

For the final measurements it seemed necessary to use a tube in which the electrons move in a uniform field, and where absorbed surface and volume charges are mostly eliminated. The tube is of the form indicated in Fig. 7. The cathode *K* and the anode *A* consist of aluminum disks 6 cm in diameter; the anode has a central opening for the beam of light, covered with a very fine wire net. The cathode *K* can be moved toward the anode by means of a magnet acting on a piece of iron within the tube. The cathode remains always parallel to the anode, being guided by a glass tube which fits exactly in the outer tube. Platinum cylinders *B* protect the anode, which is connected with the electrometer, from dark currents. The alkali metal is distilled on the cathode from a side tube *S;* the whole tube is at first heated and evacuated by the aid of a mercury pump, liquid air, and charcoal.

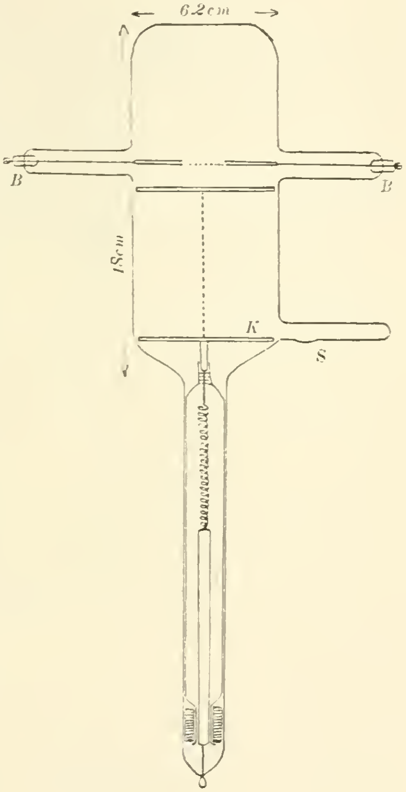


FIG. 7

Fig. 8 represents the final arrangement of the apparatus. Two electrometers are used: *E* for the measurement of the potential-difference Δp between the terminals of the graphite or cadmium sulphate resistance; the other electrometer E_2 only indicates the constancy of the electromotive force between the cathode and the anode of the photo-electric tube for all illuminations. When the current through the cell and the graphite resistance increases, the potential-difference applied partly appears at the terminals of the resistance and decreases, therefore, through the photo-electric cell. This change amounted to 1 volt for 120 volts, and was compensated by means of an additional storage cell and a variable resistance R_1. This compensation was carried out for each reading. When the reading was taken, the electrometer E_2 was disconnected from the rest by means of the key *k*. The current through the light source *L* was kept as constant as possible. *L* was a tungsten lamp with a short straight filament. The intensity could be varied from 0.1 to 110 candles. The distance between the source and the sensitive cathode was chosen between 30 and 300 cm, giving for each experiment a variation of illumination from 1 to 100.

The graphite resistance was occasionally replaced by the cadmium-sulphate resistance Cd. The solution was 1/1000 normal, the electrodes were cadmium. The capillary ended in two long tubes, one of which was filled with the solution while the other end was empty, hydrostatic equilibrium being established during about a day. The electrometer was repeatedly calibrated by means of a new Weston potentiometer. For each position of the lamp two readings were taken and for each series the lamp was moved from 300 cm to 30 cm from the cathode and back again to 300 cm. The readings for the same position coincided almost exactly within less than $\frac{1}{2}$ per cent. Table VII gives the results of three series for a potassium cell without gas. *C* means candle-power.

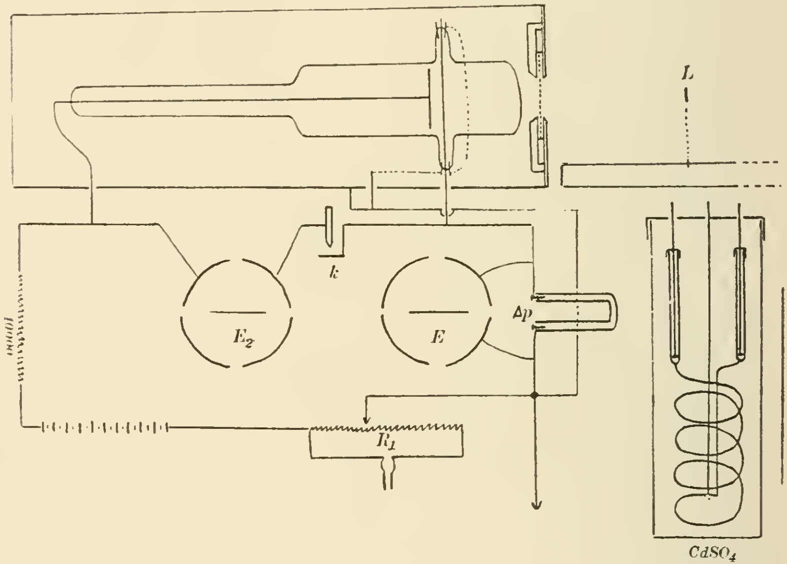


Fig. 8

The ratio between the illumination *I* and the current *d* remains constant within 1 per cent for variation of relative light-intensities from 4 to 400.

If, however, greater intensities were chosen, with increasing intensity the ratio *I*/*d* began to decrease slightly and for a source of 100 candles the deviation from a straight line was very marked, as will be seen from Table VIII.

A probable explanation of the retarded increase of the current with increasing illumination can be sought in the reflection of electrons from the anode, or in the space charge between the electrodes. If this were the case we should expect that the presence of argon in the tube would decrease the influence of reflection. We shall, in fact, see that up to the highest intensities that have been used, i.e., a source of 100 candles, there is no deviation from the straight line if the tube contains argon. For a deflection of 140 mm we find the following distribution of the applied potential-difference: $123 = \Delta p = \Delta p_1 + \Delta p_2$; i = current, Δp_1 = potential-difference across the cell, Δp_2 = potential-difference across the graphite resistance, R_1 = resistance of the photo-electric

TABLE VII

I	*C*=1		*C*=2		*C*=4	
	d	*I/d*	*d*	*I/d*	*d*	*I/d*
4	3.23	124	3.13	128	3.0	134
9	7.25	124	7.05	128	6.78	133
16	12.9	124	12.5	128	12.0	133
36	29.01	124	28.2	128	27.15	133
100	80.9	124	78.1	128	75.0	133
144	118.6	123	113.3	128	108.5	133
178	145.0	123	140.0	127	132.9	134
225	182.0	124	176.0	128	168.9	133
294	238.0	123	232.8	127	222.0	133
352	....	....	277.5	127	263.0	134
400	324.0	124	313.7	128	299.0	134

TABLE VIII

I	*d*	*I/d*	*I*	*d*	*I/d*
4	4.07	98.5	178	141.7	126
9	9.1	99.0	225	173.6	130
16	15.55	103.0	294	224.5	131
36	33.6	107.0	352	261.7	134
100	84.7	118.0	400	294.5	136

cell, R_2=resistance of the graphite. To a deflection of 140 mm corresponds a potential-difference $\Delta p_2=0.499=0.5$ volts.

$$R_2=8\cdot10^{10} \text{ ohms}; \quad i=\frac{p_2}{R_2}=6.2\cdot10^{-12}$$

$$\Delta p_1=122.5; \quad R_1=\frac{\Delta p_1}{i}=1.98\cdot10^{13}; \quad \frac{R_1}{R_2}=250.$$

These high graphite resistances are not constant above 1 volt, as is illustrated by curve 4 of Fig. 5, where the resistance decreases from 8 to $4.3\cdot10^{10}$ as the voltage rises from 4 to 123 volts. The cadmium sulphate resistance also decreases with increasing voltage, but not so strongly, for instance, when the e.m.f. changes from 2 to 128 volts, the resistance only decreased from 17.2 to 16.5. Below 1 volt both resistances are constant. The carbon resistance is also constant when it is of smaller order of magnitude. In some cases a carbon resistance as low as $1.44\cdot10^7$ ohms was used and then it was perfectly constant. It is therefore advisable to use as small a resistance as possible and an electrometer of high sensitiveness.

Tables IX, X, and XI contain the measurements made with the previous tube, with parallel electrodes, filled with argon.

$C=0.6$; $CdSO_4$ resistance

The last series was taken with a galvanometer. Within 1 per cent the ratio between the illumination and the photo-electric current is constant for an interval of intensities from 1 to 100, and for light-sources from 0.16 to 100 candle-powers. For a tungsten lamp of 100 candles there may be a slight increase of the ratio I/d for the highest intensities, or 1110 candle-meters. The smallest intensity in this series was 0.018 candle-meter. As already mentioned, readings were taken as the lamp was moving toward and away from the cathode of the cell; often a small decrease of the deflection in a given position was observed, when the source of light was receding from the photo-electric cell.

On the basis of this result we can establish a logical connection between the photo-electric, the electro-chemical, and the photochemical effects. Let us consider the following arrangement. The anode of a photo-electric cell is earthed. The cathode is connected with the anode of a voltameter, for instance a silver nitrate voltameter, the cathode of which is grounded. If light falls on the cell, the current will deposit silver according to Faraday's law. The quantity of silver deposited in a given interval of time will be proportional to the quantity of light absorbed by the photo-electric cell. But this statement is a special case of the

TABLE IX

I	*d*	*I/d*	*I*	*d*	*I/d*
4	10.02	399	36	90.3	399
9	22.6	399	100	250.6	399
16	40.1	399			

TABLE X

I	*C*=0.16		*C*=3.5		*C*=28	
	d	*I/d*	*d*	*I/d*	*d*	*I/d*
4	3.1	129	2.7	148	2.8	143
9	6.92	130	6.04	149	6.29	143
16	12.2	131	10.7	150	11.25	142
36	27.4	131	24.2	149	25.3	142
100	76.8	130	66.9	150	70.5	142
144	110.0	131	97.3	148	101.0	143
178	136.4	131	119.5	149	125.7	142
225	171.6	130	152.0	148	158.5	142
294	227.8	129	199.4	148	206.0	142
352	271.0	130	237.8	148	245.1	143
400	310.6	129	270.9	148	278.9	143

TABLE XI

I	*C*=100		*C*=34	
	d	*I/d*	*d*	*I/d*
4	3.07	130	2.31	174
9	6.92	130	5.12	175
16	12.3	130	9.15	175
36	27.7	130	20.7	174
100	77.1	130	57.7	174
144	111.0	130	83.0	174
178	137.0	130	102.8	174
225	173.0	130	129.9	174
294	224.0	131	169.8	174
352	267.0	132	203.0	174
400	304.0	132	230.5	174

fundamental law of photo-chemical action. If, therefore, light produces a chemical effect directly, we have, as it were, a combination of the photo-electric and the electro-chemical processes. But whether ionization and electric conduction can be detected in each photo-chemical reaction seems to be an open question. If in the photo-electric process $it = CIt$, and if in the electro-chemical process $M_{ag} = a_{ag} it$, then it follows for the photo-chemical reaction

$$M_{ag} = a_{ag} CIt.$$

SUMMARY

The photo-electric current is not proportional to the intensity of illumination in the older spherical form of the cell. If, however, the cell is mostly covered with a metallic layer and the anode has the form of a circular wire net, the deviation from the straight line is very small so that this cell can be used for many photometric purposes, for instance, in stellar photometry, in plant physiology, for the measurements of reflecting power and of coefficients of absorption and transmission; the Ives cell, at least for small illumination, gives a straight line. In the cell used in the final measurements, two electrodes are parallel, the field is uniform, and the current is proportional to the illumination over a very wide range if the cell contains argon, over a smaller range if the cell is evacuated. Talbot's law holds for the photo-electric phenomenon. A simple relation between the photo-electric, the electro-chemical, and the photo-chemical processes has been pointed out.

This study is being continued in several directions. The very strong, the very weak, illumination, and the action of ultra-violet light require further investigations. The Faraday cylinder method also will be used.

I am very glad to acknowledge my indebtedness to Director E. P. Hyde of the Nela Research Laboratory, where this investigation was carried out, and to his staff, for all the facilities placed at my disposal, for their helpful suggestions, and for the very good time spent in an excellent company of human scientists.

University of Illinois
Laboratory of Physics
January 1917

HIND'S VARIABLE NEBULA N.G.C. 1555[1]

BY FRANCIS G. PEASE

Hind's variable nebula N.G.C. 1555[2] lies close to the irregular variable star T Tauri. Since December 1911 seven negatives of this object have been made from time to time with the 60-inch reflector. The length of the exposures, which were on Seed 27 plates, has usually been about three hours.

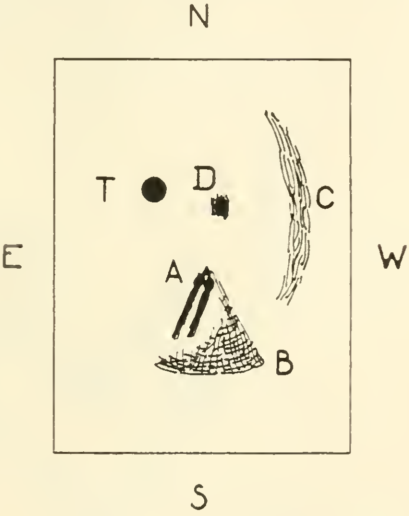


FIG. 1.—T Tauri and Hind's variable nebula.

The most prominent feature is a fan-shaped bit of nebulosity, whose apex lies 220°, 25″ southwest of T Tauri and points a little to the west of the variable (Fig. 1). Two knots at *A*, each with a streamer running to the south, are the brightest parts of the nebula. The sides of the fan include an angle of 70° and are about one minute of arc in length. The nebulosity at *B* is never strong, but on some plates shows distinct detail. A curved streamer lying to the west of *T* is also relatively faint and at times only portions of it show. A knot appears at *D* and varies in size and intensity. A nebulous wing of irregular intensity extends from the variable star about 7″ to the south. There is evidence of very faint extended nebulosity filling the whole starless region about T Tauri. Such details as can be seen on the photographs are roughly sketched in

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 127.

[2] Position for 1860: $\alpha=4^h\ 13^m\ 48^s$, $\delta=+19^\circ\ 11'.2$. N.G.C. 1554, which precedes No. 1555 by 15^s and is $0'.2$ south, is sometimes referred to as Hind's variable nebula, probably because of the note on p. 225 of the *First Index Catalogue*.

Fig. 2. The luminosity at *E*, *F*, *G*, and *H* is strongest on plate No. 55, although it is also visible in varying degrees on the other plates of the series. The regions I and K appear dark on the plates.

Below is a brief description of the appearance of the nebula on each plate. The brightness of T Tauri is about the same on all the photographs excepting Nos. 180 and 181, upon which it is about a magnitude fainter. Although both the star and the nebula increased in brightness during the interval separating Plates 181 and 217, the data now available do not establish a parallelism between the fluctuation of the two objects.

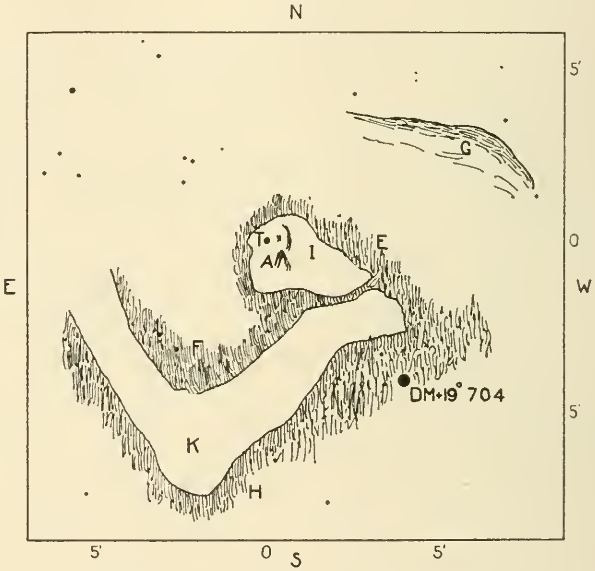


Fig. 2.—Faint nebulosity surrounding T Tauri and Hind's variable nebula

Plate No. 55, 1911, December 25, exposure 3^h 39^m. Nebula diffuse in outline.

T strong.

A knots strong; wings to south medium.

B and *C* very faint.

D medium intensity.

Plate No. 180, 1913, January 8, exposure 20^m.
T faint; has wing on south side, no other detail visible.

Plate No. 181, 1913, February 3, exposure 2^h 30^m. Nebula fan-shaped; contrast stronger than No. 55.
T faint; has strong wing on south side.
A knots stronger than in No. 55; wings to south medium.
B very faint.
C and *D* missing.

Plate No. 217, 1913, September 28, exposure 3^h. Nebula fan-shaped; sharp detail and strong contrast; the nebula is appreciably stronger in intensity than on No. 181.
T same as No. 55.
A knots about same as in No. 181; wings to south medium and longer.
B shows detail; medium intensity.
C faint.
D trace.

Plate No. 226, 1913, October 27, exposure 3^h 30^m. Nebula fan-shaped; detail softer.
T strong; shows wing on south side.
A knots blend with wings; northern half strong.
B west edge only visible; medium intensity.
C very faint.
D faint.

Plate No. 230, 1913, November 25, exposure 3^h. Nebula fan-shaped; same as No. 226.
T strong; wing on south side stronger than in No. 226.
A weaker than No. 226; point strongest; left wing strong, right weak; cross-bar halfway down.
B details; medium intensity.
C missing.
D trace.

Plate No. 280, 1916, November 27, exposure 3^h. Whole region embracing *A*, *T*, and *C* filled with diffuse nebulosity.
T strong.
A diffuse knot, appreciably weaker than on No. 226.

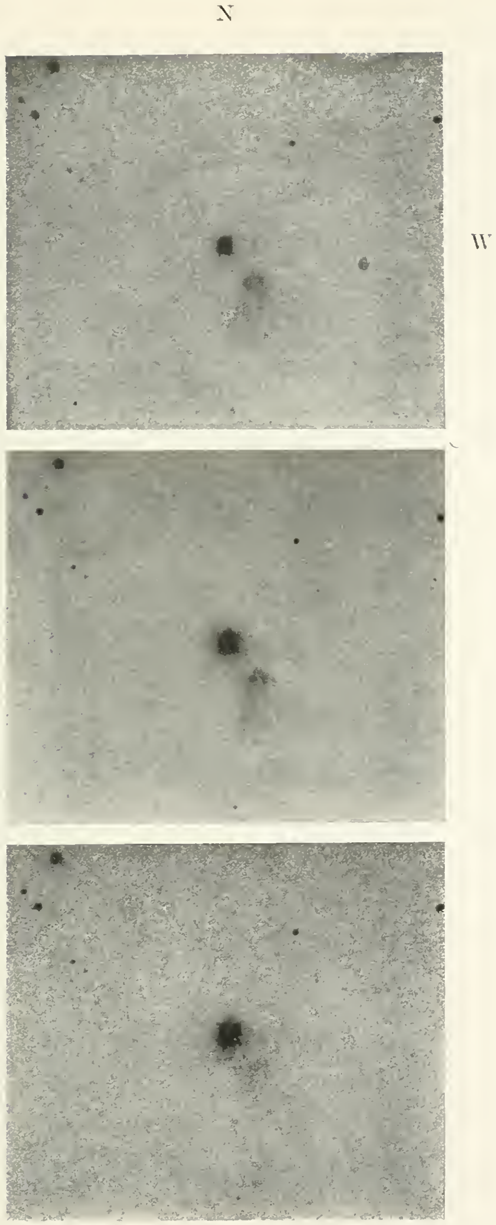
Mr. Adams finds that the spectrum of T Tauri is of type Md with additional bright lines, and that the parallax is of the order of

0".05 to 0".10. Since the bright lines of the spectrum appear to extend beyond the dark lines, they are evidently due to the surrounding nebulosity.

In Plate II are shown enlargements of photographs Nos. 181, 217, and 280. The enlargement and contrast necessary for purposes of reproduction have emphasized the grain of the photographs, but the changes in form and intensity of the nebula are clearly visible.

Mount Wilson Solar Observatory
January 1917

PLATE II



No. 181
1913, Feb. 3

No. 217
1913, Sept. 28

No. 280
1916, Nov. 27

Hind's Variable Nebula N.G.C. 1555

Exposures about 3^h with the 60-inch reflector
Enlargement 4.5, Scale 1 mm = 6″.1

SOME SPECTRA IN THE PHOTOGRAPHIC INFRA-RED

By CHARLES F. MEYER

In the summer of 1912 the author, while working at Bonn, in Professor Kayser's laboratory, photographed the spectra of a number of elements in the infra-red. The photographs were made at the suggestion of Dr. Keivin Burns.

The spectroscopy of the infra-red has not been worked up with anything like the degree of completeness which is desirable. It is important that all spectra should be investigated photographically to as great a wave-length as possible, for both the degree of accuracy which can be attained and the number of lines which can be observed are greater than by other methods. Perhaps the most complete work in this field has been done by Lehmann,[1] but his wave-lengths are well known to be very inaccurate and this makes his tables almost valueless. Many others have photographed in the infra-red, but the total amount of work which has been done is small compared to that which should be done. As a rule the wave-lengths do not go as far out as they might go, and those of many lines, even of many prominent ones, differ by several angstroms as determined by different investigators. Since the photographs made by the author were obtained, some work has been published by Lorenser,[2] and some by Eder,[3] and in the present paper these investigations will serve especially for comparison.

The spectra photographed were those of salts of lithium, potassium, calcium, barium, and strontium, in the carbon arc. For the final exposures, chlorides were always used, though other salts were occasionally tried. Some photographs of the mercury arc spectrum were also taken, but they yielded nothing worthy of comment.

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[1] *Annalen der Physik*, 5, 633, 1901; 8, 643, 1902.

[2] *Inaugural Dissertation*, Tübingen, 1913.

[3] *Sitzungsberichte d. k. Akad. der Wiss.*, Wien, 123, 1914.

A Rowland concave grating of one meter focus, and 16,000 lines to the inch, was used to analyze the spectra. The slit was placed in such a position that the region 6400 to 9600 A, of the first order, fell into the camera. The second-order iron spectrum served as comparison. While the exposure for the infra-red lines was being made, a red glass screen was placed in front of the slit, in order to eliminate the second-order lines of the spectrum which was being examined. This screen proved to be absolutely opaque to the second order, even for very long exposures. A transmission-curve of the glass, kindly made for me by Dr. Pfund, showed it to be opaque to 5800 A. At this point transmission begins, and rises gradually to about 50 per cent at 9000 A. The camera-box was made for the use of films, and accordingly these were used. They were dyed by a method which was a modification of Wallace's by the addition of dicyanin.

The width of the slit used was about a tenth of a millimeter, and the length of the exposure varied from 15 minutes to 3 hours. The great width of the slit and length of exposure interfered materially with the definition, and, consequently, with the accuracy which could be attained in measuring. The spectra were not photographed with a view to obtaining the greatest possible accuracy of wave-length, and it was decided to be content with fair accuracy. The values given by Lorenser, and by Eder, are, on the whole, better than those in this work, though there is good reason to doubt whether Eder's wave-lengths are throughout as accurate as might be supposed.

The measures were made at the Detroit Observatory, where Professor Hussey was so good as to place a measuring engine at the author's disposal. The measurement of the spectra has been delayed, as the author did not have access to an engine and other equipment for some time after making the photographs.

The wave-lengths are on the international system, being based upon Burns's table of the iron spectrum. Wave-lengths in the Rowland system are about 0.3 A higher than those on the international, in this region of the spectrum, and this should be borne in mind in making comparisons in the tables which follow.

The intensities are given on a scale of 1 to 10. Where the intensities of lines observed only by others are given, they have been roughly adapted to this scale.

All films were examined for the presence of the so-called Lyman ghosts.[1] By determining the positions of the ghosts of certain strong lines in the green and red, the positions which would be occupied by the ghosts of all strong lines could be determined, and it is felt that all such ghosts have been identified and eliminated.

LITHIUM AND POTASSIUM

(Impurities: Na, K, Ca, Rb, in the Li spectrum;
Na, Li, Ca, Ba, Rb, in the K spectrum)

The spectra of lithium and potassium showed no new lines or bands. In the lithium spectrum the lines 6708 and 8127 come out strongly, and impurities show to 8540, so that even fairly faint lines should show between 8000 and 9000 if they existed. The wave-length of the 6708 line is accurately known, and the line was not measured. The wave-lengths obtained by the author and others[2] for the 8127 line are given in Table I.

TABLE I

LITHIUM

International System		Int.	Remarks	Rowland's System		
Meyer	Eder			Paschen	Lewis	Saunders
8126.6	8126.27	10	Broad	8127.1	8126.8	8127.0

Eder's is the only previous value based on international standards. My value falls more nearly into line with the older values, based on Rowland's standards, when the correction of 0.3 A is applied.

The potassium spectrum in this region consists of two strong pairs, the longer pair reversed, and, in addition, one rather faint line. Impurities show on my films only to 8200, but are well seen out to this point. We cannot feel quite confident in this case that any lines below 9000 would have been found. The wave-lengths are given in Table II.

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[1] *Proceedings of the American Academy of Arts and Sciences*, 36, 1901; 39, 1903.

[2] All wave-lengths given for comparison will be found in Kayser's *Handbuch der Spectroscopie*, 5, 6; except those due to Lorenser and Eder, which are given in the papers already referred to.

TABLE II

POTASSIUM

International System		Int.	Remarks	Rowland's System	
Meyer	Eder			Kayser and Runge	Saunders
7698.9	7699.02	9	Reversed	7699.3	7699.08
7664.8	7664.95	10	"	7665.6	7664.91
6965.3	....	2	....	....	6966.3
6939.4	6939.07	6	Broad	6938.8	6939.5
6911.9	6911.31	5	"	6911.2	6911.8

My values for the second pair seem to be too high. The faint line has been missed by most investigators; possibly it is an impurity, but there is a line called for in about this position in the second subordinate series.

CALCIUM

(Impurities: Na ?, K)

Calcium chloride produces in the spectrum, besides the metallic calcium lines, also bands which are characteristic of the salt itself. The table of metallic lines follows (Table III). The first three lines are very faint. All efforts to explain them as spurious failed, however, so they have been retained.

Lorenser tabulates, from 7984 to 7468, eleven lines. He used in the arc at times metallic calcium and at times calcium chloride. These lines were not found either by Eder, using metallic calcium, or by me, using chloride. Why Eder did not observe the lines is not entirely clear, as their intensities range from 1 to 6 on the foregoing scale. On my films; part of the region is obscured by a band, and the remainder is not entirely free from continuous background. The strongest of these eleven lines (7610.66) is present on two of my films, but with nothing like the relative intensity assigned by Lorenser, and I had attributed it to barium, though probably falsely. There is also a faint suggestion of the line which is next, both in intensity and position, namely 7602.78. Lorenser noted that a group of five of these eleven lines, including the two just mentioned, did not appear in the vacuum arc. The group is characterized by being unsymmetrical to the red.

TABLE III

CALCIUM

International System		Int.	Rowland's System
Meyer	Eder		Lorenser
9166.2	....	1	....
9122.0	....	1	....
9103.0	....	1	....
8662.0	....	3	8662.50
8541.8	8542.25	5	8542.47
8497.8	8498.11	3	8498.35
8153.1	....	2	8153.13
......	....	....	?8077.8
7995.0	....	2	7995.31
			7984 to 7468 Eleven lines
7326.0	7326.10	9	7326.43
7202.1	7202.15	8	7202.51
7147.9	7148.14	10	7148.49

Three chloride bands appear on my films. They show very distinctly, and a number of the component lines are sharp enough to measure. The bands run to longer wave-lengths, and shade off in that direction also. The first and last are simple, that is, they have but one head. The one of intermediate wave-length is more difficult of interpretation. There are probably two heads, one at 8166 and one at 8153, about coincident with the metallic line given in the table above. So far as I know, bands have not been photographed in this region of the spectrum before. The existence of these bands was noted by Becquerel, using the phosphor-photographic method.

BARIUM

(Impurities: K, Ca)

The barium spectrum has been measured by Lehmann, Hermann, and Lorenser, basing their wave-lengths on the Rowland system. Lehmann's values are poor. Burns, Werner, George, and recently Eder, have measured the spectrum in this region on the international system, but their tables are less complete. My films show evidence of a band at about 8000, but neither the lines nor the head of the band are distinct.

TABLE IV

Calcium Chloride Bands

8855.7	8354.4	7942.4
8849.3	8349.8	7937.5
8843.3	8344.7	....
8838.3	8338.8	7908.0
8833.1	....	7903.2
....	8329.8	7899.0
....	8325.4	7894.6
8773.3	8320.8	....
8769.2	8316.1	7885.8
8763.9	8311.4	7881.7
8759.7	8307.1	....
8755.4	8302.8	7873.8
8751.7	8298.7	....
8747.9	8294.5	....
....	8290.6	7850.7
8739.1	8286.4	7846.1
8735.6	8282.4	....
8732.4	8278.1	7822.0
8715.7	8273.4	7818.5
8713.0	8268.2	7814.8
8710.1	....	....
8706.7	8257.5	7807.6
8702.6	8253.9	....
8698.6	8248.9	7770.2
....	8245.7	7766.1
....	8242.1	7762.1
8651.0*	....	....
	....	7720.3
	8231.9	....
	8228.5	7716.8*
	....	
	....	
	8166. *	
	8153. *	

* Head.

STRONTIUM

(Impurities: K, Ca)

The strontium spectrum has no lines in the photographic infra-red, except for two at the visible limit. Eder claims to have found two new lines, but these are undoubtedly the sodium pair,

TABLE V

INTERNATIONAL SYSTEM		INT.	ROWLAND'S SYSTEM	
Meyer	Eder		Hermann	Lorenser
8914. *	....	2	8915.19	8915.40
8860.5	....	1	8861.40	8861.32
8799. *	....	1	8799.86	....
....	....	1	....	8659.65
8653.6	....	2	8654.33	8654.40
8581.6	....	2	....	8582.66
....	....	1	....	8570.34
....	....	1	....	8569.60
8567.1	....	1	....	8568.08
8559.7	8559.98	5	8560.20	8560.21
....	....	1	....	8542.72
8521.9	....	1	....	....
8513.7	....	1	....	8514.50
8414.2	....	2	....	....
8350.5	....	1	....	....
....	....	3	....	8329.17
8325.2	....	2	....	....
....	....	1	....	8288.41
8284.8	....	2	....	....
8262.8	....	1	....	8264.30
8254.3	....	1	....	....
8224.0	....	1	....	....
8210.2	8210.33	4	8210.73	8210.63
8161.9	....	1	....	8162.09
8158.1	....	1	....	8158.56
8147.6	....	2	8148.32	8148.14
8120.5	....	2	8120.84	8120.88
8018.0	....	1	....	8018.64
7982.4	....	2	....	7982.75
7961.4	....	1	7961.23	7961.47
7957.6	....	1	....	7957.61
7939.3	....	1	7939.21	7939.79
7911.5	7911.35	7	7911.53	7911.67
7905.8	7905.77	7	7906.13	7906.12
7878.2	....	2	7878.13	7878.34
....	....	1	....	7865.51
....	....	1	....	7863.74
7839.8	7839.56	3	7839.57	7839.82
....	....	2	....	7829.25
7798.5	....	1	....	7798.65
....	....	1	....	?7783.84
7780.5	7780.55	7	7780.70	7780.77
....	....	3	....	7775.74
....	....	1	....	?7770.55
7766.8	7766.81	1	....	7767.19
7751.6	7751.74	5	7751.92	7752.02
7721.8	....	3	....	7722.13
....	....	....	7709.96	....
7706.6	7706.59	5	7706.82	7706.88
7672.0	7672.10	10	7672.42	7672.48
7668.2	....	1	....	....
....	....	3	....	7662.31

*Estimated

TABLE V—*Continued*

International System		Int.	Rowland's System	
Meyer	Eder		Hermann	Lorenser
7642.9	7642.88	6	7643.42	7643.31
7636.9	7636.89	6	7637.47	7637.29
....	....	1	....	7616.83
7610.5	7610.46	6	7610.74	7610.83
....	....	2	....	7575.49
7543.2	....	1	....	7543.88
7527.8	....	1	....	7528.60
7523.1	....	1	....	7523.93

present as an impurity (see below). If we subtract 0.22 and 0.23 respectively from the values given by Eder for the strontium pair (8195.14 and 8183.58), we get his values for the sodium pair, and, moreover, the character and relative intensities of the lines are the same in both cases. My films show impurities to 8660, but they show no trace of these lines, and Lorenser did not find them, either.

TABLE VI

Strontium

International System		Int.	Remarks	Rowland's System
Meyer	Eder			Lorenser
....	8195.14	3	Impurity (Na)	....
....	8183.58	3	“	....
7673.0	7673.07	3	....	7673.38
7621.5	7621.55	3	....	7621.76

There are some chloride bands present in the infra-red. The one of longest wave-length has a fairly well-marked head at 8258, but the individual lines are not distinct enough to measure. They run to longer wave-lengths and shade off in that direction also. The next is a band group, apparently. Lines are distinguishable by the aid of a lens between 8200 and 7900, but only a few are distinct enough to measure. The lines which were measured, and the heads, are given in Table VII.

SODIUM AND RUBIDIUM AS IMPURITIES

The wave-lengths of the infra-red sodium pair were in doubt by several angstroms until they were recently measured by Eder as reversed lines. I did not photograph the sodium spectrum as such, but this pair appeared quite plainly in my lithium and potassium spectra, but not reversed. The recent values, and the better of the older ones, are given in Table VIII.

TABLE VII

STRONTIUM CHLORIDE BANDS

....	Lines not distinguishable
......	
8258	Head (measured)
8044.2	Line
8040.3	“
8036.5	“
8032.5	“
8028.7	“
8024.6	“
......	
......	
7902	Head (estimated)
7883	“ (measured)
7874	“ (estimated)
7866	“ “
7860	“ “

TABLE VIII

SODIUM

International System		Int.	Rowland's System	
Meyer	Eder		Saunders	Hermann
8195.1	8194.92	10	8196.1	8195.33
8183.4	8183.35	7	8184.5	8183.74

The two strong rubidium lines also appeared in my lithium and potassium spectra, and were measured (Table IX).

In conclusion, I wish to thank Dr. Burns for suggesting this work and for the help he has given me during its progress. I also wish to thank Professors Kayser, Hussey, and Curtiss for the kindness they have shown in placing facilities at my disposal.

TABLE IX

RUBIDIUM

INTERNATIONAL SYSTEM		INT.	ROWLAND'S SYSTEM	
Meyer	Eder		Eder and Valenta	Saunders
7947.5	7947.63	5	7947.7	7947.6
7800.0	7800.30	10	7800.3	7800.2

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DEPENDENCE OF THE SOLAR APEX UPON PROPER MOTION AND CAUSE OF THE DIFFERENT POSITIONS OF THE APEX YIELDED BY RADIAL VELOCITIES AND PROPER MOTIONS

By C. D. PERRINE

In continuation of the investigations detailed in *Astrophysical Journal*, **43**, 286, 1916, which showed an apparent dependence of the position of the solar apex upon the sizes and parallactic signs of the proper motions of the stars used in the determinations (from radial velocities), apices have now been derived from proper motions of the same stars. The results from proper motions confirm satisfactorily the dependence found from the radial velocities.

The results both from radial velocities and from proper motions are given in Table I. On account of the small number of stars in the divisions of large and medium-sized proper motions of the classes A, F, and G and the uncertainty of positions determined from so few proper motions, these three classes were combined. It was necessary to reject a very few of the largest proper motions, when they came in such small groups that to have included them would have been to produce a very abnormal effect upon the result obtained. In the present case only the most extreme cases were rejected—9 out of a total of 341 stars in the three groups where rejections were made. Even large values were not rejected if there were sufficient stars in the group reasonably to reduce the effect. Such a course is unavoidable where data are limited, and is justified, in my opinion, upon the ground that the result is more representative than if such observations had been included. An examination of the solutions shows that the conclusions arrived at are in no way affected by these rejections.

The solutions were made by the method of least squares, with the use of the following formulae:

$$-X\sin\alpha + Y\cos\alpha = \mu\alpha$$

$$-X\cos\alpha\sin\delta - Y\sin\alpha\sin\delta + Z\cos\delta = \mu\delta$$

$$\tan A = \frac{Y}{X}$$

$$\tan D = \frac{Z}{(X^2+Y^2)^{\frac{1}{2}}}.$$

Right ascensions and declinations were combined. The solutions were made as in previous work of this kind, i.e., by regions of 2^h in right ascension by 30° in declination. Probable errors were not determined. The comparatively small number of observations in many of the solutions and the nature of the data seemed not to warrant it in the present preliminary work. The reality of the phenomena must rest upon other considerations.

TABLE I

APEX FROM RADIAL VELOCITIES AND FROM PROPER MOTIONS

CLASS	RADIAL VELOCITIES					PROPER MOTIONS		P.M.—R.V.
	A	*D*	$V_\odot$	Stars	Regions	*A*	*D*	*D*
			km					
2.9 and brighter.	258°.0	+41°.5	−18.9	110	33	258°.8	+24°.6	−16°.9
B.	276.0	+29.6	−20.3	193	31	272.8	+32.4	+ 2.8
A, F, G								
Large.	268.1	+ 8.6	−26.5	141	40	261.3	+22.1	+13.5
Medium.	269.0	+ 7.0	−23.7	81	29	238.5	+50.8	+43.8
Small.	254.7	+35.0	−17.8	277	47	263.5	+47.4	+12.4
K								
Large.	288.7	+18.8	−27.9	85	31	255.9	+13.0	− 5.8
Medium.	250.4	+15.1	−16.7	85	33	258.5	+42.9	+27.8
Small.	260.6	+36.8	−21.3	220	45	276.4	+69.2	+32.4
Mean.	265.7	+24.0	. . . .	. . . .	. . . .	260.7	+37.8	+13.8
Adams' 500 Radial Velocities:								
Large.	276.9	+ 3.1	−11.7	47	18	279.2	+38.4	+35.3
Medium.	244.2	+41.6	−21.9	32	12	249.6	+50.3	+ 8.7
Small.	272.0	+31.6	−15.9	349	41	266.4	+67.7	+36.1

The stars of W. W. Campbell's catalogue[1] were separated into two classes according to magnitude—those of 2.9 and brighter and those of 3.0 and fainter. Owing to the few stars of magnitude 2.9 and brighter, all spectral classes of these stars were combined. This is objectionable, but in the present state of our knowledge it seemed desirable to separate the very brightest stars.

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[1] *Lick Observatory Bulletins*, 6, 108, 1911; 7, 20, 51, 113, 1913; *Publications of Lick Observatory*, 9, 329, 1911.

Solutions were also made from the 500 stars of W. S. Adams' list[1] of radial velocities, the results of which are included in Table I. These stars also show the dependence of position of the apex upon size of proper motion.

It should be remembered in this connection, that Adams' list contains no stars south of −26° and only 94 south of the equator. Forty-seven stars are in Campbell's catalogues and were, therefore, omitted in the solutions from Adams' data.

The stars on Adams' list are all relatively faint, most of them being between magnitudes 5.5 and 6.5.

The original object of these investigations was to find the cause of the discordance of about 10° which Campbell[2] noted between the declination of the solar apex as derived from his radial velocities and the position obtained by Lewis Boss from proper motions, a discordance which is confirmed by the radial-velocity apex of Hough and Halm[3] and by many other investigations of proper motions. This discordance is discussed to some extent by Eddington.[4]

The finding of a relation to the size of $\mu\alpha$ was unexpected. The results of the present investigation bear upon this discordance in the apex of the solar motion, as well as upon the phenomena of dependence upon size of proper motion, and both will be considered together.

RELATION BETWEEN POSITION OF SOLAR APEX AND PROPER MOTION IN RIGHT ASCENSION

The classifications of the A, F, and G stars, the K stars, and Adams' list show, without a single exception, much smaller declinations for the apex from large than from small $\mu\alpha$, and no greater discordances in the intermediate values than might be expected from the data. This conclusion is equally true of the results from proper motion. This discordance of position appears to be chiefly in declination. The data do not appear at present to be sufficient to justify a conclusion as to difference in right ascension, further than to say that there are smaller variations of right ascension which appear to be systematic and that so large a discordance as that in *D* is not likely to be wholly in a single (apparently) arbitrary co-ordinate. Until more is known as to the nature of this discordance and related phenomena it does not seem advisable to attempt quantitative determinations.

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[1] *Mt. Wilson Contr.*, No. 105.

[2] *Lick Observatory Bulletin*, 6, 128, 1911.

[3] *Monthly Notices*, 70, 85, 1909.

[4] *Stellar Movements*, p. 74.

TABLE II

Comparison of Apex from Radial Velocities (Campbell) and Proper Motions (Lewis Boss)

Spectral Class	Radial Velocities				Proper Motions				P.M.−R.V.	
	A	*D*	$V_\odot$	Number of Stars	*A*	*D*	μ	Number of Stars	ΔA	ΔD
			km							
B.	276°0	+29°6	−20.3	193	274°4	+34°9	2″40	490	−1°6	+ 5°3
A.	260.9	+15.3	−18.3	185	270.0	+28.3	4.56	1647	+9.1	+13.0
F.	267.9	+11.1	−18.9	184	265.9	+28.7	7.71	656	−2.0	+17.6
G.	257.4	+20.2	−16.4	131	259.3	+42.3	5.24	444	+1.9	+22.1
K.	274.2	+25.6	−20.7	391	275.4	+40.3	5.74	1227	+1.2	+14.7
M.	269.7	+31.7	−22.9	65	273.6	+38.8	4.99	222	+3.9	+ 7.1
All classes: 2.9 and brighter. .	258.0	+41.5	−18.9	110	. . . . .	. . . . . .	. . . .	. . . . . .	. . . . .	. . . . . .
Mean.	. . . . .	+25.0	. . . . . .	. . . . . .	. . . . .	+35.6	. . . .	. . . . . .	. . . . .	+13.3

DIFFERENCES BETWEEN THE POSITIONS OF THE SOLAR APEX DERIVED FROM RADIAL VELOCITIES AND FROM PROPER MOTIONS, AND THEIR CAUSE

There are also clearly shown in the results in Table I (stars of magnitude 3.0 and fainter), differences between the declinations of the apex derived from proper motions and from radial velocities, those from proper motions being to the north of the apices from radial velocities. There is only one exception in ten groups. That one is the large $\mu\alpha$ group of the K stars, the contradiction being slight and the nature of the data such as to render their evidence of small weight in the face of all the other. It is conceivable that to some extent at least this difference of apex may be greatest among the stars of small proper motion.

There is some indication that the size of the discordance increases with spectral type, at least among the stars of small $\mu\alpha$. The evidence from Campbell's stars is very consistent and Adams' list is not necessarily contradictory. This matter is, however, confused with the question of differences between the northern and southern stars and increase with decreasing brightness.

Of great interest in this connection is a comparison[1] of these positions of the apices derived from the radial velocities of 1149 stars of magnitude 3.0 and fainter of Campbell's lists by spectral class with the apices from proper motions, also by spectral class, of 4686 stars by Lewis Boss,[2] which is given in Table II. We here find not only more northerly declinations for the apices from all six spectral classes, but a systematic curve in these varying in a general way like the *A*, *D*, $V_\odot$, and μ from both radial velocities and proper motions.

TABLE III

PROPER MOTIONS. SEPARATE SOLUTIONS FOR NORTHERN AND FOR SOUTHERN STARS

SPECTRAL CLASS	NORTHERN					SOUTHERN					ΔD
	A	*D*	*X*	*Y*	*Z*	*A*	*D*	*X*	*Y*	*Z*	N.−S.
All classes: 2.9 and brighter.......	260°0	+26°0	+27.3	+155.3	− 77.1	255°4	+21°6	+ 26.6	+101.8	− 41.6	+ 4°4
B................	281.5	+46.4	− 3.3	+ 16.3	− 17.4	269.4	+26.2	+ 0.4	+ 34.5	− 17.0	+20.2
A, F, G											
Large...........	258.0	+35.6	+48.	+228.	−167.	263.7	+11.2	+ 39.	+356.	− 71.	+24.4
Medium.........	256.3	+36.8	+ 6.3	+ 25.8	− 19.9	227.9	(+56.6)	+ 29.8	+ 32.9	− 67.3	(−19.8)
Small...........	262.2	+59.4	+ 2.4	+ 17.2	− 29.4	264.5	+40.1	+ 3.2	+ 33.6	− 28.5	+19.3
K											
Large...........	255.4	+17.9	+46.	+178.	− 59.	256.9	+ 3.9	+ 31.	+131.	− 9.	+14.0
Medium.........	269.8	+46.4	+ 0.2	+ 58.9	− 61.8	245.5	+35.9	+ 14.6	+ 31.9	− 25.4	+10.5
Small...........	267.6	+72.4	+ 0.5	+ 12.2	− 38.8	282.2	+66.2	− 3.2	+ 14.7	− 34.1	+ 6.2
Mean.............	.....	+42.6	.....	....	....	......	+32.7	....	....	....	+ 9.6
						Omitting ()	+29.3	....	....	....	+13.8
Adams' 500 Radial Velocities:											
Large...........	279.8	+33.6	−56.	+323.	−218.	275.8	+35.9	− 63.	+626.	−455.	− 2.3
Medium.........	281.9	+35.4	− 8.1	+ 38.2	− 27.7	202.8	+18.7	+115.2	+ 48.5	− 42.2	+16.7
Small...........	279.4	+70.7	− 1.2	+ 7.2	− 20.9	230.9	+50.9	+ 6.0	+ 7.4	− 11.7	+19.8
Parallax observed:											
0″06 and over....	253.7	+47.9	+97.	+332.	−383.	258.0	+24.9	+186.	+871.	−414.	+23.0

The asymmetry found in the proper motions of the B stars, together with other peculiarities observed, prompted the making of separate solutions from the northern and southern stars, the results of which are given in Table III. These results show clearly the decrease of *D* with increase of $\mu\alpha$ and in addition, in all but two cases,[1] *an appreciably larger D from the northern than from the southern stars.* Taking the differences between the general means of the north and south D, including those of Adams' list, we find

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[1] *Astrophysical Journal*, 44, 114, 1916.

[2] *Ibid.*, 26, 187, 1911.

$$\text{Northern}-\text{Southern}=+11^\circ.3.$$

Omitting Adams' list because of the lack of southern stars, and rejecting the large discordant southern D, we have

$$\text{Northern}-\text{Southern}=+13^\circ.3.$$

These values are reasonably close to +9°.1, that between the general solutions from radial velocities and from proper motions; and to +10°.6, between similar simple means of D derived from radial velocities and proper motions according to spectral class, 1259 radial velocities, and 4686 proper motions.

The difference between the simple mean of D from the radial velocities and proper motions of the *same 1194 stars* (by spectral class) is +13°.8. This indicates that the differences in D between northern and southern stars from proper motions is fundamental and exists in the brighter stars (1194) as well as in the fainter stars in Boss's solution, and that the discordance in D between the radial velocities and proper motions is not due to the 3492 fainter stars.

It is now of interest to examine the positions of the apex derived from the northern and southern radial velocities separately, as given in Table IV.

With the exception of the K and M stars of magnitude 3.0 and fainter, the individual values of D from both northern and southern regions agree quite well, and their means to the nearest degree. The difference for these K and M stars is so large and so consistent that it is difficult to believe that it is not to some extent real, when we consider the considerable number of stars in the K type.

In comparing the mean D from the radial velocities of the 1194 stars with those from the proper motions of the same stars, *the significant thing appears to be that the southern stars give closely*

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[1] An examination of the data in the A, F, G medium (southern) group showed several abnormal stars. Rejecting four of these brought the D slightly below that from the northern stars.

the same D from both, whereas for the northern stars the apex from the proper motions averages 20° to the north of that from the radial velocities.

TABLE IV

APEX AS DERIVED FROM RADIAL VELOCITIES OF NORTHERN AND OF SOUTHERN STARS

CLASS	NORTHERN STARS			SOUTHERN STARS		
	A	*D*	Number of Stars	*A*	*D*	Number of Stars
All classes: 2.9 and brighter	251°	+44°	50	258°	+40°	60
B	254	+27	70	288	+31	123
A	258	+20	97	264	+11	88
F	271	+ 9	79	264	+13	105
G	267	+19	64	249	+23	67
K	273	+ 8	151	268	+36	240
M	269	+15	24	272	+41	41
Mean	263	+20	....	266	+28	....
Mean omitting M	262	+21	....	265	+26	....
Mean omitting K and M	260	+24	....	265	+24	....

These values of *D* from the 1194 stars of the spectral classes B, A, F, G, and K are given below. Class M was omitted for the present because of its relatively few stars.

	DECLINATION OF APEX FROM	
	Northern Stars	Southern Stars
Proper motions	+43°	+29°
Radial velocities	+21	+26
P.M.−R.V.	+22	+ 3

What the declinations of the apex are for the *two regions separately* from the proper motions of the 4686 stars is not known. It may be noted, however, that the average *D* of the six spectral classes of Table II is +35°.6, as against +34°.3 obtained by Lewis Boss from a general solution from all of the stars, and that the average *D* from the proper motions of the 1194 stars of Table III (omitting one) is +36°.0. There is no evidence here that the same peculiarity in the proper motions in the two hemispheres does *not* extend to the 3492 stars as well.

If we examine the rectangular co-ordinates resulting from the solutions (Table III) we find that *nine out of the entire twelve values* *of Y and all of the values from the small $\mu\alpha$ are smaller for the northern than for the southern stars.* In other words, this indicates a smaller average *parallactic* displacement for the northern than for the southern stars. This difference decreases from stars of class B, where it is large, to the K stars, where it is slight.

If now, to illustrate, we take the results from the B stars, and *use the value of Y from the southern stars* to determine the D for the *northern* stars, we shall obtain essentially the same *D for both northern and southern stars*, a value which agrees closely with the D obtained from the radial velocities of the same stars and with that from radial velocities of all of the spectral classes. This will be true in principle also for the other cases.

As to the interpretation to be put upon this asymmetry in the proper motions, attention may be called to two other facts which bear strongly, viz.: (*a*) the solar velocity deduced from the radial velocities of the northern stars is consistently less than from the southern stars; (*b*) the proportion of stars with contrary parallactic $\mu\alpha$ is greater for northern than for southern stars.

The most reasonable interpretation of the three distinct sets of appearances above would seem to be that the parallactic asymmetry is real and that the stars in question of the northern sky are moving in general with the sun.

These conclusions are of course tentative, for any matter as obscure as the general motions of the stars, closely associated as they are with the motion of the observer, neither of which is accurately known either in direction or in amount, must of necessity require a most thorough investigation from all possible points of view and with more data, particularly for the fainter stars, than are now available. I am aware that the peculiarities discussed in this paper are susceptible of other interpretations. Taken altogether, however, it seems to me that a very simple and probable explanation of all of these observed phenomena is suggested by the forms and appearances of a class of celestial bodies which is by no means rare. I have no hesitancy, therefore, in expressing the belief that the evidence which seems very direct is sufficient to justify the assumption as a *working hypothesis* that the general motions of our stellar system are either circular or spiral.

Other peculiarities have been observed, particularly in the distribution of the residual radial velocities, which appear to be most significant and will, as soon as a fairly trustworthy general idea of the system is obtained, aid materially in improving our knowledge of the details. Fuller discussion of many points connected with the questions raised in this paper is desirable, but it seems to me that it should await other related investigations and a general advance of the whole subject.

The foregoing explanation of the cause of the difference between the positions of the solar apex from radial velocities and from proper motions indicates that the radial velocities yield the most representative position on the whole, and that until the peculiarities observed in the proper motions, particularly in the northern sky, have been thoroughly investigated, positions depending upon them alone must be adopted with especial caution.

Great care is necessary as yet in considering any facts relating to the solar motion, for all of the results so far obtained appear to be influenced by the underlying motions of the stars themselves and more than ever to emphasize its purely relative character.

CONCLUSIONS

I. The position of the apex of solar motion depends upon the proper motions in right ascension of the stars used. The differences appear to be greatest in declination, the stars with large proper motion yielding apices south of those from the stars with small proper motion. This effect is shown in the results both from proper motions and from radial velocities.

II. The differences in *D* of the apices of solar motion as derived by other investigators from radial velocities and from proper motions are consistent and they appear in general to be greater for the stars of late type than for those of early type. This discordance appears to arise chiefly from the proper motions of the northern stars and to be satisfactorily explained by the assumption that the parallactic displacement of the stars is systematically less in the northern sky.

Observatorio Nacional Argentino
Córdoba
October 1, 1916

THE CAUSE OF THE SO-CALLED POLE-EFFECT IN THE ELECTRIC ARC

BY T. ROYDS

Differences of vapor-density were first suggested in *Kodaikanal Observatory Bulletin*, No. 38, as the cause of the displacements of certain lines in different parts and conditions of the electric arc, and of the abnormal sun-minus-arc displacements of the same lines. Since, however, direct experimental proof is wanting, and has been said to give negative results, it seems desirable to discuss the evidence and experiments at the point at which work here on the subject has to be abandoned.

The cause of the displacements in the electric arc has also been treated by St. John and Babcock,[1] Gale and Whitney,[2] and Whitney,[3] none of whom discusses the evidence and conclusion in *Kodaikanal Observatory Bulletin*, Nos. 38 and 40.[4]

In the last two papers on experiments with a calcium arc, the pole displacement is ascribed to the greater amplitude of vibration of the electrons, and is said to depend on the intensity-gradient along the arc. The latter phrase is unfortunate, as, so far as I understand them, the authors do not mean the rate of change of intensity, but intensity-differences.

It must be obvious to every experimenter that the intensity of lines is great in those regions of the arc where displacement occurs; but, as it is equally true of lines which do not undergo displacement and of those which are displaced to the red and to the violet, one fails to see how the displacement can be said to depend on the intensity-differences. One might with equal or more truth say that the displacement depends on the width of the spectral lines, or on their diffuseness, but for reasons which have already been elaborated,[1] I believe that the displacement depends on the unsymmetrical character of the spectral lines. I have not met with a single case where lines whose character was known were not displaced, either not at all, to the red, or to the violet, according as they were symmetrical, unsymmetrically widened toward the red, or unsymmetrically widened toward the violet, *except under those conditions, e.g., in reversals, where the vapor-density has been kept low.* Of course these phenomena—unsymmetrical character, intensity, etc.—are not the *cause* of the displacement, but are attendant effects due probably to the same cause.

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[1] *Astrophysical Journal*, **42**, 231, 1915.

[2] *Ibid.*, **43**, 161, 1916.

[3] *Ibid.*, **44**, 65, 1916.

[4] Both these bulletins appeared in 1914.

Increased amplitude of vibration of the electrons is suggested by Gale and Whitney[2] as the cause of the displacement in the electric arc, but it is easy to see that this cannot be. The most effective and probably the only certain way known to me of increasing the amplitude of vibration of the electrons in the atom is to raise the temperature, but the displacements in the arc are not an effect of temperature, for many reasons, among which the three following seem sufficient.

1. Little is known of the variation of temperature along the arc, but it is certain that the positive pole is much hotter than the negative, whereas under normal conditions the displacement is greatest near the latter. The enhanced lines, which are high-temperature lines, appear stronger at the positive pole than at the negative,[3] also indicating that the temperature is higher there than at the negative pole.

2. The experiments described in *Kodaikanal Observatory Bulletin*, No. 40, and here, show that the displacement at the negative pole can be varied to any desired extent without reason for believing that the temperature of the arc is altered in any appreciable degree.

3. In the sun's reversing layer, where the temperature exceeds that attainable in the arc, the displacement of lines unsymmetrical in the arc is in the direction *opposite* to that of the displacement at the poles of the arc.

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[1] Royds, *Kodaikanal Observatory Bulletin*, Nos. 38, 40.

[2] *Loc. cit.*

[3] A. Fowler, *Monthly Notices*, 67, 154, 1907.

Although the evidence given in *Kodaikanal Observatory Bulletin*, Nos. 38 and 40, is strongly in favor of density as the cause of the displacements, there are many difficulties in the way of direct experimental proof, due primarily to the difficulty of controlling the vapor-density in a source of light. Experiments with different quantities of material, such as those giving Gale and Whitney's Tables III and IV, fail; or at any rate are inconclusive, because there is no reason to believe that the atoms have been separated a greater distance with the smaller amount of material. If the atoms are vaporized in clusters, they may not be removed from each other's influence any more than when a larger amount has been used. Exposure-times are not a sufficient test of vapor-density, but only an indication of the total amount of material consumed.

On account of this difficulty it was thought better to use alloys as electrodes. Presumably the molecules in an alloy are so intimately mixed with another metal that each would be surrounded by molecules of another kind, and would be removed from the influence of those of the same kind. Even so, the experiments gave negative results. The best alloys available were the coins of the Indian coinage, the silver coins containing 10 per cent of copper, and the nickel coins containing 20 per cent. As the silver coins, and the money they represent, melt away rapidly and do not give a steady arc, the experiments were conducted mostly with nickel coins (value one anna). With a nickel coin as one electrode, and the other another coin, iron, or carbon, the wave-lengths and displacements of the three copper lines λλ 4480, 4509, and 4531 were compared with those in the arc between copper electrodes with the same length of arc and current-strength. The maximum displacement of the first and last was about +0.05 A and of the second about +0.025 A. The wave-length at the center of the alloy arc was identical with that at the center of the pure copper arc, but it was found that the wave-length at the negative pole could be varied at will by varying the material of the negative electrode. With carbon as negative electrode and the nickel coin or copper as the positive, the displacement of the copper lines at the negative electrode could be made very small, especially with those conditions when the green luminosity surrounding the positive electrode did not reach up to the negative which showed the characteristic blue of the carbon arc. When the nickel coin is negative, and the positive pole a coin, iron, or carbon, there is on the other hand not the slightest difficulty in obtaining displacements at the negative quite as large as those at the negative pole of the arc between two copper electrodes.

The results with the alloy were therefore, in the main, disappointing, especially the fact that the wave-length at the center of the alloy arc was identical with that at the center of the pure metal arc. There is, however, one case, the sodium pair λλ 5682, 5688, where it is possible to obtain a displacement in the same direction as that in the sun and opposite to that usual at the negative pole. The data are given in *Kodaikanal Obersvatory Bulletin*, No. 40. The solar displacement of these lines, comparing the center of the sun's disk with the center of a very long arc, is −0.14 A (i.e., to the violet), the displacement of the unreversed line at the negative pole is +0.36 A, while the displacement of the *reversal* which occurs at the negative pole is −0.019 A. The sodium pair is very sensitive to displacement, and is only a case more extreme than many others, found in the bulletin referred to, for which the displacement at the negative pole is much smaller if the line undergoes reversal there than if the line remains unreversed. A new example of this has turned up in the first subordinate triplet of calcium near λ 4450. If the lines are reversed at the negative pole, the displacement is quite small or zero[1] and the lines appear almost, if not quite, symmetrical.[2] If, however, the lines are obtained unreversed at the negative pole, the displacement amounts to about −0.012 A and the unsymmetrical widening toward the violet is evident.

I have not met with cases such as that recorded by Whitney, where the displacement of the reversal was identical with that of the unreversed lines (it is not so for these lines on my photographs), but there is nothing impossible in it on the density hypothesis.

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[1] Royds, *Kodaikanal Observatory Bulletin*, No. 40.

[2] Royds, *Astrophysical Journal*, **41**,154, 1915, and *Kodaikanal Oberatory Bulletin*, No. 43.

The only way in which the results with the alloy and pure metal can be reconciled with the density hypothesis is to suppose that the density-differences effective in producing the displacements are of a much higher order than those obtained using the alloy containing 20 per cent of the metal investigated. It would seem that the atoms influence each other only soon after they are torn off from the electrode, as if they occur there in compact clusters which are soon dissipated so that when they reach the center of the arc the atoms are removed out of each other's influence. In the sun the density is supposed to be so small that a farther separation and displacement takes place. If the density-effect is due to the mutual electrical fields of the atoms, it is conceivable that fields of atoms of a different kind would also have an influence, thus explaining the considerable displacement at the poles of the alloy arc compared with the pure metal arc.

The considerations of the last paragraph would also explain the negative results of St. John and Babcock, but it cannot be conceded without further information that increasing the quantity vaporized increases the vapor-density in the furnace in the same ratio. One would have thought that the greater the quantity of material vaporized the greater would be the rate of its removal by condensation on the cooler parts of the tube.

The really interesting result of Gale and Whitney's and of Whitney's experiments is that they have, apparently, succeeded in obtaining arc conditions which bring the normal displacement at the negative pole of the arc down to zero, and even, for the more sensitive lines in the direction opposite to the usual one, i.e., in the same direction as the sun-minus-arc displacement.

I agree with Duffield's remarks on the influence of density- and temperature-gradients in light-scources on the displacement of spectral lines,[1] but I should like to make clear that the gradients cannot have any influence unless density and temperature are themselves causes of displacement.

Although direct experimental proof has not been obtained, I cannot find any hypothesis other than density to explain the displacement of certain spectral lines in different parts and conditions of the arc and the abnormal sun-minus-arc displacement of the same lines which have been discussed in *Kodaikanal Observatory Bulletin*, Nos. 38 and 40. Whether some additional conditions are necessary, or whether the density-differences effective in displacing lines are larger than those hitherto attempted, are points for further experiment. It is hoped that a source of light may be constructed where the vapor-density can be varied over a large range when it is possible to resume these experiments.

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[1] W. G. Duffield, *Philosophical Magazine*, 30, 385, 1915.

Kodaikanal Observatory
December 2, 1916

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

By HARLOW SHAPLEY

FIRST PART: THE GENERAL PROBLEM OF CLUSTERS[1]

An extensive investigation of the magnitudes and colors of individual stars in globular and open clusters has been undertaken at the Mount Wilson Observatory. The discussion of the results will be published in the form of a series of papers, each article dealing with a definite problem, or with the observations on a single cluster. The first communication, of which the present note is an abstract, is intended to serve as a brief introduction, reviewing our knowledge of cluster systems and treating of the value and general bearing of the investigation of clusters on the problems of stellar astronomy.

1. *The classification and distribution of clusters.*—A stellar cluster is generally defined as any stellar group, the members of which are known to be physically connected, or which from their apparent positions may be assumed to form a distinct physical system. This broad category includes the Hercules cluster as well as the Ursa Major group (or even a system of groups such as our whole galactic domain). Various classifications have been proposed, but, because of the great variety of types and the gradual transition from one type to the next, none is much more than a working convenience. Clusters have been classed as loose, irregular, ragged, coarse, open, furrowed, globular, condensed, nebulous, etc., with many qualifying adjectives for each group, but practically all designations are based on superficial appearances.

It is proposed to adopt in the present series of papers the following working classification: (1) globular clusters; (2) open clusters; (3) moving clusters.

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[1] Abstract of *Contributions from the Mount Wilson Solar Observatory*, No. 115, dated August 1915.

Although they are not mutually exclusive, there are some very good reasons which will be brought out in the later discussion for holding rather strictly to these three classes. The first class has for its type the Hercules cluster (M 13) or Messier 3, and for its distinguishing marks great condensation toward the center and numberless stars. The second contains clusters similar to Messier 11 or Messier 67 or even Praesepe, thus exhibiting a wide range in condensation and in number of stars. The third division includes the Taurus group, the Pleiades—in fact, any cluster of stars showing communal motion.

The adopted classification has on its surface the same lack of definite limits as other proposals; but in distribution we can discriminate sharply between open and globular clusters, and in probable distance we can temporarily, at least, differentiate moving groups from other open clusters. The apparent distribution in space is so striking that, taken in connection with other characteristics just coming to light, we are led to believe that there is a fundamental difference between the two main classes—the condensed globular objects and the open clusters.

Among others, Bailey, Bohlin, Hinks, Perrine, and more recently Melotte have discussed the distribution of clusters with respect to the galaxy. The most conspicuous result is that the open clusters are almost exclusively in the Milky Way and symmetrically arranged with respect to it, while the globular clusters are widely distributed in galactic latitude and quite limited in galactic longitude. The globular clusters are mostly in one hemisphere, the pole of which, according to Melotte, is at galactic latitude -9°, galactic longitude 296°. It should be noted, moreover, that no other class of celestial objects shows so marked a dissymmetrical arrangement with respect to the galactic plane.

2. *Previous observational and theoretical studies.*—Practically all the detailed observational work on individual clusters, with the exception of the investigations of cluster variables, relates to counts of the stars and to determinations of positions. No serious attempts have been made to determine accurate magnitudes, chiefly because of the lack of dependable magnitude scales for the fainter stars.

The more important investigations of clusters can be very briefly summarized. The most valuable contributions have been made by Bailey and by von Zeipel. The former has accumulated vast amounts of important information relative to cluster variables (practically all we know of these objects, in fact), as well as valuable data on classification, distribution of stars in clusters, and of clusters in the sky. In the study of Messier 3, von Zeipel has published the largest of all cluster catalogues, and has made investigations of considerable importance on the distribution of the brighter stars in several clusters, and on the possible physical relationships of the individual stars. E. C. Pickering, H. C. Plummer, See, Eddington, and Strömgren and Drachmann have contributed to the theoretical interpretation of the arrangement of stars in clusters.

The spectra of the brighter stars in a number of the brighter, very open clusters were classified by Mrs. Fleming. Of the spectra of individual stars in the fainter open clusters and in globular clusters, nothing is known aside from the work of Pease and Adams, who have classified provisionally 40 spectra in Messier 13. All spectral classes from A to K were recorded, with an average of type F, agreeing with the work of Fath and of Slipher on integrated spectra of various globular clusters.

It has been frequently suggested that certain clusters are relatively yellow, others white. But the first definite observation of color was made qualitatively by Barnard, who compared photographic magnitudes with visual and photo-visual estimates for a number of stars in the Hercules cluster.

3. *Correlation between diameters and brightness in clusters.*—From Bailey's *Catalogue of Bright Clusters and Nebulae*, which may be considered homogeneous over the field it covers, we obtain the data in Table I relative to mean diameters of globular clusters (outside the Magellanic Clouds) and the magnitudes of the brightest stars in each.

The striking result that magnitude increases with decreasing apparent diameter is to be expected if distance is correlated with apparent size. It suggests at once the possibility of deriving relative mean parallaxes for clusters from measures either of diameters or of magnitudes.

4. *The purpose of the present study of clusters.*—The object in taking up the investigation of the magnitudes and colors in stellar clusters is twofold. First, it is hoped that considerable advance can be made in our understanding of the internal arrangement and physical characteristics of these objects. Secondly, and probably of more importance, it is hoped that through the acquisition of a clearer appreciation of stellar clusters, particularly of globular systems, some contribution can be made to the knowledge of our own galactic system. For it is quite obvious that a globular cluster, whether considered as entirely extraneous to our galactic domain or not, is in itself a stellar system on a great scale—a stellar unit which without doubt must be comparable to our own galactic system in many ways, though possibly differing from it fundamentally in others.

TABLE I

Brightest Magnitude	Number of Clusters	Mean Diameter
9	1	20′
10	0	....
11	1	18
12	7	9.4
13	25	10.0
14	5	4.4
15	3	3.3
16	1	3

A. In the study of clusters, the results which may be derived only from a knowledge of magnitudes and colors are in part as follows: (1) laws of distribution or density for different magnitudes and colors; (2) relation connecting absolute magnitude and color, and its variation from cluster to cluster; (3) total number of stars in the faint globular clusters predicted from observed luminosity-curves; (4) dependence of stellar variability on color and magnitude; (5) change of color with light-variation, and the consequent opportunity of investigating on a large scale the causes underlying Cepheid variation.

B. In the study of the galactic system the advantages of an investigation of cluster magnitudes and colors include the following: (1) elimination of the factor of distance-selection; (2) the possibility of dealing with complete stellar systems—unfortunately not literally true for globular clusters, because of the great number and extreme faintness of the stars; (3) the availability of large numbers for statistical discussion; (4) the possible evaluation of absorption of light in space and of color-factors depending on absolute magnitude.

C. In addition to those suggested above the following general astronomical problems are closely connected with the present study of clusters: (1) frequency of absolute magnitudes and spectra in stellar systems—luminosity-curves; (2) the order of stellar evolution; that is, the probable character of the progression of spectral type (color) with age; (3) the distances of clusters; (4) relation of the globular cluster to the open cluster and to the galaxy.

5. *Remarks on the methods of magnitude determinations.*—The derivation of photographic and photo-visual magnitudes, and hence also of the colors of stars in clusters, has been made possible through two important factors. The first is the 60-inch reflector of the Mount Wilson Observatory; the second and more necessary one is the fundamental work by Seares in establishing photographic and photo-visual magnitude scales for the faint stars.

The 60-inch reflector allows the speed necessary for successful intercomparison of the fields of faint stars, and at the same time the scale, even at the principal focus, permits the study of the stars at the very center of many of the globular clusters.

The magnitudes determined with the reflector from focal images make no claim to high accuracy. And, indeed, for the present at least, high accuracy is not at all requisite. It is sufficient if we obtain color-indices with a probable error of a tenth of a magnitude. The crucial point is not the size of the accidental errors nor the precision of the brightness for any given cluster star, but rather the unimportance of the systematic errors and the availability of dependable magnitude scales over the entire interval of brightness investigated.

SECOND PART: THIRTEEN HUNDRED STARS IN THE HERCULES CLUSTER (MESSIER 13)[1]

1. *Preliminary remarks.*—The first paper of this series contains a brief general summary of previous investigations of stellar clusters and an outline of the object, methods, advantages, and limitations of the proposed determination of magnitudes and colors at the Mount Wilson Observatory. The second communication, which is abstracted in the following pages, treats of the results for the Hercules cluster. This well-known system was considered first for several reasons: (1) its position in the sky when observations were begun was favorable for comparison with the Pole; (2) two good catalogues of the positions of the brighter stars were available; (3) only two variable stars had been recorded among the thousand brightest objects—an obvious advantage in deriving colors from non-simultaneous determinations of photographic and photo-visual magnitudes; (4) spectra of about forty of the individual stars were known approximately from direct classification.

No other globular cluster has been studied so frequently and extensively as Messier 13. It was one of the earliest placed on record,[2] being discovered accidentally by Edmund Halley 201 years ago. Messier saw the Hercules cluster as a brilliant circular nebula, but the elder Herschel resolved it partially in 1783, and later completely with the 20-foot telescope. An account of the earlier studies by Messier, D'Arrest, and others is given by Scheiner in his monograph *Der Grosse Sternhaufen im Herkules Messier 13.*

Formerly, in the drawings and verbal descriptions, and later in the early photographs, much attention was paid to the dissymmetrical arrangement of the brighter stars in the cluster and to the possible existence of visible nebulosity. Contributions to these subjects have been made by Secchi, the Earl of Rosse, Trouvelot at Harvard working with Winlock, and Harrington at Ann Arbor assisted by a professional artist. Holden reported 13 different intersections of distinct lanes, Schaeberle interpreted the arrangement as forming a spiral with one arm clockwise, the other counterclockwise, while Scheiner and Barnard have voiced conservative views relative to the structure and nebulosity, respectively. Palmer, Miss Clerke, Perrine, and Ludendorff have also discussed different phases of these matters. The conclusion from all the former studies, which is strengthened by the results from plates made at Mount Wilson, is that no nebulous material can be detected in the Hercules cluster (at least unless it is fainter than the twentieth magnitude), and that the so-called "channels" among the brighter stars, while apparent in some form to the eye and on photographic plates of the proper exposure and contrast, do not appear among the fainter magnitudes and are probably to be considered of no particular importance in the structure of the cluster.

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[1] Abstract of *Contributions from the Mount Wilson Solar Observatory*, No. 116, dated August 1915.

[2] Messier 22 was discovered by Ihle in 1665; ω Centauri was noted as a lucid spot in the sky by Halley in 1677, though it was previously known to Bayer as a hazy star, and even to Ptolemy as the star in the Cloud on the Horse's Back; in 1681 Kirch discovered Messier 11 (not globular, but rich and considerably condensed) and Messier 5 in 1702. All of these earlier discoveries consisted in noting that the objects were nebulous spots rather than single stars. The resolution into clusters awaited in most cases the telescopes of the Herschels.

The first attempt to record the positions of the individual stars with sufficient accuracy to give future observers a trustworthy basis for the study of proper motions was made by Scheiner in 1891–1892. His catalogue contains 833 stars—all that were measurable on two plates with exposures of 2 hours and 1 hour, made with the 32.5 cm refractor of the Potsdam Observatory. A second catalogue of somewhat higher accuracy was made by Ludendorff, and contains 1136 stars. It is based on two plates taken with the same instrument at Potsdam in 1900 and 1902, but with exposures of 2.5 and 2 hours, respectively. Comparison of the positions derived by Scheiner and Ludendorff show no definite proper motion during the interval of ten years. In none of these investigations has serious consideration been paid to the determination of magnitudes.

E. C. Pickering determined the distribution of stars in Messier 13, and some valuable theoretical considerations of his results are contributed by H. C. Plummer. A much more exhaustive consideration of the star-density is made by von Zeipel, who used copies of plates made at the Yerkes Observatory and also, for the outer portions of the cluster, photographs made with the Upsala refractor. About 2500 stars were discussed, but only roughly from the standpoint of brightness. Of the many important results and conclusions derived by von Zeipel for this and other globular clusters only the following need be mentioned at this point:

1. From the ratio of the number of bright stars to faint stars in different regions it is necessary to conclude that the physical state near the center differs from that in the exterior regions.

2. So far as these brighter stars are concerned, the density in Messier 13 shows that the cluster is comparable to a spherical nebula (gaseous sphere) in convective equilibrium conforming to certain adiabatic laws.

3. It is entirely possible that the globular clusters have attained their adiabatic equilibrium solely by virtue of the mutual attraction of the constituent stars.

4. Each cluster is a gigantic system containing approximately a million stars.

This last conclusion is admittedly quite speculative. It is based upon purely theoretical assumptions that now appear to be inadmissible; but it will be shown from other considerations, which have a good observational basis, that the conclusion itself may not be far wrong. Later it will be shown why No. 1 above is a sound result, other evidence being presented to show that conditions may exist in the center of the cluster which are not evident in the outer regions. This same evidence will suggest that near the center, at least, von Zeipel and others are dealing with a distribution which is not real, and that the theoretical results, including Nos. 2 and 3 above, must be modified accordingly.

The integrated spectrum of Messier 13 has been observed by Fath at the Lick Observatory and at Mount Wilson. The work by Pease on the spectra of individual stars in the cluster is discussed in a later section of this paper.

2. *The observational data.*—The magnitudes are derived from the complete measurement of two pairs of plates, made on different nights and each consisting of one isochromatic and one ordinary photographic plate. In order to avoid, in the derivation of color-indices, errors depending on the presence of undetected light-variations in some of the stars, the plates of a pair are always taken

in succession so that only uncommonly rapid light-changes can affect the results. It is believed that errors arising from this source are negligibly infrequent and small. The results are indirectly based on the partial measurement of two other pairs of plates; and two plates containing each a long and a short exposure have been used to determine the magnitudes of fainter stars. Some preliminary plates have been of service in deriving magnitudes of newly discovered variables. The observations were made with full aperture at the principal focus of the 60-inch reflector.

TABLE II

AVERAGE PROBABLE ERROR OF A MAGNITUDE DETERMINATION

	PHOTOGRAPHIC		PHOTO-VISUAL	
	Whole Cluster	Distance > 2′	Whole Cluster	Distance > 2′
Probable error:				
One plate	±0.068	±0.057	±0.046	±0.044
Two plates	±0.048	±0.041	±0.033	±0.031
Corrected for systematic deviations:				
One plate	±0.061	±0.044	±0.040	±0.035
Two plates	±0.043	±0.031	±0.029	±0.025

The plates have been measured chiefly by the method described by Seares in *Mt. Wilson Contr.*, No. 80, Section 6. Supplementing the direct measurement with the graded scale, the writer has found it of great advantage to carry on simultaneously a series of estimates—an independent visual determination of the relative brightness of adjoining images by the Argelander method, using as steps tenths of intervals between scale-images.

In addition to this double measurement of the plates, series of completely independent duplicate measures were made to test for the various suspected systematic errors. Effects of fatigue and other personal-equation errors were detected early and avoided. During measurement the plates are illuminated artificially, so that variations in weather and sky illumination disturb the work little, if at all.

3. *Discussion of errors.*—From 1051 photographic and 244 photo-visual residuals were derived the probable errors summarized in Table II. The average probable error is less than ±0.05

mag. for 95 per cent of the magnitudes used in the statistical discussions. This error includes not only uncertainties of observation in the cluster magnitudes, but also the errors in the polar standards, uncertainties in the distance correction and zero-point, and other plate errors. When the individual residuals are corrected for systematic deviations, thus excluding the plate errors, the probable errors become as in the last two lines of the table.

The average probable error of a tabulated color-index (without correcting for systematic deviations) varies from ±0.082 to ±0.058, and for about 75 per cent of the stars is ±0.067. Excluding stars within 2′ of the center the limits become ±0.072 and ±0.051, and the median value is ±0.060.

The discussion of errors shows for the denser central region of the cluster, not only the expected larger accidental deviations, but also a systematic error apparently inherent in the plates themselves. At first it appeared not serious enough to introduce unallowable errors into the magnitudes and colors. Further investigation, however, and especially the statistical work on color-indices, suggests that a photographic phenomenon analogous to the one discovered and studied by Eberhard may influence the results to a considerable extent. Eberhard found that the intensity of a photographic image depends on the density of neighboring parts of the plate.

Further discussion of this phenomenon in its probable bearing on the magnitudes in the center of the Hercules cluster will be postponed to a later paper. For the present some reasons for suspecting its appearance may be enumerated: (1) the nature of the field photographed and the method of development; (2) the discrepancy between directly observed spectra and color-indices in the center of the cluster; (3) the conspicuous increase of average color-index in the central regions; (4) some laboratory experiments in copying and enlarging photographs of clusters.

The reasons cited for believing that a systematic photographic error may affect the magnitudes within 1′.5 or 2′ of the center, with obvious and slight changes in wording, are also valid reasons for believing that there is no such effect outside the crowded central regions. In addition we have still stronger assurance that the magnitudes of stars more distant than 2′ are reliable in the fact that the Polar Standards, which are very generally distributed on all plates throughout the outer regions of the cluster, show no trace of systematic error depending on distance.

A quantitative correction for an Eberhard effect is not possible, for as yet very little is known about its amount, variation with plate emulsions, or its control. In view of this circumstance it has been considered best to withhold publication of the magnitudes derived for the central stars of the cluster until later study can make them more definite. The data derived from them are retained to some extent, however, in the statistical tables and discussion, for it is through the analysis of this material that the peculiarity of the central region is best revealed.

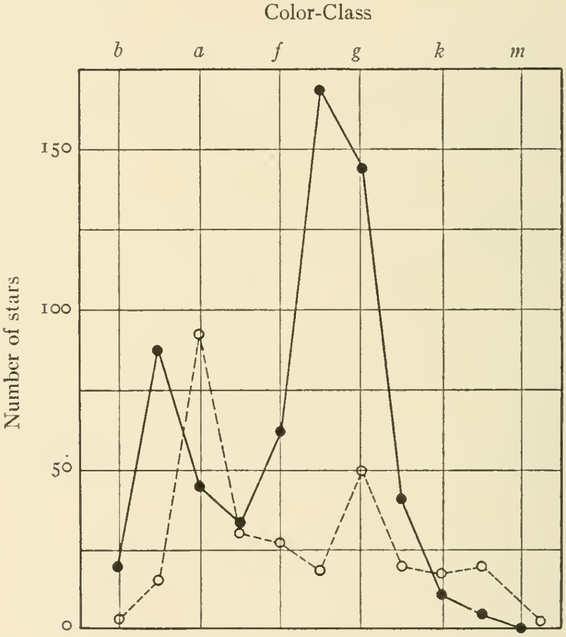

FIG. 1.—Frequency of color-classes in Messier 13 (full line), and in the sky at large

4. *On the absorption of light in space.*—The catalogue of photographic and photo-visual magnitudes contains about 1300 stars. Owing to the probable presence of systematic error in the magni-

tudes in the denser central regions, the lists now published contain only about 650 stars; publication of the remainder awaits further investigation of suspected errors. Probably not more than 18 of the magnitudes refer to superposed non-cluster stars. Discarding incomplete or doubtful results, 1049 magnitudes and color-indices are available for statistical investigation, of which 495 are outside the central region. The range of observed photo-visual magnitudes is from 11.9 to 16.8; but the results are complete only down to magnitude 15.5.

The values of the color-index in the cluster range from -0.52 to $+1.96$, with the greatest frequency about $+0.90$. Excluding the zone within 2′ of the center, the upper limit is $+1.45$, with greatest frequency near $+0.70$. A plot of the frequency of color-class for the outer portion of the cluster is shown in Fig. 1, together with the frequency-curve of spectral types at large, the data for which are taken from Parkhurst's *Yerkes Actinometry*. To plot Parkhurst's data the spectral types were interpreted directly as color-classes, and the numbers given by him were divided by two. Though differing in detail, these curves are similar in two important characters, notwithstanding considerable lack of homogeneity in the Yerkes results as regards distance and absolute brightness—each curve has a maximum for blue and for yellow stars, and the limits of color are practically the same for both.

Probably the most remarkable feature of the distribution of color-classes is not the relative paucity of *a*'s and early *f*'s, nor the great extent of the range in color, but rather the highly significant fact that there are any negative color-indices at all. Of the 495 stars at a greater distance from the center than 1′.9, more than 17 per cent have negative color-indices, and of these one-fifth are bluer than represented by color-class $b5$.

It is well known that a condition of this nature cannot exist if there is appreciable molecular scattering of light in space. It will be shown later that the parallax of the Hercules cluster cannot reasonably be greater than $+0''.0001$, and it may be but one-tenth of that amount. Using the most recent value derived by Kapteyn for the change in color-index due to space absorption, namely, $d=+0.003$ mag. per unit of stellar distance ($\pi=0''.1$), the absorption should be of the order of three magnitudes at least, and the minimum color-index +2.5. Suppose we use the very recent and smaller value, $d=+0.0015$, determined by Van Rhijn from a consideration of the stars included in the *Yerkes Actinometry*. Even then all the color-indices in Messier 13 should exceed a magnitude.

It seems necessary to conclude that in the direction of the Hercules cluster the selective extinction of light in space due to molecular scattering is negligible. If we grant a color-excess of +0.1 mag. in the cluster, due entirely to space absorption, the value of d cannot exceed +0.0001 mag. per unit of distance, an amount entirely inappreciable in dealing with the distances of the ordinary isolated stars.

Whatever the effect of intrinsic luminosity on redness may be, we can safely assume for the present that it is not sufficient to counteract or conceal the absorption of light in space if the previously found values of this quantity are adopted. It is also apparent that, over the small range in apparent magnitude (and, therefore, in absolute magnitude) observed in the cluster, the luminosity factor can have no serious influence on the observed color-index.

The interpretation of color-class directly as spectral type receives a valuable verification through the work by Pease on the spectra of the individual cluster stars, but owing to the provisional nature of the data the check cannot as yet be considered complete. The discussion of the spectra and the relevant color-indices will appear in a separate paper. For the present we only observe that for the four stars with distance greater than 1.5 the excess of observed color over that to be expected from the spectral types is but +0.08 mag. and the error possible in the magnitudes and spectral classification of a single star may easily exceed a tenth of a magnitude.

5. *Miscellaneous results.*—Hardly less striking than the result which indicates the apparent absence of light-scattering in space is the unexpected relation brought to light by plotting magnitude against color for any or all of the regions in the cluster. From the main statistical table the data are collected for the summary in Table III, which gives for successive regions the average photo-

visual magnitude for each color-class. The number of stars involved in each case is in parentheses following the mean magnitude. (The last column of this table shows the central redness in a decisive manner.) The successive lines of Table III show that for all distances from the center the average magnitude is brighter for the redder color-classes than for the blue. Not only are the g, k, and m colors associated in the mean with the brighter stars of the cluster, but the decrease of redness with decreasing brightness is distinctly progressive; in fact, nearly linear in all parts of the cluster. The phenomenon is illustrated graphically in Fig. 2.

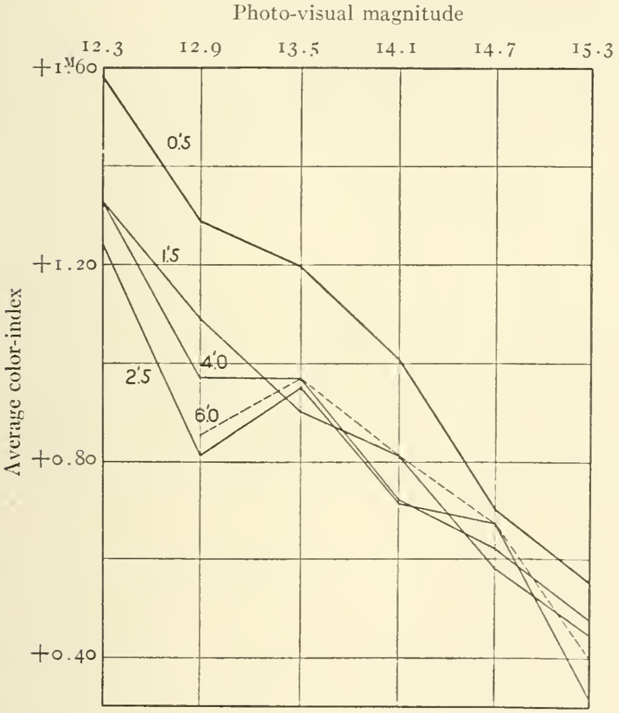

FIG. 2.—Decrease of color-index with decreasing brightness

For open clusters such as Messier 67 and the Pleiades, and for the stars in the sky at large, the relation between apparent or absolute magnitude and color has been found always in the sense of increasing redness with decreasing brightness. It is not necessary to hasten the interpretation of the contrary result obtained for a globular cluster. For the present it will suffice to observe that we are now dealing with the three or four brightest magnitudes in a stellar system in which exists a range of absolute brightness of certainly more than 10 magnitudes. Doubtless we are discussing the giant stars and only the giants among a throng of hundreds of thousands. For the stars in the galactic system Russell has come to the tentative conclusion that the giants are of comparable absolute magnitude whatever their spectral type. The present result suggests that in this globular cluster, at least, the giants are brightest when reddest. The relation between absolute brightness and color is therefore the inverse of that for dwarfs, where, without much doubt, the cooling stars are growing redder and smaller with age.

Of the four hundred faintest stars measured in Messier 13, 85 per cent are bluer than color-class *g*0, which corresponds to solar spectral type; of the four hundred brightest stars 75 per cent are redder than *g*0. An analogous result holds for the outer regions alone. Between photo-visual magnitudes 16.0 and 16.8 only four out of a special list of a hundred faint stars have color-indices greater than half a magnitude, and none is found redder than the sun.

There is some evidence that the brighter stars are more strongly condensed in the center of the cluster than the faint ones; at least the average magnitude is brighter, even without allowing for the color-excess. Outside the central zone the cluster stars seem to be very thoroughly mixed with respect to magnitude and color.

The laws connecting star-density and distance cannot be determined until the phenomenon interpreted here as Eberhard effect (which also depends on distance from the center) has been thoroughly investigated; even neglecting this effect, the distribution laws ordinarily derived must be recognized as applying to only that very small percentage of all the stars which is represented by those of exceptionally high intrinsic brightness.

Five new variable stars have been found in the Hercules cluster during the course of the present study of magnitudes. Prior to this two were known, the first discovered by Bailey at Harvard, and

TABLE III

Average Magnitudes for Different Regions and Color-Classes

Distance from Center	Color-Class						All Colors	Average Color-Class
	b	*a*	*f*	*g*	*k*	*m*		
0'.00–0'.45	....	....	14.50 (9)	14.14 (16)	13.37 (24)	13.31 (12)	13.73 (61)	*k*1
0.46–0.95	14.48 (2)	14.83 (20)	14.55 (34)	14.06 (63)	13.64 (31)	12.57 (6)	14.17 (165)	*g*1
0.96–1.45	15.12 (10)	15.12 (33)	14.78 (48)	14.06 (76)	12.83 (7)	11.92 (1)	14.46 (175)	*f*7
1.46–1.95	15.26 (15)	14.98 (20)	14.74 (51)	14.12 (54)	13.19 (3)	12.08 (1)	14.57 (153)	*f*5
1.96–2.45	15.18 (20)	15.12(14)	14.88 (38)	14.36 (30)	12.11 (3)	....	14.76 (120)	*f*2
2.46–2.95	15.22 (17)	14.75 (12)	14.80 (32)	14.03 (31)	12.74 (3)	....	14.58 (95)	*f*4
2.96–3.95	15.16 (12)	15.14 (17)	14.81 (51)	14.18 (33)	12.19 (2)	....	14.67 (115)	*f*4
3.96–4.95	15.25 (16)	15.22 (11)	14.92 (31)	14.70 (20)	13.56 (4)	....	14.91 (82)	*f*3
4.96–5.95	15.22 (6)	14.96 (5)	14.84 (18)	14.15 (20)	....	....	14.62 (49)	*f*5
≧5.96	15.16 (6)	14.03 (3)	14.74 (16)	14.54 (8)	13.69 (1)	....	14.67 (34)	*f*5
All of cluster	15.19 (113)	14.99 (153)	14.87 (328)	14.15 (357)	13.34 (78)	12.96 (20)	14.50 (1049)	*f*6
Outside distance 1'.95	15.20 (86)	15.01 (62)	14.85 (186)	14.28 (148)	12.74 (13)	....	14.71 (495)	*f*3

the second independently by Bailey and by Barnard. The periods of the two earlier variables are given by Barnard as 6.0 days and 5.10 days. Ordinarily they appear to be faint, evidently belonging to the Cepheid type. The range of variation is approximately one visual magnitude. For the five new variables the periods are not known; indeed, little is known of the light-variations beyond the fact that they appear definitely to exist. In every case, however, the smallest color-index is associated with the brightest photographic magnitude, which suggests that all seven stars are very probably Cepheid variables.

6. *On the probable parallax of the cluster.*—If, admitting a high degree of comparability between our galactic system and globular clusters, we assume that the brightest stars in Messier 13 have exactly the same absolute magnitude as the most luminous stars in our galactic system, we can compute the distance of the cluster as soon as we obtain the apparent magnitudes of the brightest stars in both systems and an assurance of negligible absorption in space. It would be better perhaps if we could assume equality of mean absolute magnitude for a number of stars of the same color-class in the two systems; or, if we could say, for instance, that the average B-type star has the same intrinsic luminosity wherever found. Then the determination of the distance of any cluster would await only measurements of magnitude.

In actual practice the derivation of the parallax of the Hercules cluster has not been simple or definite. The subject has been approached from many angles and the basic assumptions have been rather carefully examined. The complete discussion of the methods and results is deferred to a later paper, and for the present only a tabular summary will be presented.

The values in Table IV that are based on luminosity-curves fail of important weight because (1) of the provisional character of all theoretical luminosity-curves except for spectral Class B, (2) of the peculiar (perhaps chance) frequency of the brighter magnitudes among the early *b*-type stars in the cluster, and (3) of the possible necessity of distinguishing sharply between dwarf and giant stars in deriving luminosity-curves.

Incidental to the work on the Hercules cluster the following provisional parallaxes were obtained for similar objects:

Small Magellanic Cloud = 0″.00006
ω Centauri = 0.00027
Messier 3 = 0.0001

7. *On the probable dimensions in the Hercules cluster.*—We conclude from the preceding considerations of the distance of the cluster that the parallax is certainly not in excess of +0″.0001. It may be but a tenth of this value, and some of the evidence would suggest still greater distance; but a parallax less than +0″.00001 is hard to reconcile with the magnitudes of the variable stars, if they are to be considered normal Cepheids. The provisionally adopted parallax, +0″.00003, is, however, more likely to be too large than too small. On the basis of this distance some computations relative to the probable dimensions of the cluster are summarized below.

TABLE IV

SUMMARY OF DETERMINATIONS OF PARALLAX FOR MESSIER 13

Method	π	No. Stars	Suggested Weight of $\frac{1}{\pi}$
I. From variable stars	0″.00008	2	4
II. From Kapteyn's luminosity-curves:			
C.I. −0.39 to −0.20	.000005	17	1
" < −0.10	.000007	53	
" " (Pv. mag. < 15.30)	.000009	23	
" −0.10 to −0.01	.00003	33	2
All colors	.00005	495	0
III. From Russell's data for absolute magnitude:			
C.I. < −0.10	.00005	53	4
All colors	0″.00010	495	1
Provisionally adopted mean	0″.00003		

The angular diameter of that part of the cluster measured for the catalogue is less that 16′. It is readily seen, however, from the long-exposure photographs, as well as from the table of densities, that the cluster extends far beyond this limit. Von Zeipel states, on the basis of plates made especially for the purpose of fixing the limits, that the radius is not less than 17′. Accepting this result and the adopted value of the parallax, we find that the distance across the cluster is of the order of 1100 light-years. To an observer on the nearer edge of the cluster, a star on the opposite side would have a parallax of $+0''.003$, and if intrinsically one hundred times brighter than our sun, would still be nearly two magnitudes below the limit of visibility to the naked eye. To an observer on the earth, however, the difference in the apparent magnitude of two such stars, one on the nearer, the other on the farther, side of the cluster, would be only 0.02 mag., and the difference in parallax negligible.

At the distance corresponding to the adopted parallax, 100,000 light-years, the sun would appear fainter than the twenty-second magnitude, and stars of the absolute brightness of Sirius would be fainter than the eighteenth magnitude. The absolute visual magnitude of the brightest star in the cluster would be -5.7. Several stars in the galactic system of this absolute brightness are already on record; and of the giants so far known there naturally must be a selection in favor of the nearer ones. We know, in general, only the fainter limit for the absolute brightness of many if not most of the galactic giants. For instance, a second-magnitude star with a parallax not greater than $0''.001$ would have an absolute magnitude of -8 or brighter. Hence we could without seriously transgressing possibilities postulate absolute magnitudes of -10 or brighter. Since any such stars in the cluster do not exceed apparent magnitude 12.0, their distance must be nearly eight times greater than assumed above (perhaps of the order derived from the luminosity-curves of the early *b*'s) and the dimensions in the cluster would be octupled. But retaining the adopted value of the parallax, it is important to note that all the stars considered in the catalogues of Messier 13 have more than 200 times the solar brightness. In fact, probably no star as faint as the sun has yet been photographed in this cluster.

The long-exposure plates by Pease and by Ritchey would indicate, on the basis of the assumed distance, that there are probably more than 50,000 cluster stars intrinsically brighter than the sun. If all stages of stellar development are represented in the cluster as we know them in our galactic system, we must believe that the total number of luminous stars is of the order of hundreds of thousands.

Assigning for illustration a diameter of 10,000 light-years to the galactic system, it would appear from the Hercules cluster to have an angular diameter of about 5°, perhaps comparing closely in general appearance with the Greater Magellanic Cloud as seen from the earth. Our system, to the observer in the cluster, would also resemble the Magellanic Clouds in the remarkable characteristic of being rich in variables. From the cluster the thousand or so brightest apparent magnitudes would be our stars of greatest actual light, and among them, as in globular clusters and in the Magellanic Clouds, there would be a remarkably high percentage of variable stars. All types would be present, moreover, for we know they are all of high intrinsic luminosity; but those most readily discovered, for obvious reasons, would be the short-period Cepheids; that is, the cluster-type variables.

8. *Suggestions toward some working hypotheses.*—Upon the basis of the foregoing computations and from the results of preceding sections of this paper we are led, for the guidance of future work on magnitudes in clusters, to suggest some working hypotheses relative to the form, extent, and distance of stellar systems. The proposed hypotheses will be modified, no doubt, as the work progresses; their purpose now is chiefly to represent collectively and briefly those ideas concerning general stellar problems that the present study suggests.

The peculiar distribution of globular clusters and their great distances from the earth show that they do not form a part of the galactic stellar system. Each globular cluster is a complete and separate system by itself. Apparently none of them is very near, compared to the outer limits of the galaxy. The stars of a few southern clusters, particularly of ω Centauri and 47 Tucanae, are brighter than those of Messier 13, and such systems may be much nearer than a hundred thousand light-years; but many clusters are considerably fainter and presumably more distant. In observing those similar to N.G.C. 4147 (if they are built on the same model as Messier 13) we may be seeing a million light-years into space.

Our galactic system is distinctly out of the center of gravity of this higher-order system of visible globular clusters.

Until contrary evidence is brought forward we may assume that all open clusters are parts of the galactic system. This is suggested by their galactic concentration, by their motions so far as now measured, by the number of stars they contain, and more directly by the evidence of their comparatively small distances shown in the magnitudes and colors of their component stars. These galactic sub-groups are often if not generally real physical systems. The Hyades at a greater distance would resemble one of the galactic clusters, but because of the relative paucity of members could never be comparable to Messier 13, as is sometimes suggested.

Some of the globular clusters may be comparable to the galactic system in size, form, or at least in stellar constituency. For the Hercules cluster closely comparable dimensions cannot be claimed. A more appropriate comparison would be between our stellar system and the larger Magellanic Cloud. That the maximum radius of the Milky Way is probably not greater than ten thousand light-years and may be somewhat less has been deduced from many lines of evidence, the most important of which is the color of the faint stars. The shortest radius of the system is variously estimated from one-half to one-sixth of the radius in the galactic plane.

Have globular clusters such planes of symmetry? From superficial appearances a negative answer is expected. The discussion of planes of symmetry in globular clusters involves at least three important factors—the degree, orientation, and nature of the supposed oblateness. (1) The ratio of the shortest diameter to the longest may be near unity, so that only refined study from any position in space (in or out of the cluster) would be able to detect the oblateness. (2) The cluster may be so oriented in space that, whatever the ratio of its axes of symmetry, the discovery of oblateness is impossible. (3) The absolutely brighter stars may not show the oblate form, the property of condensation toward a plane being confined to the fainter stars. This latter consideration is very relevant. In our system the naked-eye stars of the second spectral type show practically no galactic concentration. In globular clusters the brightest stars, which through their apparent arrangement have given the type-name to the systems, are those that are absolutely bright and red.

Under the most favorable conditions of oblateness and orientation, therefore, the analogy to the galactic system should make the elongated form apparent only for the faint stars in the cluster. The concentration may and probably does depend on spectral type as well as magnitude. The colors in the Hercules cluster have been examined for the purpose of testing possible oblate distribution of the blue stars, but no decisive result was obtained. The counts in other clusters have also as yet yielded no positive contribution to this matter, dealing as they mainly do with only the brightest stars. A distinctly elongated or non-globular form, however, has been noted for two or three of the brightest clusters where fainter stars are involved in the photographs, and counts have verified the superficial appearance of a plane of symmetry in ω Centauri.

The conclusion is that globular clusters may very probably resemble our local stellar system in form, as well as in the properties of showing central condensation of stars, of containing all spectral types, including the giant red and yellow stars, and of having among their most luminous members a large quota of variables.

The base line used in the present case for the measurement of the amount of light-scattering in space is a very long one; those previously used have been relatively short. The hypothesis, based on the present result, that there is no appreciable space absorption, is therefore fairly safe.[1] Since it is not certain as yet, however, whether or not molecular light-scattering exists in the central part of the cluster, we must make the reservation that there is a possibility of some measurable space absorption for lower galactic latitudes. And of course we may expect patches of absorbing material here and there in space, as, for example, Barnard's dark nebulae in the Milky Way. In any case absorption should be treated as a varying quantity[1] depending on the condensation of light-scattering material, and not as a quantity that is constant, whether in the densest part of our galactic system or in the starless realm without.

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[1] The coefficient of absorption can scarcely be one-tenth the most recent previous values, even with very generous allowance for error. That being the case, the result is intimately related to the discussion of the time-honored problem of the finiteness of the stellar universe. By universe in this sense we mean the system that is formed by stars of the kind that surround the sun and appear to us in all parts of the sky. Either the extent of the star-populated space is finite, or "the heavens would be a blazing glory of light," or there is an absorption of light in space, or there is a peculiar, non-uniform distribution of stars and star-systems throughout space (Charlier), or, except in our immediate vicinity, the stars as light-giving bodies are dead or not yet born. The last two clauses are usually not included in the discussion. Omit them. Then, since the heavens are not a blazing glory, and since space absorption is of little moment throughout the distances concerned in our galactic system, it follows that the defined stellar system is finite. There is nothing novel in this conclusion. It has been the prevailing one for a long time, and has been very easily derived by making the assumption, in spite of prevailing values of the coefficient, that space absorption is really negligible.

The decision that molecular scattering of light is inappreciable in the direction of Messier 13 probably justifies the assumption that general non-selective space absorption (obstruction) is also of small importance. It also permits safer sounding of the depths of the Milky Way system in various directions, and, together with the absence of blue stars of fainter magnitudes, suggests that the limits have already been reached. There is a suggestion among the globular clusters that the stars at great distances from the center are intrinsically fainter than the nearer ones, because of either greater age, or smaller mass, or dependence of spectrum on distance. If such a condition exists in our galactic system we can better interpret the color-distribution of faint stars.

THIRD PART: A CATALOGUE OF 311 STARS IN MESSIER 67[2]

The open cluster Messier 67 was discovered about a century ago at Milan by Oriani. The early observations consisted mainly of general descriptions of the group by Messier and Sir John Herschel, and later of notes relative to the number and distribution of its members by Smyth and Fenet. More recently two photographic catalogues have been made of the positions of the individual stars, the first by Olsson at Stockholm in 1898, the second eight years later by Fagerholm at Upsala. In the positions of the 118 stars common to the two catalogues various systematic differences appear, but there is no conclusive evidence of motion during the interval of eight years.

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[1] Seeliger has considered space-absorption both as constant and as decreasing with distance (*Sitzungsberichte der mathematisch-physikalischen Klasse der königlich-Bayerschen Akademie der Wissenschaften*, 1911, Heft II, p. 413).

[2] Abstract of *Contributions from the Mount Wilson Solar Observatory*, No. 117, dated May 1916.

In the catalogues of positions the magnitudes are determined only approximately. The values derived by Olsson, evidently depending on rough estimates, are systematically too bright by more than two magnitudes. Fagerholm's magnitudes, on the other hand, are determined by careful measurements of diameters of stellar images; but they are based upon an extension of the visual scale of the *Potsdam Durchmusterung* and are systematically in error owing to the color of the stars.

In the present study the photographic magnitudes are given for all objects listed by Fagerholm, and for a few others which are within the zone covered by him but are missing from his table. Photo-visual magnitudes are determined for all but four of the stars within distance 12′ of the center. The average probable error of a color-index is about $\pm 0\overset{M}{.}1$.

After discarding a few questionable values a total of 232 color-indices is available for statistical discussion. The average color-index is +0.91, with a strong condensation of color in classes *f* and *g*. No negative indices and but five less than +0.4 were obtained. All of the *m*'s and most of the *k*'s are near the center and the former are chiefly among the brighter stars, but aside from this no definite relation exists between color, distance, and magnitude.

From a study of these colors alone, the cluster appears to consist of but a hundred stars and to be of small extent, but an examination of the large star charts (Palisa-Wolf, Franklin-Adams) shows that the cluster's influence among the background stars may extend to a distance of nearly a degree.

There is much in common between the stars of the immediate background and those of the cluster—average color, magnitude, and condensation not appearing greatly unlike. In this respect, as well as in others, there is a wide difference between Messier 67 and the condensed globular system Messier 13, discussed in the preceding part.

MOUNT WILSON SOLAR OBSERVATORY
Abstract dated January 1917

MINOR CONTRIBUTIONS AND NOTES

ON POLE-EFFECT

In our paper on "Pole-Effect in a Calcium Arc"[1] Mr. Whitney and I had intended to mention the work of Royds and his theory that differences in vapor-density at different parts of an arc are the cause of the differences of wave-length, first discovered by Goos[2] and later studied by St. John and Babcock,[3] Royds,[4] and others.[5] But some of our results seemed to be contrary to Royds's hypothesis; in particular our results gave the same values for the wave-lengths of the Ca lines at the center of an arc between carbon poles which had been soaked in a dilute solution of $CaCl_2$, and at the center of a similar arc between carbon poles which had been soaked in a strong solution of the same salt. Since the exposure times for lines of equal strength in the two cases were in a ratio of about 1 to 10, it seemed to us justifiable to assume that the density of the luminous Ca vapor was markedly different in the two cases, and therefore that differences in vapor-density alone could not be responsible for the differences in wave-length observed between center and pole. It seemed fairer, on the whole, not to mention Royds's work at all than to offer this very limited evidence in refutation of it.

In his latest article[6] Royds finds the same wave-length for the green copper lines at the center of an arc between poles of pure copper and between poles of a 20 per cent alloy of copper. This seems to confirm our view. At any rate, I find it much easier to believe that a change in vapor-density alone cannot affect a change in wave-length than to assume with him that experiments like ours "fail or at any rate are inconclusive, because there is no reason to believe that the atoms have been separated to a greater distance apart with the smaller amount of material." While I might perhaps agree with him that "exposure times are not a sufficient test of vapor-density," I find it easier to believe that radiating calcium vapor is denser in one arc than in another when plates are secured with one-tenth to one-twentieth the exposure time, than to assume, as Royds does, that when a small amount of $CaCl_2$ is present in the carbons the atoms vaporize in clusters, and are not removed from each other's influence any more than when a larger amount has been used.

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[1] *Astrophysical Journal*, 43, 161, 1916.

[2] *Ibid.*, 38, 141, 1913.

[3] *Ibid.*, 42, 231, 1915.

[4] *Kodaikanal Bulletins*, Nos. XXXVIII and XL.

[5] *Astrophysical Journal*, 43, 161, 1916; 44, 65, 1916.

[6] *Ibid.*, 45, 112, 1917.

The very tentative nature of our suggestion that amplitude of vibration might be effective in disturbing wave-lengths will, I think, be recognized at once by anyone who reads our paper. It seems unwise to enter into a discussion of the fundamental causes of pole-effect until more facts are known. This feeling is confirmed rather than otherwise by Royds's latest paper, and one must regret that he feels it necessary to abandon for the present a field in which his contributions have been so valuable.

When we have considerably more data at our disposal it may be well to analyze the fundamental causes of such changes of wave-length as occur in pole-effect and pressure-shifts. It seems to me that any discussion of vapor-density as a possible cause should be considered in the light of Sir Joseph Larmor's[1] suggestion of a modification of the dielectric constant of the medium, or Humphrey's[2] suggestion of the mutual magnetic influences of neighboring atoms, or Stark's[3] suggestion of mutual electric influences. In the meantime I do not feel that our suggestion of the possible rôle of amplitude need be taken too seriously.

I do not agree, however, with Dr. Royds that rise in temperature is the "most effective and probably the only certain way" in which to increase the amplitude of vibration of the light-emitting electrons within an atom. In fact, it seems to me that in an electric arc or spark the velocity of impinging electrons and canal rays may be the important factor, and these velocities depend on potential gradients rather than on temperatures. These are certainly the predominant factors in vacuum tubes. Perhaps the only effect of temperature is to produce the Doppler broadening, which is, of course, symmetrical.

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[1] *British Association Report*, p. 555, 1897.

[2] *Astrophysical Journal*, **23**, 233, 1906.

[3] *Jahrbuch der Radioaktivität und Elektronik*, **12**, 349, 1916.

I agree with Royds that temperature alone does not appear to be the cause of pole-effect. But it seems to me inevitable that temperature must very largely control vapor-density, and the first argument which Royds uses to show that pole-effect is independent of temperature might be used as an argument that it is independent of vapor-density as well. To my mind at least, there is a much more direct relation between temperature and vapor-density in the arc than between temperature and amplitude of vibration.

However, I do not wish to be drawn into a further discussion of these matters at present. Many related facts will occur to anyone who has thought along these lines, but the time for serious discussion does not seem to me to have arrived. Certainly I have no favorite theory to defend, and shall be quite as pleased if the final outcome points to vapor-density as I shall if it points to any other factor; but I doubt that the explanation will be a simple one, or the same for all lines. In the final solution of these questions I am sure that the interesting and valuable contributions of Dr. Royds will have great weight.

HENRY G. GALE

RYERSON PHYSICAL LABORATORY
February 6, 1917

A VARIATION IN SOLAR ROTATION

A theory has been recently advanced by R. E. DeLury which would account for discrepancies in spectroscopic determinations of the solar rotation by varying quantities of atmospheric haze. In particular[1] he has applied this theory to some results given by the writer in a paper with the above title.[2] Owing to the pressure of military duties, the writer is unable to obtain any experimental data on this interpretation of his results, but in the meantime the following comments may be made.

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[1] *Astrophysical Journal*, 44, 198, 1916.

[2] *Ibid.*, 43, 156, 1916.

1. DeLury states: "The record shows that in general high values of the rotation in the observations mentioned were obtained on the brighter days and low values on the hazier days."

During the period under consideration, namely, June 21 to August 16, DeLury made three entries in the observation book with reference to haze:

June 21, "Bright"	$V=1.911$ (low).
July 11, "Very hazy"	$V=1.975$ (high).
Aug. 16, "Bright, some water-vapor haze"	$V=1.977$ (high).

In addition to this record the writer made a visual estimate of brightness on each day that plates were taken and entered it in the observation book:

5 represents a very brilliant day—rare in Ottawa.
4 represents a bright day—normal observing weather.
3 represents a day slightly hazy.

Plates taken on days hazier than 3.5 were *not* measured. On ten days of low values (1.846 to 1.919 km) the average brightness was 4. On fifteen days of high values (1.943 to 2.007) the average brightness was 3.7. In other words, the record, if it shows anything, shows that in general high values were obtained on hazy days and low values on brighter days, in direct contradiction to DeLury's statement above.

2. DeLury bases his discussion on plates made on two dates, June 24 ("July 13" evidently an error, since the writer measured no plates made on that date) and July 20, which are described as "Hazy" and "Bright," apparently from memory, as it is not so recorded in the observation book. The measures show evidence, according to the criterion of varying velocity for different line intensity, of about 8 per cent haze. This, on his own hypothesis, should produce about an 8 per cent difference in velocity on the two dates. The difference in mean velocity is actually 12 per cent, leaving a 4 per cent change in velocity to be accounted for by

sources external to the atmosphere, which, as far as it goes, may be taken as a confirmation of the writer's results.

These experimental data are, however, both meager and inconclusive. It seems unfortunate that, with so much material on hand for discussion, plates for only two days were measured. The results for these two days are, moreover, inconclusive, since the plates measured were taken two to three hours after the writer had completed his exposures. As a consequence, on June 24 DeLury's plates were taken through clouds whose presence he notes in the observation book, whereas the writer's plates, exposed earlier, were taken with a clear sky. Furthermore, the difficulties of measurement of broad lines increase the errors and chances of prepossessions, especially with an observer whose last published measures[1] show a probable error for even well defined lines of 0.06 km, i.e., of the same order as the differences measured.

TABLE I

Day	Interval	Time First Plate	Residual First Minus Last	Mean Velocity Day	Day	Interval	Time First Plate	Residual First Minus Last	Mean Velocity Day
		A.M.	km	km			A.M.	km	km
172....	60^m	6.50	+0.022	1.911	201...	101^m	6.24	−0.053	2.026
176....	73	7.41	+ .052	1.914	208...	62	6.11	+ .034	1.996
179....	57	8.04	− .043	1.866	210...	121	6.26	− .067	1.964
182....	32	7.52	+ .049	1.871	222...	88	8.45	− .024	1.966
185....	36	6.02	+ .056	1.893	228...	184	8.41	+ .064	1.977

NOTE.—In the last four entries in this table, where more than four plates were made in a day, the early and late plates have been grouped together and averaged to diminish accidental differences without appreciably shortening the interval.

3. The plates taken by the writer contain in themselves evidence that haze plays no part in his results. In order to obtain measurable telluric lines to correct for instrumental error, the plates were all taken in the early morning. Evidently, if haze was present in sufficient quantities to effect the results, it will show its greatest effect when the sun's light was passing through the thickest layer of atmosphere. That is, for plates effected by haze the residual first plate minus last plate should be negative. In Table I are given the residuals for ten days in which the first

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[1] *Transactions Royal Society of Canada*, 1912, Sec. III, p. 1.

plate was taken before 9:00 A.M. and the interval was at least thirty minutes. A study of this table indicates that, for the first five days, when the velocities are lower than the mean, the residuals are preponderatingly positive instead of negative, as they should be if haze produced the low velocities. Similarly the last five plates of high velocity show, if anything, negative residuals, which would indicate the presence of haze.

This result is further confirmed by grouping the plates as in Table II. Here also the residuals are preponderatingly positive.

TABLE II

Groupings of Days	No. of Days	Average Values		
		Time First Plate	Time Last Plate	Residual First Minus Last
		A.M.	A.M.	km
Every day plates were made	25	7.35	8.818	−0.0016
Days first plate made before 8:00 A.M.	14	6.53	7.46	+ .0096
Grouping of Table I	10	7.16	8.30	+ .0090

Even the first grouping, which gives a small negative residual, is explained when it is remembered that on fifteen days included in that grouping the average interval between first and last plates was fifteen minutes—too small an interval to be rightly included. Removal of these days gives the last grouping, which is positive.

Summarizing: (1) There is no evidence in the observational record of the varying atmospheric haze required on DeLury's hypothesis to produce the variation. (2) His own measures of its presence are both meager and inconclusive. (3) Internal evidence from the writer's own measures does not reveal the presence of the haze.

In conclusion the writer wishes to emphasize the fact, which is also borne out by the preceding discussion, that in making exposures the greatest care was taken to avoid hazy days. Certainly plates were not taken on days with an 8 per cent to 10 per cent haze required by DeLury's hypothesis to account for the variation obtained.

H. H. PLASKETT

KINGSTON, ONTARIO
December 1916

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THE
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AND ASTRONOMICAL PHYSICS

VOLUME XLV APRIL 1917 NUMBER 3

THE EMISSIVE POWER OF TUNGSTEN FOR SHORT WAVE-LENGTHS

By E. O. HULBURT

1. *Introductory.*—The ratio of the energy per second per unit area emitted normally from a metal at a given temperature to that from a black body at the same temperature is termed the "emissive power" of the metal. If the distribution of energy in the region of short wave-lengths of the spectrum of black-body radiation is given by Wien's equation, the distribution of energy of the metal's radiation in this region is expressed by the formula

$$J=C_1\lambda^{-5}e^{-\frac{C_2}{\lambda T}}\cdot\epsilon\,, \qquad (1)$$

where J is the energy per unit wave-length radiated per second per unit area at wave-length λ by the metal at temperature T, and ϵ is the emissive power of the metal for the λ and T in question. In general ϵ is a function of λ and T. The problem of ascertaining the distribution of spectral energy of the metal's radiation thus resolves itself into a determination of the values of the emissive power for each wave-length and temperature. In the present investigation the changes in the emissive power of tungsten have been determined throughout the region of the spectrum from 3400 A to

5400 A for true temperatures from 1746° to 2785° K. A sodium photo-electric cell with an electrometer was employed for the measurements.

Previous determinations of the emissive power of tungsten have yielded results which are rather fragmentary and conflicting. Langmuir[1] has given a table which indicated that, for the visible region of the spectrum, ϵ increased as the wave-length decreased, and that ϵ increased slightly as the temperature increased. A. G. Worthing's[2] preliminary measurements showed that ϵ is greater for blue than for red light, and that ϵ unmistakably decreased with an increase in temperature. There are no data for the emissive power of tungsten in the ultra-violet.

The original plan of the present work had been to use a tungsten lamp equipped with a quartz window, and if possible to extend the measurements into that region of the ultra-violet where glass is opaque. A tungsten strip was mounted in a bulb to which was sealed normally a glass tube 1 cm in diameter and 3 cm in length. The end of the tube was closed by a quartz plate attached with red sealing-wax. Such a lamp could not be properly baked out during exhaustion, and so contained residual water-vapor. As a result, the tungsten deposited copiously on the walls of the bulb; no appreciable blackening occurred on the sides of the tube or on the quartz window. It was found that when the lamp was operated on a constant current, the temperature of the filament rose at the rate of many degrees per hour, owing to the wasting away of the metal, and a steady state was never reached. Spectrograms, Fig. 1, of a helical tungsten filament through a quartz window at various temperatures showed an appreciable amount of radiation in the ultra-violet as far as 2800 A. The temperatures assigned to each spectrogram of Fig. 1 are probably correct to within 50° K. A comparison of these spectrograms with that of a hydrogen vacuum tube, with which some experience had been had in respect to quantitative measurements in the ultra-violet, showed that there was no hope of obtaining any measurements on the tungsten radiation for wave-lengths below 3000 A, except by increasing the sensibility of the photo-electric apparatus many fold. Because of this, and because of the unsteadiness mentioned above, the plan of using the lamp with a quartz window was abandoned. All following measurements recorded in the present work have been made on the light from the tungsten in glass bulbs and have then been corrected for the transmission of the glass.

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[1] *Physical Review*, 7, 302, 1916.

[2] *Ibid.*, 7, 497, 1916.

2. *Apparatus and experimental details.*—The tungsten was in the form of a strip, 32 mm×2 mm, mounted vertically in a nitrogen atmosphere and inclosed in a glass bulb. Each lamp was carefully calibrated before the experiment, so that the relation between the current through the filament and the black-body temperature was known. The lamp was operated on storage-battery currents which were read on a calibrated ammeter, and the temperatures obtained from the calibration-curve were probably correct to 5° K. The true temperature was found from the black-body temperature by the use of Worthing's curve (*loc. cit.*).

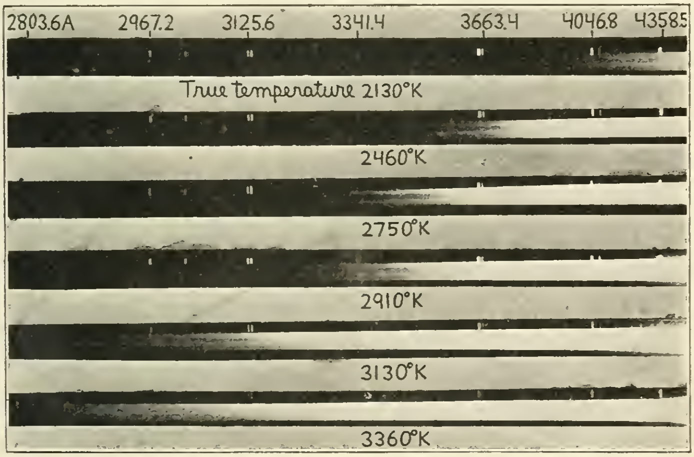

FIG. 1.—Short wave-length spectrum of tungsten at various temperatures for equal times of exposure with quartz-mercury lamp comparison spectrum (wave-lengths from R. A. Millikan, *Physical Review*, 7, 364, 1916).

The image of the tungsten strip T, Fig. 2, was focused by a quartz lens L, 2.8 cm in diameter and of focal length 15.8 cm, on the slit B of the spectrograph. Between the lamp and the quartz lens was an electrically operated shutter S controlled by a pendulum with a period of five seconds. Close to the shutter was placed a rotating sector *R*, which enabled one to reduce the intensity of the light by a known fraction. The spectrograph consisted of the Rowland mounting of a concave speculum-metal grating *C* of 16 cm radius of curvature, ruled 15,000 lines to the inch, the area of ruling being 15 mm×24 mm. The spectrum was brought to a focus at slit *A*, back of which was placed the sodium photo-electric cell *P* with a quartz window. Slit *A* and the cell were on a movable arm so that various wave-lengths could be reached. The dispersion was 106 A per mm. The cell was connected to a Dolezalek quadrant electrometer *E* of sensibility 1200 mm per volt of difference of potential between the quadrants (period 12 seconds).

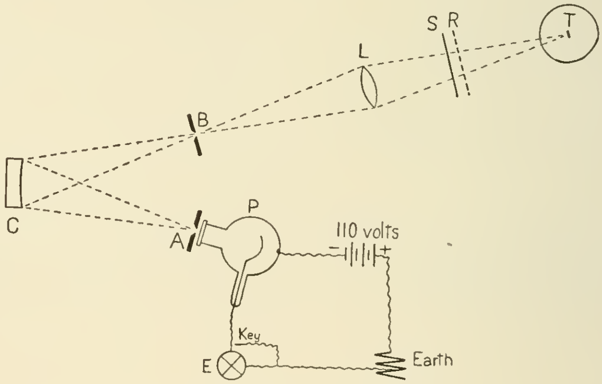

FIG. 2.—Diagram of apparatus

The steady-deflection method was employed. In this method the charge liberated in the cell during a known time of exposure to the light was collected on the insulated quadrants of the electrometer, and the deflection was taken after the needle had come to rest. The deflections recorded have in all cases been reduced to the unscreened deflection for an exposure of 10 seconds. For example, at wave-length 4326 A and temperature 2677° K., was recorded a deflection of 6930 mm. The deflection actually observed was 385 mm (the mean of 389, 383, 383, 385 mm), which was obtained by a 5^s exposure using a rotating sector which transmitted one-ninth of the light.

By means of the shutter, actuated by the pendulum, the times of exposure of the cell to the light were probably accurate to 0.5 per cent. The drift of the electrometer needle was negligible, being less than 1 mm per minute. In spite of these favorable conditions it was found impossible to repeat readings within an accuracy of 1 per cent; the deflections obtained under ostensibly identical experimental conditions often varied by as much as 4 per cent. The cause of this might have been due in part to fluctuations of the intensity of the light, and in part to unsteadiness in the cell. All the measurements recorded in this paper were the means of three, or more, observations, and were considered to be correct within 1 per cent for large deflections, the error being greater for the smaller deflections. The inability to repeat observations with precision was a serious defect of all the measurements. The most probable direction of improvement would have been in the use of a more sensitive cell, which would allow the sensibility of the electrometer to be decreased with a corresponding gain in steadiness. The cell used in the present work was one which had been prepared two years ago by the distillation of sodium *in vacuo*, and it seemed to have lost none of its sensitiveness. An attempt was made to use the cell in connection with a high resistance, and to employ the electrometer as a delicate galvanometer. It was found that this method was of a much lower order of sensitiveness than the steady-deflection method, and could not be used in the present work.

3. *The linear relation test of the photo-electric cell.*—The relation between the energy incident on the cell and the deflection of the electrometer was found to be a linear one for monochromatic radiation. The test was made in the following way: The transmissions of two neutral tint glasses for the blue mercury line at 4358 A were found to be 13.5 per cent and 13.9 per cent, respectively. The deflection due to the unscreened light was 2541 mm, to the light through the 13.5 per cent screen 343 mm, and to the light through the two screens 47.9 mm. The ratio of electrometer deflection to incident energy in each case was $\frac{2541}{1.00}$ or 2541, $\frac{343}{0.135}$ or 2541, and $\frac{47.9}{0.135\times0.139}$ or 2553, respectively. The ratio

was constant within the error of experiment, and hence the linear relation was considered satisfactorily established. Several sets of measurements were carried out with similar results.

4. *Scattered light.*—The spectrograph was inclosed in a light-tight metallic case blackened on the inside, and fitted with suitable diaphragms of smoked zinc. A visual examination with a transmission grating of the supposedly monochromatic beam of light issuing from slit *A*, when a tungsten lamp was used as a source, showed that the beam also contained light of all the other wave-lengths of the spectrum. This dilution of the beam with white light was due to the grating itself, which scattered the incident light in a manner similar to the effect produced by a rough surface. The intensity of this extraneous light was small, being about as strong as the light scattered by a camphor-smoked surface placed in front of the grating, and seemed to be roughly constant throughout the spectrum. When a mercury lamp was used as a source there appeared, in addition to general scattered light, at 5230 A a weak ghost of the green mercury line 5461 A. Data (Fig. 3) taken at the beginning of the work, and intended primarily for calibration purposes, have been used to obtain the corrections for scattered light. From curve 1, Fig. 3, the spectral photo-electric curve of a quartz-mercury lamp from 2000 A to 6000 A, it is seen that the deflections at dark portions of the spectrum, between the lines, dropped down to an approximately constant value, which indicated that the scattered light was fairly constant at all wave-lengths. On the strength of this the correction for scattered light has been taken to be the same for each wave-length. The intensity of the scattered light has been obtained from curve 2, Fig. 3. This curve gives the deflections taken throughout the spectrum of tungsten through glass at a constant temperature, from 2000 A to 6000 A. Below 3000 A the deflections were assumed to be due entirely to scattered light, and amounted to 30 mm at each wave-length, which was 1.1 per cent of the maximum deflection. The curve was then corrected by subtracting 30 mm from each deflection; this is shown in Fig. 3 by the dotted line drawn under curve 2. The corrections for the spectral photo-electric curves at the other temperatures were found by comparing their three greatest deflections with the corresponding deflections of curve 2,

Fig. 3; this amounted to a comparison of the areas under the curves with an accuracy sufficient for the purpose. In this way all the deflections have been corrected for the effect of scattered light.[1]

5. *The change in emissive power with wave-length.*—This was determined by measurements on a black body at the melting-point of palladium, 1822°5 K.,[2] and on the tungsten at 2143° K. Columns 1, 2, and 3, Table I, give the experimental data, corrected for scattered light. Column 4 gives the values of the "relative emissive power (uncorrected)" computed from the relation

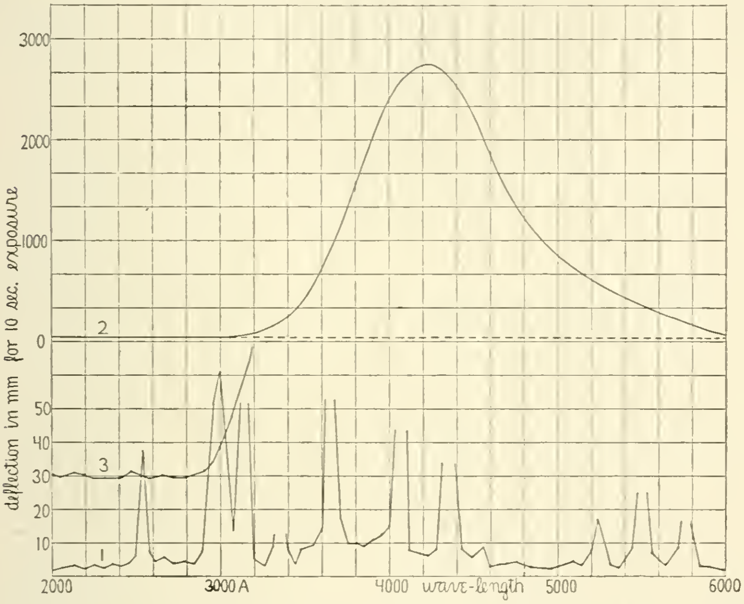

FIG. 3.—Spectral photo-electric curves. 1, quartz-mercury lamp; 2, tungsten lamp; 3, part of 2, enlarged. The dotted line under 2 shows the percentage attributed to scattered light.

$$\text{Relative } \epsilon \text{ (uncorrected)} = \frac{d_t}{d_b} e^{\frac{C_2}{\lambda}\left(\frac{1}{2143}-\frac{1}{1822.5}\right)}, \qquad (2)$$

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[1] An abstract of this work appeared in the *Journal of the Franklin Institute*, **182**, 695, 1916. At that time no corrections for scattered light were made. Therefore the conclusions stated there have been modified somewhat in the present paper. In the abstract, by mistake, the value for ε at 2143° for 4677 A was taken to be 0.470. The correct value is 0.475.

[2] A. L. Day and R. B. Sossman, *American Journal of Science*, **29**, 93, 1910.

where d_t and d_b are the deflections produced by the tungsten and black-body radiation, respectively, at wave-length λ, and C_2 is 14460 $\mu\times$deg. K.[1] The values of column 4, already corrected for scattered light, were corrected for three other effects: (*a*) the finite width of slits, (*b*) the difference in the width of slit *A* in the two sets of measurements, (*c*) the loss of energy produced by the glass.

a) The corrections due to the finite width of slits, computed by the graphical method,[2] were negligible except for wave-lengths 3478 A and 5641 A, in which cases the correction amounted to an increase of 0.5 per cent in "relative ϵ (uncorrected)."

TABLE I

Emissive Power of Tungsten, at Temperature 2143° K., as Function of the Wave-Length

1	2	3	4	5	6
Wave-Length	Electrometer Deflection for 10 sec. Exposure Corrected for Scattered Light				
	Black-Body Temp. 1822°K. Slit A. 0.464mm Slit B. 0.502 mm	Tungsten Temp. 2143°K. Slit A. 0.279 mm Slit B. 0.502 mm			
	d_b Means of Four Series	d_t Means of Four Series	ϵ Relative (Uncorrected)	ϵ Relative (Corrected)	ϵ (Absolute Value)
3240 A......	0.2 mm	2.5 mm	....	....	....
3478........	4.1	23.9	0.191	0.533	0.485
3717........	11.6	79.7	.281	.543	.495
3956........	26.0	151.7	.289	.540	.493
4196........	44.0	223.1	.299	.552	.503
4435........	65.3	266.0	.279	.516	.470
4677........	70.6	252.7	.282	.521	.475
4916........	61.6	189.9	.275	.507	.463
5158........	41.0	110.4	.269	.498	.453
5400........	26.6	62.5	.260	.481	.438
5641........	21.0	42.4	.246	.457	.415
5890........	17.6	29.0	....	....	....

b) Because of the feebleness of the light from a black body at 1822°5 K., slit *A*, Fig. 2, was opened wider for the measurements on the black body than for those on the tungsten. This produced an increase of 66.3 per cent in each value of column 4, Table I.

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[1] E. P. Hyde, F. E. Cady, W. E. Forsythe, *Astrophysical Journal*, 42, 294, 1915.

[2] E. P. Hyde, *ibid.*, 35, 237, 1912.

c) In order to determine the transmission of the glass, the tungsten lamp was mounted between a quartz-mercury lamp and the spectrograph. The transmission of the two walls of the bulb was determined for certain mercury lines, and the transmission of one wall calculated. This was done for the glass at room temperature and for the glass when heated. The heating was effected by lighting the lamp and allowing the glass to become hot. Then the lamp was turned out and the transmission of the glass was measured immediately afterward. The data are shown in Fig. 4. The correction for the effect of the glass amounted to an increase in "relative ϵ (uncorrected)" of 66.6 per cent for 3478 A, 11.6 per cent for 3717 A, 11.3 per cent for 3956 A, and 11.1 per cent for all the other wave-lengths. The values of the emissive power thus corrected, "relative ϵ (corrected)," are given in column 5, Table I. The values of column 5 are 9 per cent greater than other observers have found in the regions of the spectrum where a comparison is possible. Several factors, the chief of which are (1) the difference in the width of slit *A* used in the two sets of measurements, (2) the existence of general scattered light, (3) the difficulty of lining up the apparatus so that the tungsten strip was accurately replaced by the black body, contribute to this discrepancy. Therefore the values in column 5 have been used only in determining the relative

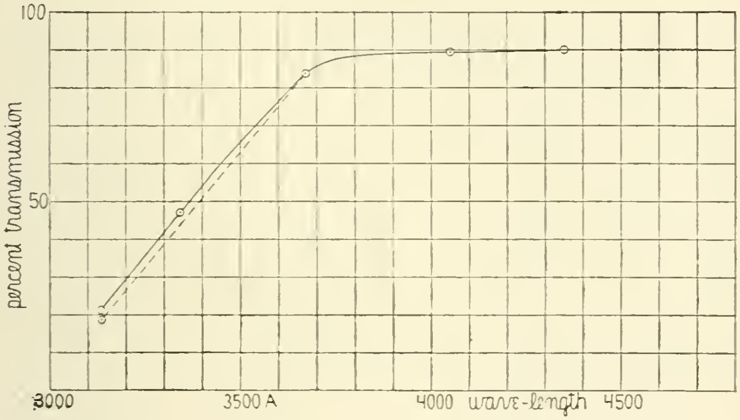
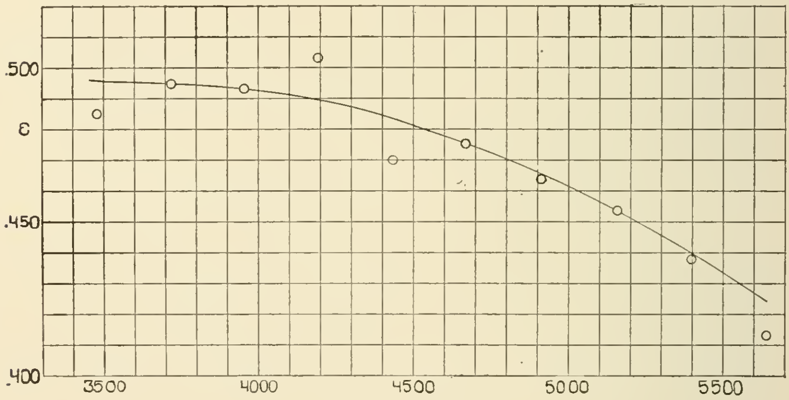


FIG. 4.—Transmission of glass of lamp bulb; full line, cold glass; dotted line, hot glass.

changes in the emissive power with wave-length. In column 6 (see also Fig. 5) the absolute values of ϵ are given. These depend upon the choice of the value $\epsilon=0.475$ for wave-length 4677 A at 2143° K. (from unpublished measurements by A. G. Worthing).

6. *The change in the emissive power with temperature.*—The experimental procedure was as follows: with the photo-electric cell set for a certain wave-length the deflections were observed when the filament was operated at various temperatures. The logarithmic isochromatic curves, i.e., the curves formed by plotting the logarithm of the deflection at constant wave-length against the reciprocal of the temperature, were obtained for a series of wave-lengths from 3295 A to 5532 A, for two lamps. The curves for lamp 2 are shown in Fig. 6; the curves for 4566, 4806, and 5047 A have been shifted arbitrarily along the X-axis to prevent them from intersecting. The deflections have been corrected for scattered light. The width of slit was 0.279 mm for slit A, and 0.502 mm for slit B; the correction for finite slit-width was negligible. The range of temperature was from 2103° K. to 2785° K. for all the lines, except for those for 4086, 4326, 4566, and 4806 A. In the case of these four it was possible to get readings for a temperature as low as 1746° K., because in this region of the spectrum of tungsten the



Wave-lengths in A

FIG. 5—Emissive power of tungsten at temperature 2143° K.

deflections were greatest. Fig. 6 shows that within the error of experiment the logarithmic isochromatic lines are rectilinear. Therefore, for each line

$$ln\ d = ln\ (J \times \text{const.}) = a - \frac{b}{T},$$

or

$$J = a'e^{-\frac{C_2'}{\lambda T}}, \tag{3}$$

d is the deflection produced at wave-length λ by J, the energy per unit wave-length, radiated per second per unit area by the tungsten at temperature T. Since the isochromatic lines are taken to be rectilinear for the range of temperatures under investigation, it follows that a' and C_2' are independent of T, but may be functions of λ. C_2' has been computed from the slope of each isochromatic line, and the value of C_2' for each wave-length is shown graphically in Fig. 7. The observed values of C_2' for 3295 A and 3372 A have been increased by 1.3 per cent and 0.3 per cent respectively, because of the temperature coefficient of transmission of the glass.

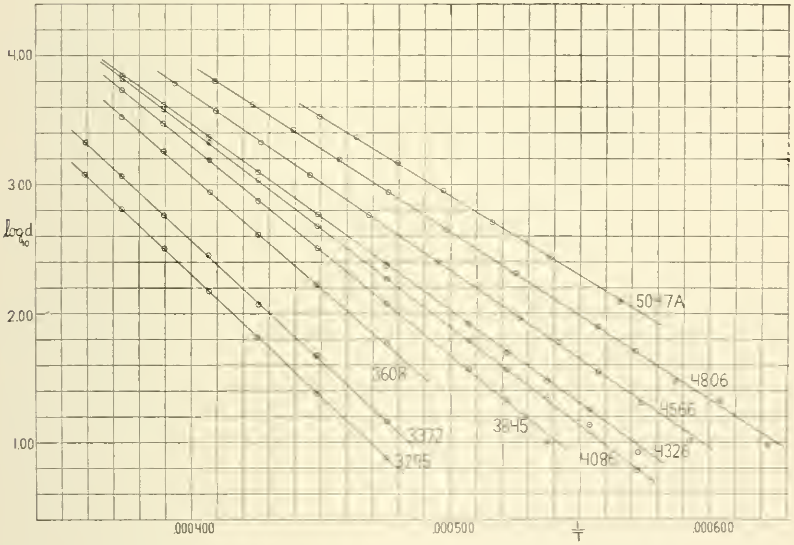


FIG. 6.—Logarithmic isochromatic curves of tungsten, lamp 2

From equations (1) and (3) we obtain

$$ln\frac{\epsilon}{f(\lambda)}=\frac{C_2-C_2'}{\lambda T}. \qquad (4)$$

Equation (4) has been used to compute the change in ϵ with T, using 14460 $\mu\times$deg. K. for C_2, and taking the values of C_2' for each wave-length from the straight line of Fig. 7.

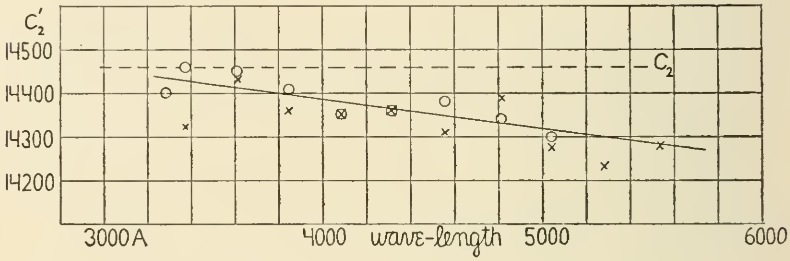


FIG. 7.—Values of C_2' in $\mu\times$ deg. K.
Crosses, lamp 1.
Circles, lamp 2.
Dotted line, C_2 = 14460$\mu\times$deg. K.
Values given by circles have been given greater weight.

7. *Emissive power of tungsten.*—The final values for the emissive power have been obtained by taking the values of ϵ at 2143° K. for each wave-length from the smooth curve of Fig. 5, and computing the values at the other temperatures by the use of equation (4) and the values at C_2' for each wave-length from Fig. 7. The results are shown graphically in Fig. 8. The change in ϵ with T, shown in Fig. 8, is in fair agreement with Worthing's measurements at 468 $\mu\mu$ and 665 $\mu\mu$.

It is worthy of mention that by means of Fig. 8, together with formula (1), the distribution of spectral energy of tungsten radiation can be computed. This enables one to use a tungsten lamp for the purpose of calibrating a spectro-photometric apparatus in energy terms.

8. *Accuracy of the results.*—The values of ϵ, given in Fig. 5, for those wave-lengths at the ends of the region of the spectrum under investigation were open to an error of perhaps 5 per cent, because in these cases the deflections were small and the corrections large.

The change in ϵ with T has been determined with greater precision; it is considered that given the value of ϵ for one temperature the data of this paper enable the computation of ϵ at another temperature with an error of about 1 per cent, for the range of temperatures and wave-lengths covered, the error being greater for the shorter wave-lengths.

9. *The effect upon the results of uncertainties in the values of C_2 and the melting-point of palladium.*—The values of ϵ given in Fig. 8 have seemingly depended upon C_2 and the melting-point of palladium through the values of the temperatures used in the computations, and through the explicit occurrence of C_2 in equations (2) and (4). It is shown, however, in the following paragraphs that the values of ϵ are independent of any knowledge of the values of the two constants, C_2 and the melting point of palladium.

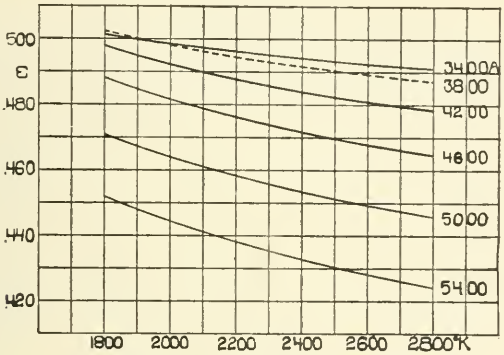


Fig. 8.—Emissive power of tungsten

The true temperature, T°K., of the tungsten has been found by the use of Worthing's scale from the known black-body temperature S°K. T, on Worthing's scale, is defined by the equation

$$\frac{1}{T}=\frac{1}{S}-\frac{\lambda_r\, ln\, \epsilon_r}{C_2},$$

where ϵ_r is the emissive power for wave-length λ_r, and the temperature S, as measured by Worthing (*loc. cit.*).

S is defined by the equation

$$\frac{1}{S}=\frac{1}{P}-\frac{\lambda_r\, ln\, R}{C_2},$$

where P°K. is the melting-point of palladium, and R is the ratio of the brightness of a black body at T to its brightness at P, measured at wave-length λ_r.

Combining these two equations

$$\frac{1}{T}=\frac{1}{P}-\frac{\lambda_r \, ln\epsilon_r R}{C_2},$$

or

$$\frac{1}{T}=\frac{1}{P}-\frac{a}{C_2}, \tag{5}$$

where a is a quantity independent of C_2 and P.

Consider the change in ϵ with λ as given by (2). Writing (2) in a more general form, by putting T for 2143° K. and P for 1822°.5 K., gives

$$\epsilon=\frac{d_t}{d_b}\,e^{\frac{C_2}{\lambda}\left(\frac{1}{T}-\frac{1}{P}\right)}.$$

Substituting for $\frac{1}{T}$ from (5),

$$\epsilon=\frac{d_t}{d_b}\,e^{-\frac{a}{\lambda}}$$

which is free from C_2 and P.

Consider the change in ϵ with T as given by (4). C_2' in (4) may be considered to be computed from the co-ordinates of two points, $\left(ln\ d_1, \frac{1}{T_1}\right)$ $\left(\text{and } ln\ d_2, \frac{1}{T_2}\right)$, which lie on the logarithmic isochromatic line for wave-length λ, by the relation

$$C_2'=\frac{\lambda\ ln\frac{d_2}{d_1}}{\frac{1}{T_1}-\frac{1}{T_2}},$$

where d_1 and d_2 are the deflections at T_1 and T_2, respectively. Substituting for $\frac{1}{T_1}$ and $\frac{1}{T_2}$ from (5),

$$C_2'=C_2\frac{\lambda\ ln\frac{d_2}{d_1}}{a_2-a_1}.$$

a_1 and a_2, given by (5), correspond to T_1 an T_2, respectively. Using (4), we may write the relation between the values of the emissive powers ϵ_1 and ϵ_2 for temperatures T_1 and T_2, respectively, at wave-length λ:

$$ln\frac{\epsilon_1}{\epsilon_2}=\frac{C_2-C_1'}{\lambda}\left(\frac{1}{T_1}-\frac{1}{T_2}\right).$$

Substituting for C_2', $\frac{1}{T_1}$ and $\frac{1}{T_2}$, this becomes

$$ln\frac{\epsilon_1}{\epsilon_2}=\frac{a_2-a_1}{\lambda}-ln\frac{d_2}{d_1},$$

which is free from C_2 and P.
Therefore the present method of determining the changes in the emissive power is independent of a knowledge of the values of C_2 and the melting-point of palladium.

10. *Summary.*—The radiation from tungsten has been studied by means of a sodium photo-electric cell and electrometer throughout the region of the spectrum from 3400 to 5400 Å, for true temperatures, on Worthing's scale (*loc. cit.*), from 1746° to 2785° K. From the measurements the values for the emissive power of tungsten have been computed; these values are independent of a knowledge of the radiation constants C_2 and the melting-point of palladium.

In conclusion, it is a pleasure to acknowlege my indebtedness to the director and members of the staff of the Nela Research Laboratory. Dr. W. E. Forsythe has calibrated lamps and manipulated the black body for me. I desire to express especially my appreciation of the many helpful suggestions and kindly criticism of Dr. A. G. Worthing. The work was carried out under the Charles F. Brush Summer Fellowship in Physics.

Nela Research Laboratory
National Lamp Works of the General Electric Company
Nela Park, Cleveland, Ohio
December 1916

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

By HARLOW SHAPLEY

FOURTH PAPER: THE GALACTIC CLUSTER MESSIER 11[1]

Of the many striking stellar clusters in the great galactic clouds extending through Ophiuchus, Sagittarius, and Scorpio, none has received as much attention as Messier 11,[2] the very rich open aggregation of stars in the small constellation of Scutum Sobieski. Discovered by Kirch two hundred and thirty-five years ago, it has since been observed and described by nearly all students of clusters, nebulae, and comets.[3] In 1836–1839 a catalogue of micrometrically determined positions of 184 of its stars was undertaken by Lamont at Munich.[4] Thirty years later Helmert at Hamburg repeated the measures, forming a catalogue of 200 stars and comparing his results with those of the earlier astronomer.[5] The favorable length of the interval since the work of Lamont and the serviceable check by Helmert's later catalogue led Stratonoff at Tachkent in Turkestan to redetermine in 1896 with the more accurate methods then available the relative position of the stars, and to deduce proper motions for the cluster as a whole and for its individual members.[6] To accomplish this the positions were measured on photographic plates, not only for the stars in the cluster, but for several hundred stars outside its bounds. The 861 objects, whose co-ordinates to the hundredth of a second of arc are tabulated by Stratonoff, include practically all the stars in an area of 700 square minutes which are brighter than a certain magnitude (near the limit of his plates). That the comparison of his positions with those of sixty years earlier gave only negative results for proper motion does not detract from the value of the work, for his result indicates the great distance of the stars involved, and the new measures of position afford a much sounder basis for the future investigation of motions. Stratonoff also determined the photographic magnitudes; but since his values are based on estimates of brightness taken from the *B.D.* catalogue, no especial accuracy is claimed for them.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 126.

[2] Messier 11 = h.2019 = G.C.4437 = N.G.C.6705; position for 1900 (from *Harvard Annals*, **60**, 214, 1908), $\alpha = 18^{\rm h}\,45^{\rm m}.7$, $\delta = -6°23'$; galactic longitude = 355°, galactic latitude = −3°. The conspicuous bright star in the south-following edge of the cluster is Lalande 35043 = B.D. −6°4929, etc.

[3] For instance, see the paper by Barnard in *Monthly Notices*, **57**, 15, 1896, and the descriptive and historical notes by Miss Clerke, *The System of the Stars*, p. 229 (London, 1905).

[4] *Annalen der königlichen Sternwarte bei München*, **17**, 305.

[5] *Publicationen der Hamburger Sternwarte*, No. 1, 1874.

[6] *Publications de l'Observatoire astronomique et physique de Tachkent*, No. 1, 1899.

The present study of colors and magnitudes in Messier 11 was undertaken chiefly because of the cluster's low galactic latitude and its position with reference to the star clouds of the southern Milky Way. The open cluster discussed in the preceding paper of this series,[1] Messier 67, is situated in galactic latitude +34°. Its constituency differs little from the non-cluster stars of that neighborhood, and the conclusion was reached that, unlike the globular clusters, it is nothing more than a condensation in an ordinary stellar field. The faint stars in that locality are of the redder color-classes, in agreement with Seares's result for the North Polar stars,[2] galactic latitude +28°. A different result might be anticipated for stars in Messier 11 and for the fainter members of the rich galactic cloud which forms its background. The determination of color-classes in such regions thus contributes directly to the problem of the extent of the Milky Way, and bears as well on the relation of open stellar groups to the surrounding clouds of stars.[3]

The large angular distance of Messier 11 from the reference field of standard magnitudes at the North Pole has made it difficult to secure good intercomparison photographs, and it has been necessary to reject many plates without measurement, either because of deformed images, or because of marked differences in the seeing in the directions of the cluster field and the North Pole. The plates finally adopted and used for the determination of magnitudes are described in Table I. The time and hour-angle for each plate refer to the middle of the exposure on the cluster.

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[1] *Mt. Wilson Contr.*, No. 117, 1916.

[2] *Ibid.*, No. 81; *Astrophysical Journal*, **39**, 361, 1914.

[3] Excellent photographs by Barnard of the star clouds in the region of Messier 11 are published in *Lick Observatory Publications*, **11**, 1915.

TABLE I

LIST OF PLATES

Plate Number	Date, G.M.T.	Kind of Plate	Number of Exposures	Exposure Time	Hour-Angle	Extinction Correction
2467.....	1915, June 7.965	Iso	3	10^{m}	23°.5 W	$+0^{M}.06$
2468.....	7.960	Seed 27	3	2	21.5 W	+0.11
2638.....	Aug. 16.804	Iso	3	5	34 W	+0.02
2639.....	16.803	Seed 27	3	2	33 W	+0.06
3180.....	1916, July 5.710	Iso	2	10	40 E	0.00
3181.....	5.704	Seed 27	3	1	42 E	−0.02
3235.....	8.753	Iso	3	15	22 E	+0.06
3236.....	8.746	Seed 27	3	3*	25 E	+0.11

* Duration of exposure on cluster uncertain; the measures on Polar Standards rejected and scale of plate and zero point determined by comparisons with results from other Seed 27 plates.

The method of measurement and reduction is sufficiently indicated in the preceding papers.[1] In order to abbreviate the catalogue of magnitudes, the residuals are omitted and a brief discussion of them is given in Tables II and III. An error in the timing of one exposure on Plate 3236 (not discovered until the plates were measured) made it necessary to determine the scale from the results of the other Seed 27 plates. To do this the mean magnitudes in the eighth column of Table II were plotted against the scale-readings for the corresponding stars on Plate 3236, and the resulting curve was employed to determine magnitudes for all stars measured on that plate.

The residuals for the different plates in Tables II and III are the deviations (in hundredths of a magnitude) from the tabulated means after the systematic deviations at the bottom of the table have been applied. The stars included in these two tables are chiefly near the center and are the most difficult in the cluster. The average errors for the more distant stars are appreciably smaller. The tabulated residuals show that no important errors are present. For one plate, which showed diversely irregular images for the more

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[1] *Mt. Wilson Contr.*, Nos. 115, 116, 117.

TABLE II

Residuals for Photographic Magnitudes

Star	Plate 2468		Plate 2639		Plate 3181		Mean of Three Plates	Plate 3236		Mean of All Plates*
	Mag.	Res.	Mag.	Res.	Mag.	Res.		Mag.	Res.	
449	14.37	− 4	14.18	−10	14.21	+ 1	14.25	14.43	+13	14.30
452	......	.....	12.71	+12	12.46	− 5	......	12.66	+ 5	12.61
459	14.27	+12	13.88	−14	13.79	−15	13.98	14.21	+17	14.04
462	12.84	− 6	12.79	+ 2	12.63	− 6	12.75	12.90	+11	12.79
466	13.90	− 7	13.77	− 7	......	.....	......	13.90	+ 4	13.86
466*a*	14.89	− 7	14.87	+ 4	......	.....	......	14.78	− 7	14.85
472	13.06	+10	12.84	+ 1	12.71	− 4	12.87	12.80	− 5	12.85
472*a*	15.74	+ 3	15.67	+ 9	15.39	−11	15.60	15.60	0	15.60
476	13.09	−18	13.15	+ 1	13.25	+19	13.16	13.14	− 2	13.16
491	12.37	− 8	12.38	+ 6	......	.....	......	12.28	− 6	12.34
501	12.07	0	12.11	+17	11.71	−15	11.96	11.95	− 1	11.96
503	14.76	+13	14.40	−10	14.48	+ 6	14.55	14.44	− 8	14.52
516	12.20	− 6	12.12	− 1	......	.....	......	12.13	− 2	12.15
517	13.73	+12	13.48	0	13.29	−11	13.50	13.49	− 1	13.50
520	15.13	− 8	14.96	−12	15.03	+ 3	15.04	15.28	+18	15.10
520*a*	15.40	−13	15.53	+13	15.39	+ 7	15.44	15.38	− 4	15.42
523	12.44	+ 8	12.30	+ 7	12.02	−13	12.25	12.23	− 2	12.25
523*a*	15.50	0	15.30	− 7	15.33	+ 4	15.38	15.43	+ 4	15.39
524	14.13	− 2	13.92	−10	14.00	+ 6	14.02	14.11	+ 7	14.04
533	13.57	+ 1	......	.....	13.24	−11	......	13.53	+ 8	13.45
533′	14.43	+10	14.11	− 9	14.16	+ 4	14.23	14.16	− 6	14.22
534	12.67	−14	12.82	+14	......	.....	......	12.61	− 9	12.70
538	......	.....	12.62	+ 8	12.50	+ 4	......	12.55	− 1	12.56
538*a*	14.60	+10	14.23	−14	14.23	− 6	14.35	14.48	+ 9	14.39
539	13.30	−11	......	.....	13.10	−10	......	13.49	+19	13.30
542	14.23	+ 8	13.84	−18	......	.....	......	14.04	0	14.04
542*a*	15.00	+13	14.51	−23	......	.....	......	14.78	+ 2	14.76
543	15.19	−22	15.49	+21	15.23	+ 3	15.30	......	.....	15.30
544	12.07	− 1	......	.....	11.71	−16	......	12.13	+16	11.97
551	12.77	+ 5	12.59	0	12.46	− 5	12.61	12.63	+ 2	12.61
553	15.09	+ 4	14.91	− 1	14.91	+ 7	14.97	14.84	−10	14.94
554	14.06	+ 1	......	.....	13.90	+ 6	......	13.85	− 9	13.94
556	13.90	+13	13.66	− 2	13.54	− 2	13.70	13.53	−13	13.66
558	14.56	+ 3	......	.....	14.41	+ 9	......	14.29	−13	14.42
562	14.53	−14	......	.....	14.65	+19	......	14.49	− 7	14.56
565	13.84	+ 7	13.68	+ 4	13.54	− 2	13.69	13.57	− 9	13.66
570*a*	15.65	+16	15.41	+ 5	15.22	− 6	15.43	15.23	−15	15.38
572	13.47	+ 4	13.36	+ 6	13.14	− 8	13.32	13.33	+ 1	13.32
572*a*	15.43	−24	15.67	+13	15.51	+ 5	15.54	15.64	+ 8	15.56
581	11.90	− 5	......	.....	11.72	− 2	......	11.91	+ 7	11.84
582	......	.....	14.26	− 3	14.29	+ 8	......	14.39	+ 8	14.31
586	13.93	+10	13.68	− 2	13.58	− 4	13.73	13.71	− 1	13.72
589	15.22	0	15.19	+10	15.06	+ 5	15.16	14.95	−16	15.11
593	13.90	− 9	13.95	+ 9	13.72	− 6	13.86	13.93	+ 5	13.88
593*a*	15.53	− 5	15.49	+ 4	15.41	+ 4	15.48	15.43	− 4	15.47
600	14.43	+13	14.04	−13	14.21	+12	14.23	14.08	−11	14.19
600*a*	15.97	−10	15.91	− 3	16.16	+30	16.01	15.81	−15	15.96
604*a*	15.87	+ 8	15.59	− 7	15.49	− 9	15.65	15.76	+ 8	15.68
607	13.64	+ 9	13.41	− 1	13.26	− 8	13.44	......	.....	13.44
622	14.37	−13	......	.....	14.56	+27	......	14.25	−14	14.39
No. of values	47		42		43			48		
Av. dev	±0.085		±0.079		±0.085			±0.076		
Syst. dev	+0.11		−0.02		−0.10			....		

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* In some cases these means differ slightly from the final mean photographic magnitudes in the catalogue, because of the application of the systematic corrections derived with the aid of this table.

distant stars, only images near the center were measured; and for Plate 3235 the relatively long exposure of 15 minutes made impossible the accurate measurement of the brighter stars.

TABLE III

RESIDUALS FOR PHOTO-VISUAL MAGNITUDES

Star	Plate 2467		Plate 2638		Plate 3180		Plate 3235		Mean Pv. Mag.	Color-Index
	Mag.	Res.	Mag.	Res.	Mag.	Res.	Mag.	Res.		
449	14.19	+ 1	14.16	+ 2	14.26	+ 2	14.20	− 4	14.20	+0.10
457	13.18	−10	13.36	+12	13.25	− 9	13.40	+ 6	13.30	+0.36
459	13.77	+ 1	13.56	−16	13.80	− 2	14.00	+18	13.78	+0.26
462*a*	14.66	+ 6	14.53	− 3	14.63	− 3	14.66	0	14.62	+0.39
472*a*	14.64	+ 3	14.57	0	14.73	+ 6	14.57	−10	14.63	+0.97
474	14.04	+12	14.17	+19	14.00	− 8	13.96	−12	14.04	+0.40
481	13.68	− 8	13.71	− 1	13.84	+ 2	13.89	+ 7	13.78	+0.10
485	13.97	−11	14.14	+10	14.07	− 7	14.21	+ 7	14.10	+0.44
492	13.72	− 2	13.76	+ 6	13.80	0	13.74	− 6	13.76	+0.34
502	13.97	− 6	13.92	− 7	14.10	+ 1	14.21	+12	14.05	+0.19
503	14.24	+ 5	14.23	+ 8	14.33	+ 8	14.04	−21	14.21	+0.31
513	14.22	+ 2	14.05	−11	14.29	+ 3	14.32	+ 6	14.22	+0.24
514	14.76	+ 5	14.61	− 6	14.83	+ 6	14.72	− 5	14.73	+0.40
520	14.47	+ 6	14.29	− 8	14.60	+13	14.35	−12	14.43	+0.67
520*a*	14.84	− 2	14.88	+ 8	15.04	+14	14.69	−21	14.86	+0.57
522	12.96	− 6	12.99	+ 1	13.03	− 5	13.18	+10	13.04	+0.57
523*a*	14.96	+ 5	14.79	− 8	14.91	− 6	15.06	+ 9	14.93	+0.46
524	14.00	+ 8	13.77	−11	13.95	− 3	14.04	+ 6	13.94	+0.10
532	14.06	+ 1	14.11	+10	14.10	− 1	14.00	−11	14.07	+0.03
533′	14.13	+ 3	14.03	− 3	14.13	− 3	14.20	+ 4	14.12	+0.10
538*a*	14.17	−14	14.31	+ 4	14.42	+ 5	14.42	+ 5	14.33	+0.06
543	14.90	+ 6	14.76	− 4	14.84	− 6	14.93	+ 3	14.86	+0.34
553	14.76	+11	14.53	− 8	14.63	− 8	14.77	+ 6	14.67	+0.27
452	12.08	+21	11.70	−13	11.89	− 8	......	.....	11.89	+0.76
461	12.74	+ 1	12.65	− 4	12.86	+ 3	......	.....	12.75	+0.43
462	12.23	+ 1	12.18	0	12.32	0	......	.....	12.24	+0.55
472	12.31	− 1	12.33	+ 5	12.39	−3	......	.....	12.34	+0.52
476	12.34	0	12.33	+ 3	12.42	− 2	......	.....	12.36	+0.80
477	12.65	− 3	12.72	+ 8	12.72	− 6	......	.....	12.70	+0.54
482	13.04	0	12.99	− 1	13.16	+ 2	......	.....	13.06	+0.43
486	12.31	−12	12.44	+ 5	12.59	+ 6	......	.....	12.45	+0.40
491	11.21	−10	11.41	+14	11.36	− 5	......	.....	11.33	+1.00
493	12.01	+11	11.76	−10	12.00	0	......	.....	11.92	+0.63
498	12.71	+ 2	12.59	− 6	12.83	+ 4	......	.....	12.71	+0.50
516	11.42	− 6	11.65	+21	11.42	−16	......	.....	11.50	+0.62
523	11.45	− 6	11.33	−14	11.81	+20	......	.....	11.53	+0.72
534	11.26	+ 1	11.16	− 5	11.40	+ 5	......	.....	11.27	+1.40
538	12.05	+ 2	11.98	− 1	12.11	− 2	......	.....	12.05	+0.55
551	12.11	+ 3	12.05	+ 1	12.15	− 3	......	.....	12.10	+0.52
No. of values	39		39		39		23			
Syst. dev.										
>13 Mag.	−0.02		−0.06		+0.04		+0.04			
<13 Mag.	−0.02		−0.06		+0.08		....			
Av. dev.	±0.052		±0.071		±0.053		±0.087			

With a few exceptions, all the magnitudes of the main catalogue (Table IV) depend upon at least two plates. The numbers in the first column are those assigned by Stratonoff. Stars marked with an asterisk are omitted from the statistical discussion, either because neighboring images are so close that the Eberhard effect may have influenced their magnitudes, or because of uncertainty in the measures. Except for close groups and double stars, the general concentration of images on the plates used is not sufficient to cause fear of perceptible errors from the Eberhard effect or similar phenomena. The residuals give no evidence of a differential effect of this kind between the plates; and, even for stars nearly in contact, whatever errors enter affect mainly magnitudes and not colors, for the densities of the stellar images on the two kinds of plates are closely comparable. In choosing stars for statistical discussion, however, liberal exclusion was made wherever possibility of such systematic errors was suspected. The results tabulated in Tables VI and VII should, therefore, be completely free from systematic or accidental errors depending on stellar condensation.

The numbers with postscript letters in Tables II, III, and IV refer to objects not listed by Stratonoff—those with appended Greek letters indicating the components of double or triple stars, and those with italic letters referring either to close faint companions or merely to neighboring uncatalogued stars of a photovisual brightness comparable with that of the stars catalogued. Using Stratonoff's stars for reference points, identification positions of the postscript stars were determined by measurements with a millimeter scale on enlarged positives. The right ascensions and declinations so obtained are listed in Table V. For positions of the Stratonoff stars reference must be made to the Tachkent catalogue.

As a matter of convenience in discussing colors and magnitudes, the cluster was divided into several concentric regions. The numbers in the second column of Table IV refer to these divisions, Region 1 including all stars within $0'.5$ from the center, Region 2 including all stars between $0'.5$ and $1'.0$, Region 3 those between $1'.0$ and $1'.5$, etc. The star at the point adopted as center of the cluster is No. 462, for which the position for 1900 is:

$$\text{R.A.} = 18^{\text{h}}\ 45^{\text{m}}\ 42^{\text{s}}.75, \qquad \text{Dec.} = -6^{\circ}\ 23'\ 12''.8.$$

TABLE IV

Photometric Catalogue of 458 Stars in Messier 11

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
204...	7	13.38	13.12	+0.26	253..	5	14.50	14.31	+0.19
205...	9	13.86	13.76	+0.10	257..	6	15.26	14.84	+0.42
206...	7	14.50	14.34	+0.16	258*.	5	14.24	13.86	+0.38
207...	8	12.75	12.24	+0.51	258*a*.	5	15.80	15.00	+0.80
208...	7	14.39	13.98	+0.41	260*.	6	14.06	13.72	+0.34
210...	8	14.76	14.58	+0.18	261*.	7	14.40	14.19	+0.21
211...	7	12.26	11.62	+0.64	262..	7	13.54	13.31	+0.23
212...	9	13.68	13.61	+0.07	263..	5	14.82	14.69	+0.13
213...	7	14.37	14.08	+0.29	264..	5	14.82	14.62	+0.20
214...	7	13.48	13.18	+0.30	265*.	7	13.34	13.32	+0.02
217...	8	14.70	14.54	+0.16	266*.	6	13.89	13.53	+0.36
218...	8	14.42	14.20	+0.22	266*a*.	6	15.24	14.82	+0.42
219...	6	14.64	14.54	+0.10	266*b*.	6	15.17	13.44	+1.73
221...	6	13.07	12.86	+0.21	267..	5	13.26	12.78	+0.48
224...	6	13.56	13.58	−0.02	267*a*.	5	14.76	13.30	+1.46
225...	6	13.78	13.64	+0.14	267*b**	5	15.20	14.70	+0.50
226*..	8	14.09	13.76	+0.33	268..	6	14.46	14.25	+0.21
226*a*..	7	14.40	14.50	−0.10	272..	5	13.76	13.36	+0.40
226*b*..	7	14.90	13.46	+1.44	273..	4	13.00	12.54	+0.46
226*c*..	8	15.38	13.50	+1.88	273*a*.	4	15.41	14.91	+0.50
227...	6	12.06	11.31	+0.75	274..	5	14.32	14.12	+0.20
229...	6	13.57	13.48	+0.09	275..	7	14.86	14.72	+0.14
230...	6	14.82	14.71	+0.11	276..	7	14.43	14.26	+0.17
232...	6	12.22	11.89	+0.33	277..	8	14.54	14.28	+0.26
233α*.	6	14.52	12.40	....	278..	4	14.32	14.11	+0.21
233β*.	6	14.68		....	280..	7	13.01	11.92	+1.09
234*..	8	14.57	14.36	+0.21	283..	4	14.08	13.92	+0.16
236...	6	13.07	12.61	+0.46	284..	5	14.49	14.26	+0.23
237...	7	11.74	11.02	+0.72	285..	4	13.08	12.38	+0.70
238...	8	14.58	14.32	+0.26	287..	4	14.56	14.28	+0.28
239...	5	13.24	13.13	+0.11	287*a*.	4	15.23	14.69	+0.54
240...	5	13.21	11.81	+1.40	288..	4	12.75	12.13	+0.62
242...	5	14.00	14.16	−0.16	291..	6	14.82	14.50	+0.32
243...	8	14.07	13.92	+0.15	292*.	4	13.22	12.80	+0.42
245...	5	12.15	11.89	+0.26	293*.	4	12.56	11.71	+0.85
247...	7	13.12	11.07	+2.05	293*a**	5	15.29	14.76	+0.53
248*..	5	14.64	14.48:	+0.16:	296..	5	13.89	13.64	+0.25
248*a**.	5	15.50	14.17:	+1.33:	296*a*.	5	14.86	14.70	+0.16
249...	6	13.92	13.80	+0.12	297..	4	12.44	11.68	+0.76
249*a*..	5	15.28	14.82	+0.46	298..	4	13.91	13.63	+0.28
249*b*..	5	15.14	14.56	+0.58	300..	6	12.28	11.86	+0.42
250...	5	13.08	12.60	+0.48	301..	6	15.16	14.64	+0.52
251...	6	13.34	13.12	+0.22	302..	4	15.24	14.77	+0.47
251*a*..	5	15.11	14.82	+0.29	303..	4	14.69	14.55	+0.14
252...	7	13.08	11.12	+1.96	307..	5	14.20	14.02	+0.18

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
308*..	4	14.96	14.65	+0.31	358..	3	13.84	13.66	+0.18
310...	4	13.04	12.52	+0.52	359..	3	14.33	14.25	+0.08
310*a*..	3	15.60	14.34	+1.26	360*.	4	14.62	14.25	+0.37
314...	5	14.28	14.16	+0.12	362..	5	13.68	13.28	+0.40
315...	4	14.48	14.19	+0.29	363..	3	14.00	13.86	+0.14
317*..	6	14.36:	14.14	+0.22:	364..	4	12.73	11.44	+1.29
320...	5	14.84	14.67	+0.17	365*.	3	13.32	12.55	+0.77
321...	6	14.86	14.56	+0.30	366..	3	13.02	12.52	+0.50
322*..	6	13.46:	13.12	+0.34:	368..	3	14.08	13.60	+0.48
323*..	4	13.54	12.82	+0.72	369*.	3	14.44	13.76	+0.68
323*a**.	4	14.66	14.51	+0.15	370*.	3	15.04	14.64	+0.40
324...	4	13.80	13.70	+0.10	372*.	3	14.56	14.10	+0.46
324*a*..	4	15.07	14.76	+0.31	373..	2	14.86	14.68	+0.18
325...	3	13.04	12.48	+0.56	374..	2	14.46	14.10	+0.36
328...	3	14.11	13.84	+0.27	375..	3	12.54	11.97	+0.57
329...	3	14.65	14.30	+0.35	376..	4	14.10	13.58	+0.52
330...	4	14.66	14.26	+0.40	376*a*.	4	15.11	14.78	+0.33
330*a*..	4	15.84	14.68	+1.16	377..	4	13.64	13.27	+0.37
331...	4	12.24	11.14	+1.10	378..	2	14.18	13.87	+0.31
333...	6	14.06	13.86	+0.20	379..	4	14.76	14.31	+0.45
334...	3	14.18	13.95	+0.23	380..	3	14.49	14.16	+0.33
335...	3	15.24	14.84	+0.40	381..	4	12.44	11.81	+0.63
336...	3	12.08	11.29	+0.79	381*a*.	4	15.29	14.27	+1.02
336*a*..	3	15.26	14.66	+0.60	383..	4	14.12	14.04	+0.08
337...	4	14.61	14.24	+0.37	384..	6	14.58	14.52	+0.06
338...	4	13.36	12.90	+0.46	385..	5	12.96	12.18	+0.78
338*a*..	5	15.60	15.00	+0.60	386..	5	14.64	14.37	+0.27
341...	3	12.42	11.78	+0.64	387..	3	12.84	11.30	+1.54
342...	3	13.22	11.47	+1.75	389..	2	14.67	14.36	+0.31
343...	4	14.18	13.74	+0.44	391..	2	13.18	12.76	+0.42
344...	5	13.23	12.62	+0.61	392..	3	15.04	14.74	+0.30
345...	4	12.06	11.32	+0.74	393..	3	13.54	13.18	+0.36
345*a*..	4	15.30	14.80	+0.50	394..	2	14.16	13.92	+0.24
346...	3	12.82	12.46	+0.36	394*a*.	2	15.34	15.00	+0.34
347...	4	13.32	11.74	+1.58	395*.	5	12.19	11.31	+0.84
348...	4	14.84	14.60	+0.24	396..	3	15.16	14.92	+0.24
348*a*..	4	15.13	14.62	+0.51	397..	4	14.98	14.62	+0.36
350...	6	14.42	14.40	+0.02	397*a*.	5	15.68	15.02	+0.66
351...	5	13.45	13.06	+0.39	398..	2	13.84	13.51	+0.33
352...	5	13.69	13.32	+0.37	400*.	3	14.68	14.28	+0.40
353...	3	13.68	13.31	+0.37	402..	4	13.06	12.53	+0.53
354...	6	14.55	14.52	+0.03	403..	3	13.53	13.24	+0.29
355...	3	14.49	14.04	+0.45	404..	3	14.36	14.02	+0.34
356*..	4	12.60	11.94	+0.66	405..	7	13.18	12.93	+0.25
357...	5	14.78	14.52	+0.26	406..	6	14.99	14.70	+0.29

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
407...	3	11.93	11.02	+0.91	456..	3	12.30	11.44	+0.86
408...	3	13.00	12.52	+0.48	456*a*.	3	15.01	14.64	+0.37
409...	2	13.74	13.11	+0.63	457*.	1	13.66	13.30	+0.36
410*..	5	14.80	14.18	+0.62	458..	2	14.09	13.84	+0.25
411...	1	14.04	13.84	+0.20	459..	1	14.04	13.78	+0.26
411*a*..	2	15.42	15.10	+0.32	460..	2	12.97	12.63	+0.34
412...	2	13.54	12.18	+1.36	460*a*.	3	15.50	14.90	+0.60
414*..	3	13.96	13.54	+0.42	461..	2	13.18	12.75	+0.43
417*..	5	14.64	14.13	+0.51	461*a*.	2	15.59	14.90	+0.69
419...	3	15.36	14.80	+0.56	462..	1	12.79	12.24	+0.55
419*a*..	4	15.00	14.72	+0.28	462*a*.	1	15.01	14.62	+0.39
420...	2	13.42	13.28	+0.14	464..	3	13.94	13.74	+0.20
420*a*..	3	15.43	14.99	+0.44	465..	3	13.12	12.62	+0.50
421...	3	13.13	12.60	+0.53	466*.	1	13.82	13.34	+0.48
421*a**.	3	15.42	14.56:	+0.86:	466*a**	1	14.82	13.40	+1.42
422*..	3	13.22	12.66	+0.56	467..	4	15.14	14.64	+0.50
423...	1	13.46	11.83	+1.63	468..	4	13.91	13.62	+0.29
424...	6	12.72	12.23	+0.49	468*a*.	4	15.10	14.67	+0.43
425...	2	12.62	12.12	+0.50	469..	2	13.78	13.42	+0.36
426*..	3	13.21	12.67	+0.54	470*.	1	14.48	13.85	+0.63
427...	1	13.26	11.80	+1.46	471..	4	12.62	12.04	+0.58
428*..	2	14.89	14.30	+0.59	472..	1	12.86	12.34	+0.52
430...	4	13.32	12.88	+0.44	472*a*.	1	15.60	14.63	+0.97
431...	1	12.55	12.11	+0.44	474*.	1	14.44	14.04	+0.40
431*a*..	1	16.00	15.28	+0.72	475..	6	14.42	14.40	+0.02
432*..	1	13.64	13.32	+0.32	476..	1	13.16	12.36	+0.80
433...	2	13.79	12.16	+1.63	477*.	2	13.24	12.70	+0.54
436...	4	14.12	13.74	+0.38	480..	5	13.52	13.28	+0.24
437*..	3	11.46	10.69	+0.77	481..	2	13.88	13.78	+0.10
438*..	2	14.84	14.38:	+0.46:	482*.	2	13.49	13.06	+0.43
439...	6	13.64	13.49	+0.15	483..	6	14.32	14.23	+0.09
440*..	5	14.18	14.10	+0.08	484..	4	14.15	13.89	+0.26
442*..	5	14.00	13.82	+0.18	485..	1	14.54	14.10	+0.44
443...	5	13.58	13.32	+0.26	486..	1	12.85	12.45	+0.40
444...	4	14.61	14.25	+0.36	487..	2	12.39	11.59	+0.80
445...	1	12.74	12.42	+0.32	488..	6	14.52	14.51	+0.01
445*a*..	1	15.24	14.78	+0.46	489..	3	13.40	12.91	+0.49
446...	3	13.58	13.27	+0.31	490*.	3	13.12	12.62	+0.50
447...	2	14.06	13.64	+0.42	490*a**	3	14.56	14.08	+0.48
448...	5	14.02	13.94	+0.08	491*.	2	12.33	11.33	+1.00
449...	1	14.30	14.20	+0.10	492..	1	14.10	13.76	+0.34
451...	6	13.09	12.84	+0.25	493..	2	12.55	11.92	+0.63
452...	1	12.65	11.89	+0.76	494..	3	12.32	11.47	+0.85
454...	6	12.66	12.16	+0.50	494*a*.	3	15.14	14.80	+0.34
455...	6	14.84	14.62	+0.22	495*.	2	12.42	11.59	+0.83

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
496α*.	3	14.70	13.87	+0.83	540*.	3	12.08	11.02	+1.06
496β*.	3	13.14	12.36	+0.78	541..	5	14.35	14.16	+0.19
496γ*.	3	14.80	13.76	+1.04	542*.	2	14.00	13.61	+0.39
497...	3	14.82	14.43	+0.39	542a*	2	14.74	14.00	+0.74
498...	1	13.21	12.71	+0.50	543..	2	15.20	14.86	+0.34
499*..	5	13.50	13.22	+0.28	544..	3	11.97	11.35	+0.62
501...	4	11.96	11.11	+0.85	545*.	2	9.3:	8.5	+0.8:
502...	2	14.24	14.05	+0.19	546*.	4	13.47	12.68	+0.79
503*..	2	14.52	14.21	+0.31	548*.	4	12.23	11.32	+0.91
504*..	3	12.31	11.94	+0.37	551..	2	12.62	12.10	+0.52
505*..	2	12.80	11.92	+0.88	552..	6	14.20	14.28	−0.08
507*..	3	13.02	11.81	+1.21	553..	2	14.94	14.67	+0.27
508...	6	13.23	12.93	+0.30	553a.	2	15.77	15.12	+0.65
509*..	2	13.28	12.46	+0.82	554..	3	13.93	13.84	+0.09
513*..	2	14.46	14.22	+0.24	555*.	3	14.74	12.91	+1.83
514...	2	15.13	14.73	+0.40	556..	4	13.66	13.50	+0.16
515...	7	14.09	14.08	+0.01	556a.	4	16.17	15.08	+1.09
516...	3	12.12	11.50	+0.62	557..	4	13.56	11.50	+2.06
517...	2	13.50	13.07	+0.43	557a.	5	15.14	14.85	+0.29
519...	4	13.30	12.88	+0.42	557b.	5	15.14	14.68	+0.46
519a..	4	15.56	15.00	+0.56	558..	3	14.42	14.32	+0.10
520...	2	15.10	14.43	+0.67	558a.	4	16.68	14.68	+2.00
520a..	2	15.43	14.86	+0.57	559*.	3	15.08	14.38	+0.70
522...	2	13.61	13.04	+0.57	560*.	3	12.80	11.11	+1.69
523...	3	12.25	11.53	+0.72	561*.	2	14.78	13.92	+0.86
523a..	3	15.39	14.93	+0.46	562..	3	14.55	14.28	+0.27
524...	2	14.04	13.94	+0.10	563..	2	13.12	12.61	+0.51
525*..	3	12.08	11.14	+0.94	563a.	3	15.48	15.01	+0.47
525a*.	3	14.58	14.21	+0.37	564*.	3	14.28	13.40:	+0.88:
526...	4	13.63	13.06	+0.57	564a*	2	15.36	14.56	+0.80
526a*.	4	15.41	14.66	+0.75	564b*	3	15.36	14.52	+0.84
528...	7	13.54	13.48	+0.06	565..	4	13.66	13.62	+0.04
529...	5	13.48	13.36	+0.12	567..	3	14.61	14.40	+0.21
532...	3	14.10	14.07	+0.03	568*.	3	13.31	12.16	+1.15
532a..	4	15.47	14.76	+0.71	568a*	3	15.46	14.67	+0.79
533...	3	13.44	13.22	+0.22	569..	6	13.26	12.96	+0.30
533a..	3	15.93	14.92	+1.01	570*.	4	13.20	12.87	+0.33
533′..	2	14.22	14.12	+0.10	570a.	5	15.38	14.69	+0.69
534*..	2	12.67	11.27	+1.40	571*.	3	13.16	12.16	+1.00
536...	5	14.58	14.22	+0.36	572..	4	13.33	12.94	+0.39
536a..	5	15.37	14.80	+0.57	572a.	4	15.54	14.96	+0.58
538...	2	12.60	12.05	+0.55	573..	3	13.13	12.74	+0.39
538a..	2	14.39	14.33	+0.06	573a.	3	15.34	14.88	+0.46
539...	3	13.29	11.31	+1.98	574..	3	12.94	12.41	+0.53
539a*.	3	15.50	14.57	+0.93	576*.	3	12.39	11.65:	+0.74:

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
576*a**.	3	14.30	14.02:	+0.28:	614*a**	5	16.12	14.12	+2.00
577...	4	12.22	11.44	+0.78	615..	4	13.92	13.70	+0.22
579α*.	4	14.72	14.81	−0.09	616..	6	13.89	13.66	+0.23
579β*.	4	14.38	13.73	+0.65	617..	8	13.25	12.78	+0.47
579*a*..	4	15.32	14.90	+0.42	618..	7	14.09	13.84	+0.25
580*..	3	14.34	14.06	+0.28	619..	5	13.86	13.36	+0.50
581...	4	11.85	11.08	+0.77	620..	7	13.33	13.06	+0.27
581*a*..	4	14.56	14.35	+0.21	621..	7	14.10	13.80	+0.30
582...	4	14.35	14.32	+0.03	622..	4	14.39	13.78	+0.61
583...	6	13.84	13.69	+0.15	622*a*.	4	15.36	14.78	+0.58
585...	3	14.05	13.76	+0.29	623..	5	13.83	13.62	+0.21
586...	4	13.73	13.60	+0.13	623*a*.	5	16.00	15.10	+0.90
588...	3	15.14	14.88	+0.26	624..	4	14.04	14.00	+0.04
588*a*..	4	15.30	15.00	+0.30	630..	5	13.54	13.52	+0.02
589...	4	15.11	14.73	+0.38	631..	7	12.46	11.94	+0.52
590...	3	13.04	11.20	+1.84	633..	8	13.37	13.18	+0.19
591...	7	14.64	14.15	+0.49	634..	6	13.46	13.22	+0.24
592*..	5	14.22	13.88	+0.34	635..	5	13.23	13.01	+0.22
593...	3	13.88	13.58	+0.30	637..	7	15.08	14.69	+0.39
593*a*..	3	15.47	14.92	+0.55	638..	5	14.88	14.72	+0.16
594...	4	13.28	12.80	+0.48	639..	6	14.64	14.44	+0.20
596...	6	14.43	14.10	+0.33	640*.	6	13.12:	13.04:	+0.08:
597...	7	12.78	11.92	+0.86	643..	6	15.08	14.60	+0.48
598...	4	13.02	11.46	+1.56	644..	5	14.30	13.94	+0.36
598*a*..	4	15.40	14.73	+0.67	645..	7	15.15	14.67	+0.48
599...	5	12.40	11.80	+0.60	647..	5	12.66	11.98	+0.68
600...	5	14.19	13.97	+0.22	648..	6	13.54	13.16	+0.38
600*a*..	5	15.96	15.00	+0.96	649*.	5	14.77	14.32	+0.45
601*..	8	11.69	10.68:	+1.01:	650..	6	14.54	14.58	−0.04
602...	4	14.02	13.80	+0.22	652..	8	13.16	12.81	+0.35
603...	7	12.83	12.14	+0.69	653..	6	13.52	13.12	+0.40
604...	4	13.34	12.94	+0.40	654..	6	13.36	11.92	+1.44
604*a*..	3	15.68	14.90	+0.78	655..	6	14.97	14.62	+0.35
606...	4	13.72	13.31	+0.41	656..	7	14.31	13.67	+0.64
606*a*..	4	16.38	15.57	+0.81	657..	6	14.49	14.04	+0.45
607...	4	13.48	13.34	+0.14	658..	6	14.28	13.93	+0.35
608...	5	14.45	14.23	+0.22	660..	6	14.39	14.14	+0.25
609...	5	13.21	11.20	+2.01	662..	7	12.47	12.00	+0.47
610...	4	13.48	13.27	+0.21	663..	6	11.90	11.22	+0.68
610*a*..	4	15.52	14.84	+0.68	664..	7	12.79	12.30	+0.49
610*b*..	4	15.26	14.54	+0.72	666..	8	13.45	12.01	+1.44
611...	4	13.39	12.95	+0.44	668..	7	14.82	14.64	+0.18
612...	6	14.82	14.58	+0.24	670..	7	13.92	13.60	+0.32
613...	4	14.15	13.99	+0.16	671..	7	13.66	13.40	+0.26
614*..	5	15.48	14.62	+0.86	672..	6	12.50	12.12	+0.38

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
674...	7	13.28	11.84	+1.44	680..	7	13.58	12.84	+0.74
675...	7	13.62	13.30	+0.32	681..	6	14.90	14.51	+0.39
677...	8	15.17	14.76	+0.41	682..	9	13.05	12.72	+0.33
679...	6	15.10	14.66	+0.44	685..	7	14.92	14.62	+0.30

Notes to the Catalogue

Star

233. Components not separated on isochromatic plates.

248. Components not separated on Seed 27 plates.

421*a*. A close double, components unequal.

461. Chosen as central star by Stratonoff.

462. Position of this star adopted as center of cluster; it is 32″ south of No. 461.

470. Close double, components not separately measured.

545. This bright star, *B.D.* −6°4929, is No. 1 in the catalogues of Lamont and Helmert, and is the star upon which the Mount Wilson plates were centered. It is a minute of arc distant from the center of the cluster, and probably is not physically connected with the group. The star is too bright for accurate measurement on photographs taken with the full aperture of the 60-inch reflector. The visual magnitude according to the *B.D.* catalogue is 8.8, and, according to Lamont and to Helmert, 9.0 and 8.6, respectively. Stratonoff's photographic magnitude, which, however, is based on visual standards, is 8.6. No proper motion has been detected in this or in any of the reference stars (Stratonoff, *op. cit.*, p. 24).

555. May be variable. About half a magnitude fainter July 8, 1916, than June 7, 1915. The value of the color-index given in the table is of the right order.

560. Close double, components not separately measured.

570. Close double, components not separately measured.

640. A close double; components equally bright, both photographically and photo-visually but not separately measured.

The arrangement of the material in Tables VI and VII follows that of the analogous tables in preceding contributions.[1] The 14 stars fainter than photo-visual magnitude 15.0 are excluded from Table VII. In the region covered, which is a square 6′ on a side with star No. 462 as center, all stars brighter than 15.0 are included. The cluster itself is apparently limited to a little more than half of this area, and is also limited chiefly to the brighter magnitudes here considered. The density of the stars fainter than magnitude 14.0 or 14.5 is little, if any, greater in the cluster than outside, and there

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[1] Tables XII and XIII of *Mt. Wilson Contr.*, No. 116, and Tables VII and VIII of No. 117.

TABLE V

Positions of Postscript Stars

Star	R.A. 1900	Decl. 1900	Star	R.A. 1900	Decl. 1900
226*a*........	$18^{h}45^{m}31^{s}.8$	$-6^{\circ}21'12''$	494*a*.......	$18^{h}45^{m}44^{s}.0$	$-6^{\circ}24'39''$
226*b*........	32.8	21 4	496α.......	43.9	21 51
226*c*........	32.3	20 30	496β.......	44.0	21 52
233α........	32.6	23 18	496γ.......	44.1	21 53
233β........	32.6	23 20	519*a*.......	45.3	24 46
248*a*........	33.4	22 55	520*a*.......	44.4	23 28
249*a*........	34.9	21 58	523*a*.......	45.9	22 16
249*b*........	34.7	22 4	525*a*.......	45.2	24 33
251*a*........	34.3	24 28	526*a*.......	45.5	21 38
258*a*........	34.5	24 16	532*a*.......	46.0	21 57
266*a*........	34.7	21 32	533*a*.......	45.1	24 18
266*b*........	34.8	21 37	536*a*.......	46.5	21 29
267*a*........	34.8	23 37	538*a*.......	45.5	23 2
267*b*........	34.2	23 55	539*a*.......	46.0	24 19
273*a*........	35.3	23 10	542*a*.......	45.9	23 25
287*a*........	37.0	24 13	553*a*.......	46.3	22 57
293*a*........	36.2	22 5	556*a*.......	46.6	22 6
296*a*........	37.0	21 39	557*a*.......	45.9	25 6
310*a*........	37.3	23 40	557*b*.......	46.4	25 3
323*a*........	37.6	21 55	558*a*.......	47.5	24 26
324*a*........	38.2	22 6	563*a*.......	46.8	22 57
330*a*........	37.2	24 9	564*a*.......	46.6:	23 16
336*a*........	38.2	22 32	564*b*.......	46.8	23 18
338*a*........	37.7	21 23	568*a*.......	46.8	23 47
345*a*........	38.2	24 54	570*a*.......	48.1	21 41
348*a*........	38.7	21 32	572*a*.......	47.6	21 43
376*a*........	38.9	24 46	573*a*.......	47.3	22 58
381*a*........	39.7	24 41	576*a*.......	47.5	22 34
394*a*........	40.9	22 48	579α.......	47.5	24 47
397*a*........	40.5	21 17	579β.......	47.6	24 45
411*a*........	41.3	22 53	579*a*.......	47.1	24 50
419*a*........	41.3	24 50	581*a*.......	47.5	23 26
420*a*........	41.9	22 11	588*a*.......	48.6	22 42
421*a*........	41.9	24 22	593*a*.......	48.2	23 50
431*a*........	41.2	23 29	598*a*.......	48.6	22 47
445*a*........	41.8	22 54	600*a*.......	48.4	21 24
456*a*........	42.9	22 4	604*a*.......	48.3	23 30
460*a*........	42.6	24 19	606*a*.......	49.5	22 17
461*a*........	42.2	22 31	610*a*.......	50.0	22 46
462*a*........	42.6	23 17	610*b*.......	50.2	22 33
466*a*........	42.9	23 25	614*a*.......	49.4	21 54
468*a*........	43.9	24 57	622*a*.......	50.0	23 36
472*a*........	43.2	23 30	623*a*.......	50.2	24 48
490*a*........	44.1	24 15			

TABLE VI

Frequency of Colors and Magnitudes

Distance from Center	Limits of Pv. Mag.	Color-Class							All Colors
		b	*a*0–*a*4	*a*5–*a*9	*f*	*g*	*k*	*m*	
0′.0 to 1′.0	11.0–11.5..	......	......	......	......	......	......	......	......
	11.5–12.0..	......	......	......	2	1	1	1	5
	12.0–12.5..	......	......	1	7	1	1	1	11
	12.5–13.0..	......	......	1	4	......	......	......	5
	13.0–13.5..	......	1	1	3	......	......	......	5
	13.5–14.0..	......	2	7	1	......	......	......	10
	14.0–14.5..	......	4	2	2	......	......	......	8
	14.5–15.0..	......	1	3	4	1	......	......	9
	≥15.0..	......	......	2	2	......	......	......	4
1.0 to 2.0	11.0–11.5..	......	......	......	5	5	3	3	16
	11.5–12.0..	......	......	......	6	......	1	1*	8
	12.0–12.5..	......	......	1	5	......	......	......	6
	12.5–13.0..	......	......	2	14	......	......	......	16
	13.0–13.5..	......	1	7	2	......	......	......	10
	13.5–14.0..	......	9	11	4	......	......	......	24
	14.0–14.5..	......	6	12	3	1	1	......	23
	14.5–15.0..	......	1	11	23	2	......	1	38
	≥15.0..	......	......	1	2	2	......	......	5
2.0 to 3.0	11.0–11.5..	......	......	......	2	......	......	1†	3
	11.5–12.0..	......	......	2	3	......	2	......	7
	12.0–12.5..	......	......	1	3	......	......	......	4
	12.5–13.0..	......	......	4	4	......	......	......	8
	13.0–13.5..	......	4	8	4	......	1	1	18
	13.5–14.0..	1	5	7	......	......	......	......	13
	14.0–14.5..	2	7	9	1	......	......	......	19
	14.5–15.0..	1	9	11	10	......	......	......	31
	≥15.0..	......	......	......	2	3	......	......	5
≥3.0	11.0–11.5..	......	......	......	1	......	......	2‡	3
	11.5–12.0..	......	......	......	2	2	1	......	5
	12.0–12.5..	......	......	......	4	......	1	......	5
	12.5–13.0..	......	......	3	2	......	......	......	5
	13.0–13.5..	......	2	6	......	......	1	......	9
	13.5–14.0..	......	3	3	2	......	......	1	9
	14.0–14.5..	......	3	4	1	......	......	......	8
	14.5–15.0..	1	4	2	2	......	......	......	9
	≥15.0..	......	......	......	......	......	......	......	0
All of cluster	11.0–11.5..	......	......	......	8	5	3	6	22
	11.5–12.0..	......	......	2	13	3	5	2	25
	12.0–12.5..	......	......	3	19	1	2	1	26
	12.5–13.0..	......	......	10	24	......	......	......	34
	13.0–13.5..	......	8	22	9	......	2	1	42
	13.5–14.0..	1	19	28	7	......	......	1	56
	14.0–14.5..	2	20	27	7	1	1	......	58
	14.5–15.0..	2	15	27	39	3	......	1	87
	≥15.0..	......	......	3	6	5	......	......	14
	All magnitudes....	5	62	122	132	18	13	12	364

* Color-Index = +2.06. † Color-Index = +2.01. ‡ Contains one star with color-index of +2.05.

TABLE VII

DISTANCE AND COLOR CLASS

DISTANCE FROM CENTER	QUANTITY	COLOR-CLASS						ALL COLORS
		a0–a4	*a5–a9*	*f*	*g*	*k*	*m*	
0′.0 to 0′.5	No. stars...	1	5	8 (1)	2	1	1	18
	Av. Pv. mag.	14.20	13.68	12.83	13.50	11.80	11.83	13.10
	Av. C. I....	+0.10	+0.30	+0.51	+0.88	+1.46	+1.63	+0.59
0.5 to 1.0	No. stars...	7	10	15 (4)	1	1	1	35
	Av. Pv. mag.	14.03	13.92	13.21	11.59	12.18	12.16	13.47
	Av. C. I....	+0.12	+0.31	+0.53	+0.80	+1.36	+1.63	+0.45
1.0 to 1.5	No. stars...	6	23	24 (8)	4	2	3	62
	Av. Pv. mag.	14.00	13.91	13.29	12.21	12.82	11.33	13.41
	Av. C. I....	+0.10	+0.30	+0.56	+0.91	+1.40	+1.86	+0.53
1.5 to 2.0	No. stars...	11	20	38 (12)	5	3	2*	79
	Av. Pv. mag.	13.87	14.02	13.46	13.26	11.55	13.09	13.56
	Av. C. I....	+0.11	+0.30	+0.55	+1.04	+1.48	+2.03	+0.53
2.0 to 2.5	No. stars...	13†	19	14 (5)	0	2	1	49
	Av. Pv. mag.	14.12	13.85	13.39	....	12.56	11.20	13.68
	Av. C. I....	+0.11	+0.27	+0.55	....	+1.43	+2.01	+0.39
2.5 to 3.0	No. stars...	16‡	23	13 (2)	0	1	1	54
	Av. Pv. mag.	14.15	13.72	13.30	....	11.92	13.44	13.71
	Av. C. I....	+0.06	+0.28	+0.49	....	+1.44	+1.73	+0.32
3.0 to 3.5	No. stars...	7§	13	11 (5)	2	2	2‖	37
	Av. Pv. mag.	14.29	13.61	12.76	11.92	12.65	11.10	13.21
	Av. C. I....	+0.09	+0.29	+0.57	+0.98	+1.44	+2.00	+0.53
>3.5	No. stars...	6	5	3 (0)	0	1	1	16
	Av. Pv. mag.	13.93	13.67	13.26	....	12.01	13.50	13.58
	Av. C. I....	+0.14	+0.28	+0.46	....	+1.44	+1.88	+0.44
All of cluster	No. stars...	67	118	126 (38)	14	13	12	350
	Av. Pv. mag.	14.07	13.83	13.27	12.68	12.20	12.04	13.50
	Av. C. I....	+0.10	+0.29	+0.54	+0.96	+1.44	+1.88	+0.467

* Color-indices are +2.00 and +2.06.
† Contains one star with color-index of −0.16.
‡ Contains three stars with color-indices of −0.08, −0.04, and −0.02.
§ Contains one star with color-index of −0.16.
‖ Contains one star with color-index of +2.05.

is no evidence on any of the photographs of the existence of the highly concentrated background of faint stars that is typical of truly globular clusters. In fact, the cluster is apparently a condensation, on a rich background, of 150 or 200 stars whose magnitudes are between 11.0 and 14.5. The distribution (number of stars per square minute for successive regions) is illustrated in Fig. 1.

No conspicuous difference in average color or average magnitude is evident between the center of the cluster and the outermost region. In this similarity of cluster stars to the non-cluster stars in the immediate neighborhood, the conditions here are like those in Messier 67, the open cluster discussed in the foregoing paper. The result supports the hypothesis that open clusters are an integral part of the galactic system and in most characteristics differ little from their environment.

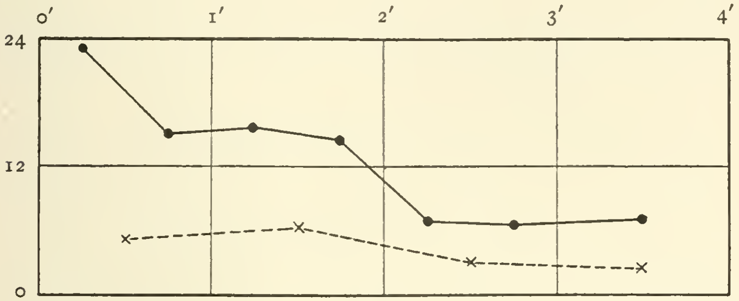

FIG. 1.—Distribution of stars in Messier 11. Full line, all stars brighter than pv. mag. 15.0; broken line, stars between pv. mag. 14.0 and 15.0.

Ordinates: Number of stars per square minute
Abscissae: Distance from center of the cluster

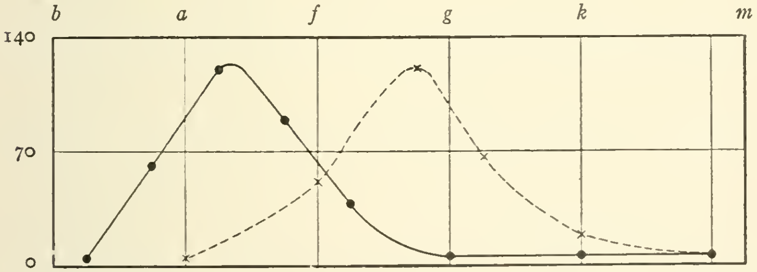

FIG. 2.—Relative frequency of colors. Full line, Messier 11; broken line, Messier 67. Ordinates: Numbers of stars. Abscissae: Color classes.

Curves of relative frequency of colors are shown in Fig. 2. The data for Messier 67 are taken from Table VII of *Contribution* No. 117. The distribution of color-classes is seen to be totally different in the two open groups. The color-index of maximum frequency is about $0\overset{M}{.}6$ bluer for the cluster in the star clouds than for the one of higher galactic latitude. Moreover, the small color-indices in Messier 11 are not confined to the cluster itself and its immediate surroundings, but occur throughout that region of the sky. The point will be treated further in the following paper of this series, which deals with the colors in several parts of the surrounding galactic clouds.

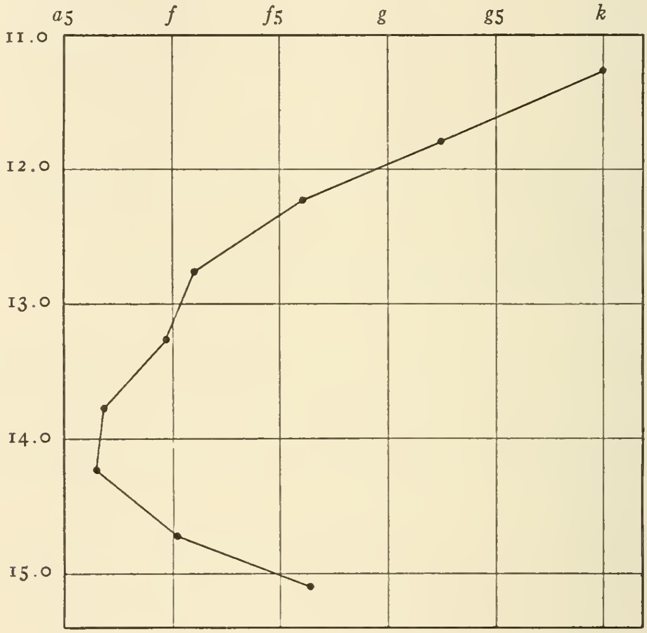

FIG. 3.—Relation between color and luminosity in Messier 11
Ordinates: Color classes. Abscissae: Apparent magnitudes

In the same paper will appear a discussion of the decrease of brightness with decreasing color-index—a point wherein Messier 11 bears a resemblance to the Hercules cluster. For the present the average color-index for successive intervals of magnitude is shown in Table VIII, and a plot of the results appears in Fig. 3. To supplement the lower part of the curve, provisional color-indices of a number of faint stars have been determined from Plates 3235 and 3236. For 34 objects the average photo-visual magnitude is 15.34, and the average color-index +1.03; but because of a selection, uncertain in amount, of some stars peculiarly red and others peculiarly blue, the result for this group of fainter stars is not of high weight.

TABLE VIII

MAGNITUDE AND AVERAGE COLOR

Interval of Pv. Mag.	Number of Stars	Mean Pv. Mag.	Mean C.I.
11.0 to 11.5..	22	11.26	+1.20
11.5 to 12.0..	25	11.80	+0.90
12.0 to 12.5..	26	12.22	+0.64
12.5 to 13.0..	34	12.75	+0.44
13.0 to 13.5..	42	13.25	+0.39
13.5 to 14.0..	56	13.75	+0.27
14.0 to 14.5..	58	14.23	+0.26
14.5 to 15.0..	87	14.71	+0.41
≥15.0..	14	15.09	+0.66

SUMMARY

1. A catalogue has been made of the photographic and photovisual magnitudes of 458 stars within four minutes of arc of the center of the open galactic cluster Messier 11, which is situated in one of the densest star clouds in the sky. Although considerable difficulty was experienced in obtaining satisfactory series of plates, the average probable error of a resulting color-index does not exceed a tenth of a magnitude.

2. It seems probable that the cluster proper is composed of not more than 200 stars, nearly all of which are brighter than magnitude 14.5. The diameter of the group appears to be less than five minutes of arc.

3. In absolute values and frequency of color-indices, the stars in the cluster are not unlike the stars in its neighborhood; but they average half a magnitude bluer than those of the same magnitude in and near the open cluster Messier 67. The galactic latitude of the latter is +34°, of the former −3°.

4. The presence of small color-indices among the faint stars in this low galactic region must have a significant bearing on the extent of the Milky Way.

MOUNT WILSON SOLAR OBSERVATORY
November 1916

THE COLOR-CURVE OF XZ CYGNI[1]

By MARTHA BETZ SHAPLEY

The progressive change of spectral type throughout the period of light-variation, recently found for Cepheid variables, denotes a synchronous and continuous change of color-index. As detailed studies of spectral variation will be possible for only the brighter stars, the color-curves afford the best means of inferring the changes in spectral class of the fainter Cepheids, particularly of those with half-day periods. As yet only one or two variables of this type have definitive color-curves. A further contribution to the subject is made in the present paper, which deals with the photographic and photo-visual light-curves of XZ Cygni,[2] a cluster-type variable with unusually rapid change of light.

Ninety-five multiple-exposure photographs, of which 50 are on Seed "27" and 45 on Cramer "Instantaneous Iso" plates, were obtained by Mr. Seares and Mr. Shapley, and kindly placed at my disposal. The plates are described in Tables I and II. An asterisk after the number in the first column indicates that in addition to the exposures on the field of the variable, the plate was also exposed on the field of the North Polar Standards to determine the magnitudes of the comparison stars. For such plates the number after the comma in the sixth column refers to the exposures on the Pole. The magnitude in the fifth column is the mean deduced from all the exposures on one plate. The light-elements used in computing phases are those of Enebo as corrected by Martin and Plummer:[3]

$$\text{Maximum} = \text{J. D. } 2417201.2542 + 0^{d}466586 \cdot E$$

which represent the Mount Wilson observations with sufficient accuracy.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 128.

[2] XZ Cygni = B.D. + 56°2257 = A.Oe. 19376 = A.G. Helsingfors 10573. The position for 1900.0 from *Hamburger Astronomische Abhandlungen*, **1**, No. 3, p. 95, 1909, is: R.A. = $19^{h}\ 30^{m}\ 24^{s}92$, Dec. = $56°10'22''8$.

[3] *Monthly Notices*, **74**, 226, 1914.

Because of the great difference in the brightness of the available comparison stars, diaphragms, as indicated in the seventh column, were employed to make the images comparable with those of the variable. The duration of exposure was one minute for the Seed "27" and three minutes for the Isochromatic plates.

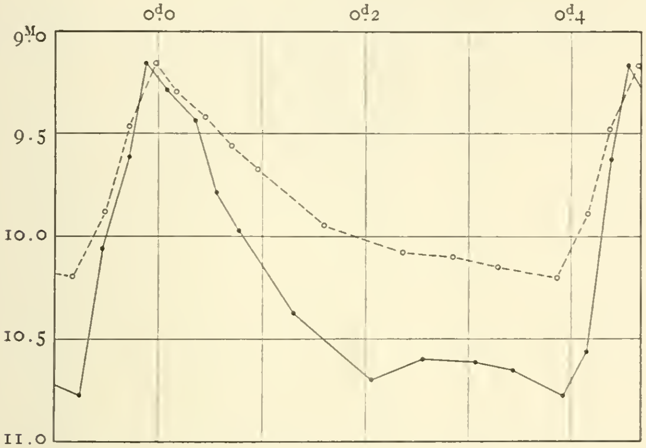

FIG. 1.—Mean photo-visual (broken line) and photographic light-curves of XZ Cygni

The methods of measurement and reduction were those generally employed in photometry with the 60-inch reflector.[1] The North Polar plates yield photographic and photo-visual magnitudes for the comparison stars as shown in Table III. For these determinations, Plates 1904, 1926, 1939, 2860, and 1911, 1927, 1954, 2861 were not used because of the distorted images of polar stars.

The observations in Tables I and II were combined into normals in order of phase, generally four in a group, giving the data of Table IV. Plotting the mean magnitudes against phase, we have the light-curves represented in Fig. 1. Whatever irregularities exist in the shape of the maximum at different epochs and in the color-change are necessarily obscured in a mean curve; and the period of XZ Cygni is too long to permit a determination of the complete color-variation at a single epoch.

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[1] Seares, *Mt. Wilson Contr.*, No. 80; *Astrophysical Journal*, **39**, 307, 1914.

TABLE I

Photographic Observations of XZ Cygni

Plate Number	Date	Julian Day and Gr. H.M.T.	Phase	Pv. Mag.	No. Exposures	Diaphragms	Normal Number
1715....	1914 July 19	2420333.901	$0^{d}.455$	9.22	5	14, 9, 6	13
1717....		.915	0.002	9.45	5	32, 14, 9, 6	1
1719....		.932	0.019	9.45	5	14, 9, 6	1
1721....		.949	0.036	9.82	5	32, 14, 9	2
1723....		.964	0.051	10.04	5	32, 14, 9	3
1725....		.979	0.066	9.90	5	32, 14, 9	4
1727....		.993	0.080	9.93	5	14	4
1851....	Aug. 26	2420371.656	0.417	10.23	8	14, 9	11
1853....		.674	0.435	9.49	8	14, 9	12
1855....		.693	0.454	9.14	8	14, 9	13
1857....		.712	0.006	9.13	8	14, 9	1
1859....		.732	0.026	9.30	8	14, 9	2
1861....		.751	0.045	9.42	8	14, 9	3
1863....		.769	0.063	9.97	8	14, 9	3
1865....		.787	0.081	10.12	8	14, 9	4
1867....		.805	0.099	10.29	8	14, 9	5
1869....		.825	0.119	10.25	8	14, 9	5
1871....		.849	0.143	10.36	8	14, 9	5
1873....		.867	0.161	10.64	8	14, 9	5
1875....		.885	0.179	10.83	8	14, 9	6
1877....		.906	0.200	10.78	8	14, 9	6
1879....		.924	0.218	10.78	8	14, 9	6
1904*...	Sept. 18	2420394.799	0.230	10.42	8, 2	14, 9	6
1906....		.817	0.248	10.68	8	14, 9	7
1908....		.837	0.268	10.62	8	14, 9	7
1910....		.855	0.286	10.62	8	14, 9	8
1926*....	Sept. 19	2420395.844	0.342	10.58	2, 2	14, 9	9
1939*....	Sept. 20	2420396.737	0.302	10.57	8, 2	14, 9	8
1941....		.758	0.323	10.52	8	14, 9	8
1943....		.775	0.340	10.63	8	14, 9	9
1945....		.793	0.358	10.66	8	14, 9	9
1947....		.813	0.378	10.69	8	14, 9	10
1949....		.836	0.401	10.69	8	14, 9	10
1951....		.853	0.418	10.32	8	14, 9	11
1953....		.871	0.436	9.47	8	14, 9	12
2027....	Sept. 22	2420398.758	0.457	9.12	8	14, 9	13
2029....		.775	0.007	9.12	8	14, 9	1
2031....		.795	0.027	8.95	8	14, 9	2
2033....		.813	0.045	9.70	8	14, 9	2
2035....		.832	0.064	9.72	8	14, 9	3
2037....		.854	0.086	9.93	8	14, 9	4
2043....	Sept. 23	2420399.623	0.388	11.07	4	32, 14	10
2045....		.638	0.403	10.84	4	32, 9	11
2047....		.652	0.417	10.85	4	32, 9	11
2051....		.682	0.447	9.89	4	32, 6	12
2860*....	1915 Nov. 30	2420832.624	0.398	10.68	2, 2	14	10
3214*....	1916 July 7	2421052.703	0.248	10.52	3, 2	14, 9	7
3215*....		.708	0.253	10.60	3, 2	14, 9	7
3232*....	July 8	2421053.704	0.316	10.72	3, 2	14	8
3234*....		.714	0.326	10.72	2, 2	14	9

TABLE II

PHOTO-VISUAL OBSERVATIONS OF XZ CYGNI

Plate Number	Date	Julian Day and Gr. H.M.T.	Phase	Pg. Mag.	No. Exposures	Diaphragms	Normal Number
1716....	1914 July 19	2420333.909	$0^{d}.463$	9.28	3	32, 14, 9	12
1718....		.924	0.011	9.30	3	32, 14, 9	1
1720....		.941	0.028	9.32	4	32, 14, 9, 6	1
1724....		.972	0.059	9.52	3	32, 14	3
1726....		.987	0.074	9.62	3	40, 32, 14	3
1728....		2420334.000	0.087	9.73	3	32	4
1852....	Aug. 26	2420371.665	0.426	9.62	8	32, 14	10
1854....		.682	0.443	9.32	8	32, 14	11
1856....		.703	0.464	9.16	8	32, 14	12
1858....		.721	0.016	9.34	8	32, 14	1
1860....		.742	0.036	9.11	8	32, 14	2
1862....		.760	0.054	9.61	8	32, 14	2
1864....		.778	0.072	9.50	8	32, 14	3
1866....		.796	0.090	9.62	8	32, 14	4
1868....		.814	0.108	9.68	8	32, 14	4
1870....		.834	0.128	9.84	8	32, 14	5
1872....		.858	0.152	10.04	8	32, 14	5
1874....		.876	0.170	9.95	8	32, 14	5
1876....		.894	0.188	9.96	8	32, 14	5
1878....		.915	0.209	9.99	8	32, 14	6
1905....	Sept. 18	2420394.808	0.239	10.05	4	32, 14	6
1907....		.826	0.257	10.08	4	32, 14	6
1909....		.846	0.277	10.05	4	32, 14	7
1911*....		.864	0.295	10.06	4, 2	32, 14	7
1927*....	Sept. 19	2420395.857	0.355	10.16	2, 2	32, 14	9
1940....	Sept. 20	2420396.745	0.310	10.03	4	32, 14	7
1942....		.767	0.332	10.07	4	32, 14	8
1944....		.783	0.348	10.08	4	32, 14	8
1948....		.826	0.391	10.29	4	32, 14	9
1950....		.844	0.409	10.00	4	32, 14	10
1952....		.862	0.427	9.78	4	32, 14	11
1954*....		.879	0.444	9.32	4, 2	32, 14	11
2028....	Sept. 22	2420398.767	0.466	9.04	4	32, 14	12
2030....		.786	0.018	9.22	4	32, 14	1
2032....		.804	0.036	9.43	4	32, 14	2
2034....		.823	0.055	9.54	4	32, 14	2
2036....		.844	0.076	9.59	4	32, 14	3
2038....		.864	0.096	9.67	4	32, 14	4
2044....	Sept. 23	2420399.631	0.396	10.20	4	32, 14	9
2046....		.647	0.412	10.06	4	32, 14	10
2861*....	1915 Nov. 30	2420832.628	0.402	10.13	2, 1	14	9
3213*....	1916 July 7	2421052.699	0.244	10.19	2, 2	14	6
3216*....		.712	0.257	10.26	2, 2	14	7
3231*....	July 8	2421053.701	0.313	10.28	3, 4	60, 14	8
3233*....		.711	0.323	10.17	1, 2	14	8

From Fig. 1 it is apparent at once that the photographic range is much in excess of the photo-visual. At maximum the star is of about the same brightness in blue and in yellow light, indicating a spectrum of perhaps A0. At minimum the difference in brightness reaches nearly 0.6 mag., and at the corresponding point the spectral class is probably F4 or F5. Three spectrograms made by Mr. Adams show an actual range of A0 to A8; but the length of the exposures on his plates (two to two and one-half hours) tends to obscure and lessen the range, though not affecting the type at maximum.

TABLE III

MAGNITUDES OF COMPARISON STARS

Star	B.D. Number	Photographic Magnitude	Photo-visual Magnitude
1*	....	11.43±0.03	10.96±0.02
2	+56°2259	11.88±0.03	10.09±0.07
3	+56°2261	7.79±0.03	6.98±0.07
4†	....	12.16±0.01	10.64±0.03

*Position, 1855: R.A. $=19^{h}\,29^{m}\,46^{s}$, Dec. $= +56^{\circ}\,4'.5$.

†Position, 1855: R.A. $=19^{h}\,28^{m}\,55^{s}$, Dec. $= +56^{\circ}\,5'.4$.

TABLE IV

MEAN LIGHT-CURVES

PHOTOGRAPHIC			PHOTO-VISUAL		
No.	Mean Phase	Mean Mag.	No.	Mean Phase	Mean Mag.
1	$0^{d}.008$	9.29	1	$0^{d}.018$	9.30
2	.034	9.44	2	.045	9.42
3	.056	9.79	3	.070	9.56
4	.078	9.97	4	.095	9.68
5	.130	10.38	5	.160	9.95
6	.206	10.70	6	.237	10.08
7	.254	10.60	7	.285	10.10
8	.307	10.61	8	.329	10.15
9	.342	10.65	9	.386	10.20
10	.391	10.78	10	.416	9.89
11	.414	10.56	11	.438	9.47
12	.439	9.62	12	0.464	9.16
13	0.455	9.16			

The photometric data concerning maximum and minimum light and color of XZ Cygni are as follows:

	Pg. Mag.	Pv. Mag.	C.I.	Color-Class
Maximum	9.16	9.16	0.00	*a*0
Minimum	10.78	10.20	0.58	*f*4
Range	1.62	1.04	0.58	....

The range and shape of the photographic curve are in fair agreement with the results of Martin and Plummer.[1]

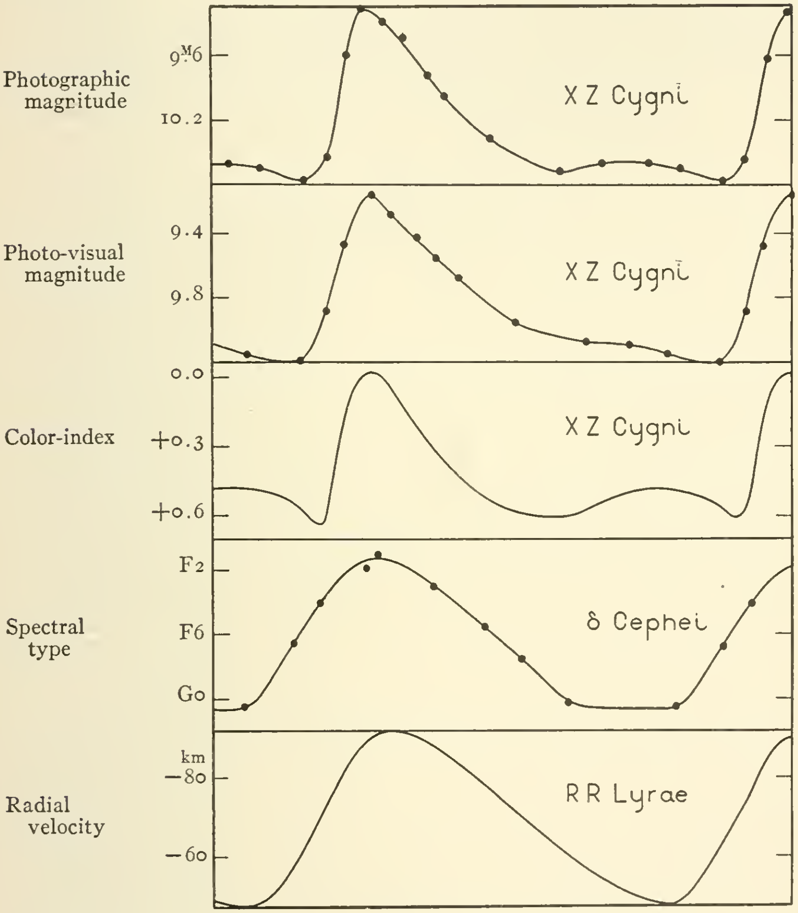

FIG. 2.—Five aspects of Cepheid variation

In conclusion, Fig. 2 is presented as an interesting summary of the various periodic changes which Cepheid variables undergo from one epoch of maximum light to another: (1) variation in photographic light, (2) variation in photo-visual light, (3) variation in color, (4) variation in spectrum,[1] (5) variation in radial velocity.[2] Since data relating to (4) and (5) are not available for XZ Cygni, the curves from other objects of the Cepheid class have been inserted for comparison. There is a striking similarity in the shape and fluctuations of all the curves. It is impossible not to conclude that all these phenomena are due to one underlying cause.

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[1] *Monthly Notices*, 74, 230, 1914.

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[1] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916.

[2] *Lick Observatory Bulletins*, **7**, 143, 1913.

THE APPLICATION OF THE PLANE GRATING TO THE DETERMINATION OF THE INDEX OF REFRACTION OF A GAS, WITH VALUES FOR AIR FROM λ 2500 TO λ 6500[1]

By ROBERT WILLIAM DICKEY

INTRODUCTION

There are many methods applicable to the determination of the index of refraction of a gas for wave-lengths in the visible spectrum and the near ultra-violet spectrum. Any method, to be of universal use, must give results in the extreme ultra-violet which are accurate to the same degree as those in the visible region. It was the purpose of the present investigation to study the use of the plane grating in extending as far as possible into the ultra-violet region the dispersion curve of a gas. The test of any method is the value of the results derived by it, consequently the index of refraction of air has been obtained for different wave-lengths between 2493 and 6412 A. A careful study of the possibilities and difficulties of the method has been made.

HISTORICAL

As the problem is an old one, it has been undertaken by a great many investigators. A few of the most important methods will be reviewed along with the results found.

Ketteler,[2] using a Jamin interferometer, observed the fringe-shift for definite change in pressure of the gas. His difficulty was that of obtaining efficient sources of monochromatic light, although the method gave absolute values. He obtained the index of refraction for yellow sodium light for air, carbon dioxide, hydrogen, sulphur dioxide, and cyanogen.

L. Lorenz,[3] working with a similar apparatus, determined the refractive indices of air freed of moisture and carbon dioxide, and of oxygen and nitrogen for sodium and lithium light. Mascart,[1] making use of Talbot bands, measured the absolute value of the index of refraction of air and chlorine for sodium light and the relative values for the four cadmium lines. Perreau[2] used the actual tubes of Mascart, but discarded the method involving Talbot bands and obtained a banded spectrum by analyzing white light which had passed through a Jamin refractometer. Kayser and Runge[3] placed a hollow prism between a photographic plate and a large concave grating and measured the displacement of various spectral lines when the pressure of the air in the prism was changed. They worked between $\lambda = 5630$ and $\lambda = 2360$ with a pressure-change of 10 atmospheres.

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[1] Dissertation, Johns Hopkins University.

[2] *Poggendorf's Annalen*, **124**, 390–406, 1865.

[3] *Annalen der Physik und Chemie*, N.F. **11**, 70, 1880.

Rentschler[4] used a Fabry and Perot interferometer and a concave grating and determined the indices of refraction of air, nitrogen, oxygen, carbon dioxide, and carbon monoxide from $\lambda = 5769$ to $\lambda = 3341$. The interferometer plates were silvered, and owing to the low reflecting power of silver between $\lambda = 2280$ and $\lambda = 3341$ Rentschler's results were not carried below $\lambda = 3341$. Among the recent publications of work in the visible spectrum is that of C. and M. Cuthbertson, who used a Jamin refractometer and determined the indices of refraction of air, oxygen, nitrogen, hydrogen,[5] neon,[6] krypton, xenon, helium, and argon[7] between $\lambda = 6563$ and $\lambda = 4861$. Quite recently an investigation of this problem has been carried out by Miss Howell,[8] using a Fabry and Perot interferometer (with plates nickeled by cathode discharge), in connection with a quartz spectrograph. She has extended the dispersion-curve of air to $\lambda = 2652$ and those of hydrogen, oxygen, and carbon dioxide to $\lambda = 2753$. The advantage of this method is that it is not necessary to count the number of interference bands going by for a change of pressure, thus making the method applicable to the ultra-violet region.

[1] *Comptes rendus*, **86**, 321, 1878.

[2] *Annales de Chimie et de Physique*, **7**, 289, 1896.

[3] *Annalen der Physik*, **50**, 293, 1893.

[4] *Astrophysical Journal*, **28**, 345, 1908.

[5] *Proc. Roy. Soc.*, **83**, 151, 1909.

[6] *Ibid.*, **83**, 149, 1909.

[7] *Ibid.*, **81**, 440, 1908.

[8] *Physical Review*, **6**, 81, 1915.

The method here described consists in determining the index of refraction of a gas by finding the ratio of the wave-lengths in vacuum and in the gas by means of a Rowland plane grating, using a Littrow type of mounting. The general idea involved is not new, for Osborne and Lester[1] used this method for the determination of the index of refraction of a liquid. Its application to a gas is new and there are many details in which this method differs from the one mentioned above.

THEORETICAL CONSIDERATIONS

If v_0 is the velocity of light-waves in the pure ether, v, the velocity in a definite gas, then the index of refraction of the gas with reference to the ether is defined as the ratio of v_0 to v. Calling the index of refraction n, we have

$$n=\frac{v_0}{v}=\frac{\nu\lambda_0}{\nu\lambda}$$

where ν is the frequency corresponding to the wave-length λ.

$$\therefore n=\frac{\lambda_0}{\lambda} \text{ and } n-1=\frac{\lambda_0-\lambda}{\lambda}=\frac{\Delta\lambda}{\lambda}\cdot \qquad (1)$$

Since λ is known, it is necessary to measure $\Delta\lambda$ for a change in pressure of the gas. To accomplish this, the fundamental principle of the grating is made use of. The condition under which we obtain a bright line of the mth order, when white light is allowed to fall upon the grating, is that the path difference between rays reflected from adjacent grooves shall be $m\lambda$, where λ is the wave-length of the line in question. The equation for the Littrow mounting is

$$m\lambda=a(\sin\theta+\sin i) \qquad (2)$$

where θ is the angle of incidence of the light, and this is kept constant, and i is the angle of the diffracted beam. i differs very slightly from θ. If the grating is surrounded by a gas, let the path difference for some particular line in the spectrum be $m\lambda_1$. Then when the same grating is placed in vacuum, the path difference between adjacent grooves for the same line will be $m\lambda_2$, according to equation (1), where λ_2 is greater than λ_1. Since θ is constant in equation (2), the change in λ is accompanied by a change in i and a shift of the line is observed. If a photographic plate is placed in the focal plane of the observing telescope, this shift can be recorded by making two exposures, one with the grating in vacuum and the second with the grating in the gas, keeping the grating fixed during the two exposures.

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[1] *Ibid.*, 35, 210, 1912.

APPARATUS AND METHOD

The Littrow form of spectrometer (Fig. 1) was used. To avoid instrumental shifts due to the possible relative motion of the parts of the apparatus, it was mounted as rigidly as possible. Two 3-inch iron I-beams, 300 cm in length, were fastened together and parallel to each other so that the flanges gave a firm horizontal surface in which to mount the various parts. At one end a cast-iron base-plate 22 cm in diameter and 3.2 cm thick was mounted and insulated from the I-beams by hard rubber supports so as to prevent as much as possible the transference of heat to or from the plate. At the center of this plate the grating-holder was attached with the proper adjustments. Just behind the grating-holder the connection to the pump and outside air was made as shown in Fig. 1. At the other end of the beams an iron plate, carrying the slit, reflecting prism, and photographic plate-holder, was fastened. The collimator lens was mounted on a slide fastened between the I-beams. In focusing for any definite region of the spectrum the collimator lens alone was moved. A heavy brass bell-jar, 12.5 cm inside diameter, 14 cm deep, with walls 1 cm thick and a heavy flange at the bottom, covered the grating. The front was arranged so that a quartz window could be inserted and made air-tight. The opening for the window was 2.5×4 cm. Through a hole in the top of the bell-jar a thermometer reading to tenths of a degree was inserted.

A Rowland 2.5-inch grating, ruled with 15,000 lines to the inch, was used. A grating with bright third and fourth orders was selected. On account of the high coefficient of expansion of speculum metal, every precaution was taken to keep the temperature of the grating constant during a series of exposures. A heating coil wound on the inside walls of a large wooden box (40×55×75 cm), surrounding the bell-jar, helped to keep the temperature constant. Then the massiveness of the bell-jar tended to oppose any rapid changes in the temperature of the inside. This problem of temperature has proved a very serious trouble, and it will be discussed more fully in a later paragraph.

The entire optical system, consisting of a small condensing lens *C*, a totally reflecting prism *P*, the collimator lens *L*, and the plane parallel plate *W*, was made of quartz. As a source of light the Pfund iron arc, operated on 110 volts with 5 or 6 amperes, was used. An image of the arc was thrown on the slit *S* by means of the condensing lens *C*, and a larger image was projected on the wall of the room at a distance of 5 meters so as to make certain that the same part of the arc was being used in a series of exposures. The observations were recorded on photographic plates.

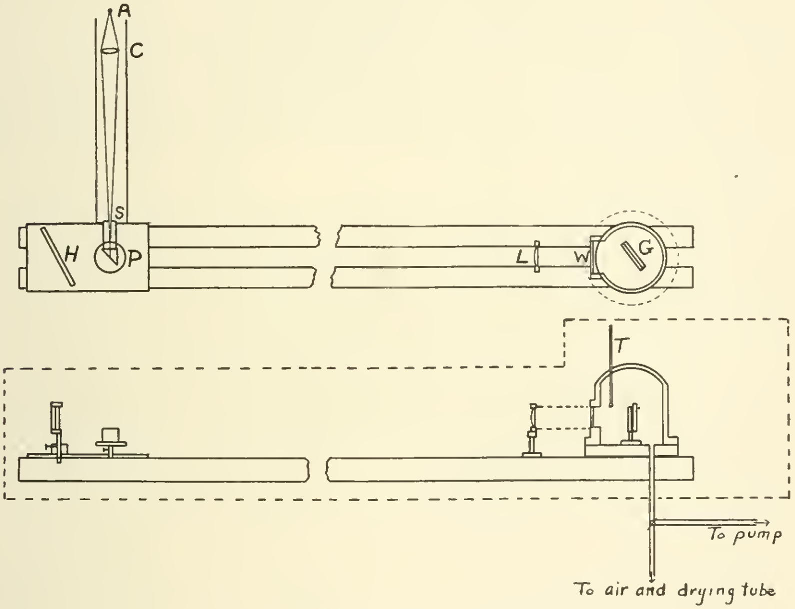

FIG. 1.—Arrangement of apparatus

The general procedure was to pump out the bell-jar until the pressure was a small fraction of a millimeter of mercury, take an exposure on the photographic plate, then let in the air through drying solutions and make a second exposure on the same plate, being very careful not to disturb it between exposures. The greatest difficulty to be overcome was the change in temperature of the grating between exposures, due to a thermodynamic increase or decrease in the temperature of the surrounding air when the pressure was increased or decreased. The resultant change in the temperature of the air itself is negligible so far as the density of the air is affected, but the high coefficient of expansion of the metal of the grating causes change in the grating space. This in turn causes a considerable shift of the spectrum lines for even a small fraction of a degree change in temperature. Upon the reduction of the pressure in the bell-jar by about one atmosphere, a sudden drop of about 1° C. was indicated by the thermometer whose bulb was inside the inclosure.

If one could tell when the grating reached a steady state, the difficulty would be overcome, but it is evident that there is a considerable lag of temperature in the grating with reference to the thermometer. A thermo-couple with one set of junctions insulated and imbedded in a brass block attached to the grating was used in an attempt to determine the steady state, but this had to be abandoned owing to the heat conducted to or away from the junction by the leads. An attempt was then made to avoid the difficulty by lowering the temperature of the air to that of liquid air before letting it into the bell-jar, but this method failed and was discarded.

Although this error due to temperature changes could not apparently be done away with, it was decreased to a certain extent by the filling of all the unnecessary space in the bell-jar with pieces of metal of high specific heats and large absorbing surfaces.

Finally, the scheme of lowering and raising the pressure of the air at intervals of twenty minutes and taking all exposures three minutes before and three minutes after letting the air into the bell-jar was adopted. In this way the error due to temperature-changes will be the same for all observations. The foregoing cycle of operations was repeated several times before any observations were made. As the total change in the shift due to thermodynamic changes was only a few thousandths of a millimeter, it was impossible to determine the actual value to a reasonable degree of accuracy by taking exposures at different times before and after letting the air into the bell-jar. The only course left open was to take the mean value of the index of refraction of air for some line in the visible spectrum and refer all observations to this one value. The exposures for different regions of the spectrum varied from 15 seconds to 5 minutes, so it was necessary to adopt the three-minute intervals mentioned above.

The spectrum of the third order was used between $\lambda=6400$ and $\lambda=4300$, of the fourth order from $\lambda=4100$ to $\lambda=2700$, and of the third order for the extreme ultra-violet observations. Even though the overlapping spectra were not in focus, much trouble was experienced by the fog produced on the plates. By the proper selection of absorbing solutions and screens this was avoided to a certain extent.

In the design of the apparatus any mechanism for setting the grating without removing the bell-jar was purposely avoided, for it was thought that such an arrangement might cause an instrumental shift in the grating when the pressure in the inclosure was changed. The grating was set at the desired angle by means of a telescope and scale, the grating being used as a mirror, after the sides of the constant temperature box and the bell-jar had been removed. When the bell-jar was replaced over the grating it was necessary to make the plane-parallel quartz window perpendicular to the incident light from the lens. This was done by adjusting the bell-jar until the image of the part of the incident light reflected from the front surface of the quartz plate fell in the center of the photographic plate and just below the spectrum. The location of this image plays an important part in one of the corrections which will be discussed later.

The air was freed from moisture by passing it through sulphuric acid and over phosphorous pentoxide. The pressure in the bell-jar was measured by a mercury manometer, the readings being reduced to mercury at 0° C. Readings of temperature were made to 0°.05 C. and of pressure to one-tenth of a millimeter.

RESULTS

Observations were made at intervals of about 200 A, three or four plates being made for each region. This gave a means of checking the measurements taken from a single plate. In every case tried the values from two different plates were found to agree.

The position of the lines on the plates and their corresponding shifts were measured by a traveling microscope, the screw of which was made in the same way as the screws for the Rowland ruling machines. Measurements were made to 1/1000 mm. In cases where the lines on any one plate were numerous, only those best adapted to measurement were selected. Each line and its corresponding shift were measured on an average of six or seven times and the mean of the results taken.

Position correction.—In the preliminary experiments it was noticed that the shift was not only horizontal, but vertical to a small degree. In looking for an explanation of this, it was found that a correction, depending upon the distance of the line from the center of the plate, had to be made. This correction can be deduced with the aid of Fig. 2. The only essential parts to be considered are the quartz plate *W*, which serves as a boundary between the outside air and the inclosure, and the photographic plate. If, when the pressure in space 2 is the same as that in space 1, the path of the ray from the grating *G* to the photographic plate is *ABC*, its path, when the pressure in 2 is less than that in 1, will be *ABD* owing to the deviation of the ray as it passes from a rarer to a denser medium. At the center of the optical system this will not occur, for the ray is normal to the quartz plate *W*.

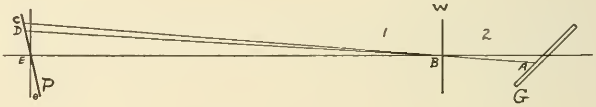

FIG. 2

Let n' = the index of refraction of space 1 with reference to space 2 at the observed temperature and pressure. This value of n' is derived from measurements at the center of the photographic plate where no correction is necessary.

Let l = distance *ED*, or distance of line from center of plate

Let θ = tilt of photographic plate

Then $CD=\left(\frac{n'l\cos\theta}{EB}-\frac{l\cos\theta}{EB}\right)EB$, approximately, $=(n'-1)\,l\cos\theta$

On account of this error the measured shifts taken from the half of the plate toward the red end of the spectrum are too small, and those of the other half too large. In referring the shifts to the center of the plate, the foregoing correction had to be added to or subtracted from the measured values of the "red" end or "blue" end respectively of the plate. Obviously it was very necessary to determine the center of the photographic plate, and this was done by means of the image of the slit reflected from the front surface of the quartz window. This has been discussed in the section on the apparatus and method.

The values of the shifts, after the foregoing correction had been made, were reduced to standard conditions. As a result of many experiments the following relation between the index of refraction of a substance and its density,

$$\frac{n-1}{\rho}=\text{constant},$$

has been well established by many investigators. Further, the shift for any definite wave-length is such that

$$\frac{n-1}{\Delta\lambda}=\text{constant}.$$

Hence, putting

$$\Delta\lambda=s,$$

it follows that

$$\frac{s}{\rho}=\text{constant}.$$

Or, expressing ρ in terms of pressure and absolute temperature,

$$\frac{sT}{p}=\text{constant}.$$

Hence, if s_1 is observed at temperature T_1 and pressure p_1 (expressed in millimeters of mercury), the value of s, which would have been observed if the temperature had been 0° C. and the pressure 760 mm, is given by the formula

$$\frac{s273}{760}=\frac{s_1(273+t_1)}{p_1}$$

or

$$s=s_1\frac{760}{p_1}\,\frac{273+t_1}{273}.$$

As the shifts reduced to standard conditions were given in millimeters, it was then necessary to reduce them to angstrom units. To do so it was necessary to compute the dispersion for each line of the plate. The positions of three lines, one at the center of the plate, x_0, and one near each end of the plate, x_1 and x_2 respectively, were carefully measured. It was assumed that any two wave-lengths with their corresponding positions obeyed the following relation,

$$\lambda_0-\lambda=A(x_0-x)+B(x_0-x)^2,$$

in which A and B are constants, x_0 the position of λ_0 on the plate, and x the position of any other line λ. λ_0 is taken as near as possible to the center of the plate. To obtain the values of A and B the three lines mentioned above were used. Differentiating the expression, we have

$$\frac{d\lambda}{dx}=A+2B(x_0-x).$$

Then if x_0 is selected, the dispersion at any other point on the plate indicated by x is known from the above relations. All the shifts were then multiplied by the value of $\frac{d\lambda}{dx}$ for the corresponding value of x. The resultant values are the respective changes in the wave-lengths of the different lines of the spectrum as the pressure is changed by the amount observed. According to equation (1), $n-1=\frac{\Delta\lambda}{\lambda}$; hence the values of $n-1$ could be calculated. The wave-lengths of the lines were taken from Kayser's *Handbuch.*

A specimen of the results is given in Table I. This is one of the seventeen tables obtained. In this table column I gives the relative positions of the lines on the photographic plate; column II, the corresponding wave-lengths as taken from Kayser's *Handbuch;* column III, the mean values of the shifts measured; column IV, the shifts corrected for the distance of the line from the center of

TABLE I

I x(mm)	II λ(A)	III S_2(mm)	IV S_1(mm)	V S(mm)	VI $\frac{d\lambda}{dx}\frac{(A)}{mm}$	VII $\Delta\lambda$(A)	VIII $n-1$	IX
$\lambda_1$256.188	4199.25	0.5501	0.5338	0.5943	2.1285	1.2650	0.0003012	For $\lambda=4219.52$ $n-1=0.0003011$
258.483	4204.15	.5506	.5349	.5956	2.1276	1.2671	.0003014	
259.964	4206.87	.5509	.5355	.5962	2.1256	1.2673	.0003012	
263.039	4213.82	.5505	.5360	.5968	2.1232	1.2671	.0003007	
264.243	4216.33	.5508	.5366	.5984	2.1223	1.2699	.0003012	
265.740	4219.52	.5514	.5376	.5986	2.1211	1.2696	.0003009	
277.985	4245.43	.5540	.5433	.6049	2.1115	1.2772	.0003009	
288.707	4268.93	.5565	.5486	.6108	2.1032	1.2847	.0003009	For $\lambda=4315.29$ $n-1=0.0003005$
290.574	4271.95	.5562	.5488	.6111	2.1017	1.2843	.0003006	
295.643	4282.59	.5575	.5514	.6139	2.0978	1.2879	.0003007	
297.103	4285.60	.5567	.5510	.6135	2.0966	1.2863	.0003001	
301.239	4294.32	.5587	.5541	.6170	2.0934	1.2916	.0003008	
306.647	4305.63	.5590	.5558	.6188	2.0892	1.2928	.0003003	
307.818	4308.09	.5598	.5569	.6201	2.0883	1.2950	.0003006	
$\lambda_0$311.262	4315.29	.5605	.5585	.6219	2.0856	1.2970	.0003006	
319.000	Center of plate							
321.803	4337.24	.5624	.5631	.6270	2.0774	1.3025	.0003003	
329.352	4352.90	.5637	.5664	.6306	2.0715	1.3063	.0003001	
337.578	4369.95	.5661	.5709	.6357	2.0651	1.3128	.0003004	
340.558	4376.11	.5662	.5718	.6367	2.0628	1.3134	.0003001	
344.242	4383.71	.5668	.5734	.6384	2.0600	1.3151	.0003000	For $\lambda=4408.58$ $n-1=0.0003001$
354.535	4404.95	.5700	.5792	.6449	2.0519	1.3232	.0003004	
356.323	4408.58	.5697	.5794	.6451	2.0505	1.3238	.0003001	
363.212	4422.74	.5713	.5828	.6489	2.0451	1.3271	.0003001	
365.530	4427.50	.5726	.5847	.6510	2.0433	1.3302	.0003004	
367.145	4430.79	.5713	.5838	.6500	2.0420	1.3273	.0002996	
$\lambda_2$372.878	4442.52	0.5736	0.5876	0.6543	2.0376	1.3332	0.0003001	

$t_1=24.^\circ6$ C.; $p_1=744.1$ mm.; angle of tilt of camera $=12°$; third order of grating.

the plate; column V, the shifts reduced to standard conditions; column VI, the dispersion in angstrom units per millimeter as measured on the photographic plate; column VII, the shifts in angstrom units, or rather the change in wave-lengths due to the change in pressure; and column VIII the values of $n-1$ as calculated by equation (1). The calculation of the index of refraction from one line alone is not very reliable, for the physical characteristics of the line may be such as to influence the results. For this reason the practice of determining the final values from a number of lines has been adhered to. This is shown in column IX. The values of $n-1$ were reduced to one particular line by taking as nearly as possible the same number of evenly spaced lines on each side of a central line, and taking the mean of the values. This practice is permissible in the region of the spectrum where the dispersion is comparatively small, but in the ultra-violet the separate values of $n-1$ were reduced to one particular line by the use of Kayser and Runge's dispersion formula, and the mean of these reduced values was taken as a final value. Table I represents one of the better sets of observations. In some cases values of $n-1$ had to be determined from a less number of lines.

Since the temperature correction for the shifts was not determined, it was necessary to select some iron line whose index of refraction for air has been well determined by other observers, and refer all values to this line. For this purpose the line $\lambda=4315.29$ was selected. The index of refraction was calculated from the dispersion formulae of Kayser and Runge,[1] Rentschler,[2] and Miss Howell,[3] the values obtained for $n-1$ being 0.0002961, 0.0002959, and 0.0002958. The mean of these is 0.0002959. The value obtained in the present investigation was 0.0003005, thus giving a difference of 0.0000046, which will be called the temperature correction. From the following discussion it will be seen that this correction is constant for all regions of the spectrum; and the corrected values of the index of refraction for air for all wave-lengths is found by subtracting this correction from the computed values.

[1] *Annalen der Physik*, **50**, 293, 1893.

[2] *Astrophysical Journal*, **28**, 345, 1908.

[3] *Physical Review*, **6**, 81, 1915.

The formula for the Littrow type of mounting of the plane grating is

$$m\lambda = a(\sin\theta + \sin i)$$

where m is the order of the spectrum, λ the wave-length of the spectrum line, a the grating space, θ the angle the incident light makes with the normal to the grating, and i the angle of the refracted beam. For the present discussion θ and i can be considered as remaining constant. Suppose a temperature-change in the grating occurs, then the expansion produces a change, δa, in the grating space. The corresponding change in the wave-length, $\delta\lambda$, is given by the relation

$$m\delta\lambda = \delta a(\sin\theta + \sin i).$$

Dividing this expression by the one above, we have

$$\frac{\delta\lambda}{\lambda} = \frac{\delta a}{a} = \text{constant},$$

if the change δa is a constant in every case. The exposures were carried out under exactly the same conditions so that δa would be a constant. Therefore the correction to $n-1$ for temperature changes is a constant for all values of λ.

The indices of refraction of air for the different wave-lengths obtained in this manner are given in Table II. The first column contains the values of the wave-lengths, the second, the observed values of $n-1$, and the third the corrected values of $n-1$. In the fourth column are recorded the number of lines observed in determining $n-1$ for the specified wave-length, λ.

The dispersion-curve is shown in Fig. 3. Curve I indicates the values obtained from the observed values of the shifts of the lines on the photographic plate, while curve II represents the values of $n-1$ referred to the known value for the line $\lambda = 4315.29$.

It is customary to express dispersion-curves in terms of the Cauchy formula,

$$n-1 = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4},$$

although this is of no theoretical importance. It is given here so that the results can be compared with those of other observers. According to Cuthbertson (*loc. cit.*)

$$n-1=10^{-7}\left(2885.4+\frac{13.38}{\lambda^2}+\frac{.50}{\lambda^4}\right);$$

Kayser and Runge (*loc. cit.*) find

$$n-1=10^{-7}\left(2878.7+\frac{13.16}{\lambda^2}+\frac{.316}{\lambda^4}\right);$$

Rentschler (*loc. cit.*) finds

$$n-1=10^{-7}\left(2903+\frac{3.80}{\lambda^2}+\frac{1.23}{\lambda^4}\right);$$

Miss Howell (*loc. cit.*) finds

$$n-1=10^{-7}\left(2881.7+\frac{11.83}{\lambda^2}+\frac{.46}{\lambda^4}\right)$$

where λ is expressed in thousandths of a millimeter. In the present investigation the values of the constants a, b, and c in the Cauchy formula were obtained by the method of least mean squares, and the following formula was found:

$$n-1=10^{-7}\left(2878.12+\frac{11.59}{\lambda^2}+\frac{0.551}{\lambda^4}\right).$$

This is in good agreement with the formula of Kayser and Runge and that of Miss Howell.

DISCUSSION OF RESULTS

A remarkably smooth dispersion-curve was obtained, only one point failing to touch the curve. This was at $\lambda=4879.39$, and the reason for this can be found in the physical characteristics of the lines in this region. As they were broad and not well defined, accurate settings on them were almost impossible. For the converse reason the values below $\lambda=4500$ are more accurate.

The magnitude of the shift is also a determining factor in the accuracy of the results as well as the sharpness of the lines. To increase the magnitude of the shift, higher orders of the grating were used as far as possible, in keeping with the intensity of the

TABLE II

λ (A)	$n-1$ (Observed)	$n-1$ (Corrected)	No. Lines Used in Determining $n-1$
6411.90......	0.0002956	0.0002910	3
6230.93......	.0002960	.0002914	4
5615.89......	.0002965	.0002919	8
5406.02......	.0002968	.0002922	8
5328.21......	.0002971	.0002925	4
5269.70......	.0002971	.0002925	3
5202.49......	.0002972	.0002926	9
5139.64......	.0002975	.0002929	1
4879.39......	.0002986	.0002940	7
4710.46......	.0002988	.0002942	2
4618.95......	.0002991	.0002945	7
4517.70......	.0002997	.0002951	9
4408.58......	.0003001	.0002955	7
4315.29......	.0003005	.0002959	12
4219.52......	.0003011	.0002965	7
4118.70......	.0003015	.0002969	18
3941.03......	.0003023	.0002977	12
3867.35......	.0003027	.0002981	10
3651.61......	.0003043	.0002997	12
3575.60......	.0003049	.0003003	3
3450.46......	.0003058	.0003012	12
3292.73......	.0003076	.0003030	11
3037.51......	.0003107	.0003061	9
2727.64......	.0003165	.0003119	14
2664.77......	.0003184	.0003138	4
2613.91......	.0003205	.0003159	11
2567.01......	.0003228	.0003182	8
2493.34......	0.0003268	0.0003222	4

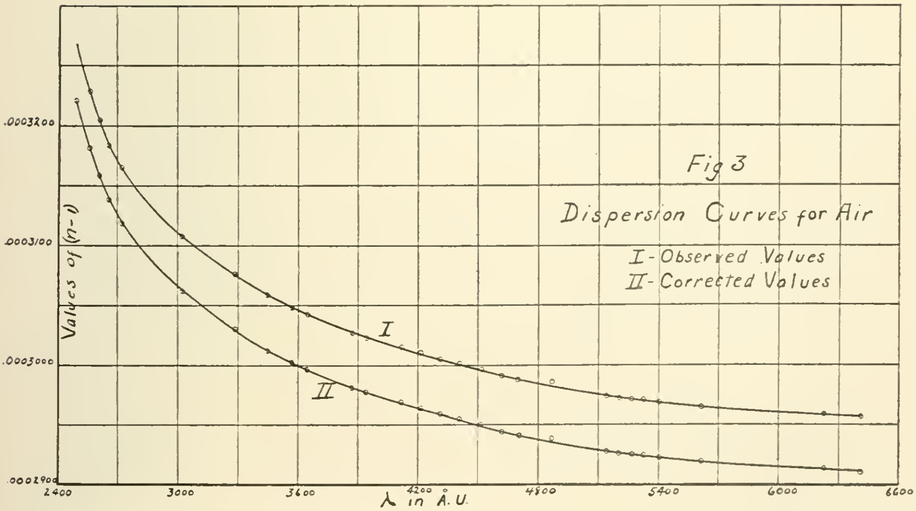

FIG. 3

lines. The large angle of tilt of the photographic plate in the extreme ultra-violet made it possible to use the third order in this region and obtain the same magnitude of shift. The shifts varied from 0.45 mm to 0.92 mm, the particular value depending upon the order used and the region of the spectrum.

The dispersion-curve was not extended below $\lambda = 2493$, owing to the arrangement of the apparatus. The opening in the bell-jar was such as greatly to reduce the amount of the grating space used and consequently to cut down the intensity of the spectral lines. From a consideration of the number of separate measurements which are associated with a single value of $n-1$ and the good agreement of these measurements, it is safe to say that the fourth significant figure is determined to a fair degree of accuracy. It can be relied upon to be correct to within several units.

As far as the method is concerned, there seems to be only one serious difficulty—the effect of temperature-changes as the pressure of the gas is changed. It is believed that, with several improvements in the present apparatus, the value of this temperature correction can be determined and consequently the absolute value of $n-1$. The author hopes to continue this work in the near future, using other gases. In the present form this method is applicable to gases which do not injure the grating and for which a single value of the index of refraction is known.

Among the advantages of this method the following may be mentioned: first, any source of light may be used which gives fairly sharp lines, the number being immaterial provided there are enough in any one region to determine the dispersion; second, the final value of $n-1$ is not obtained from observations on a single line; third, values of $n-1$ can be obtained for wave-lengths of all lines that can be photographed in a reasonable length of time using an ordinary grating spectrometer. This last fact makes the method a very desirable one in extending into the extreme ultra-violet the dispersion-curves of gases.

SUMMARY

1. The application of the plane grating to the determination of the index of refraction of gases has been thoroughly studied and

the only serious difficulty is found to be one due to changes of temperature.

2. The dispersion-curve of air has been obtained from

$$\lambda=6412 \text{ to } \lambda=2493.$$

3. The accuracy of the corrected values of the index of refraction of air agrees with those obtained by other methods.

In conclusion I wish to thank Dr. Anderson, who suggested the present problem and whose interest, help, and constant encouragement have been invaluable.

I am indebted to Professor Ames for his interest and advice throughout the course of the work. I wish also to thank Dr. Pfund for his advice and helpful suggestions.

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LEXINGTON, VA.
March 1917

A STATISTICAL STUDY OF CERTAIN SOLAR PHENOMENA

BY OLIVER J. LEE

The frequent disappointment in following a group of spots or flocculi to the limb of the sun, and in finding either no eruption at all or else merely a few series of small jets where prominences of considerable size and interest had been expected, has led the writer to examine the H calcium plates taken with the Rumford spectroheliograph in the last thirteen years to ascertain whether or not there is any basis for such expectancy. In the first examination of 3200 solar negatives of more recent dates the writer was ably assisted by Mr. Max H. Petersen, of the University of Wisconsin, who was a research scholar at this observatory in the summer of 1916.

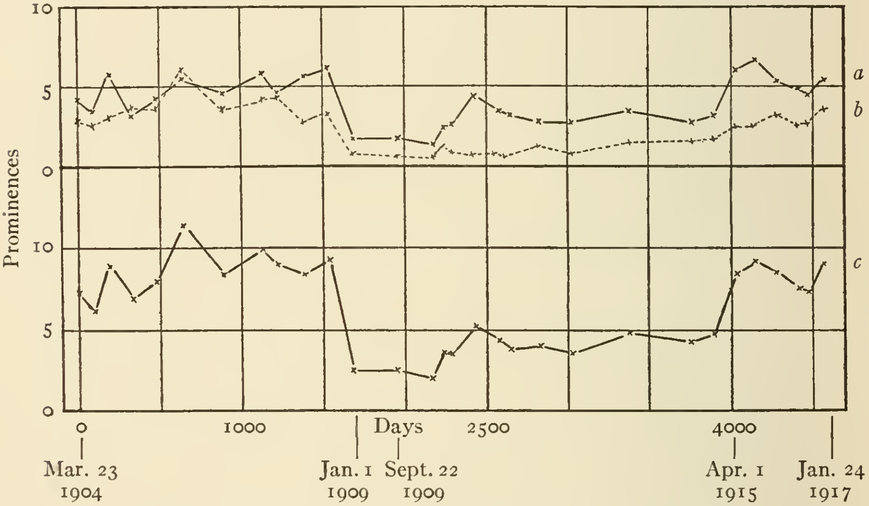
The object has been to determine the total number of prominences and the number of prominences probably connected with spots, flocculi, and filaments.

To aid in tracing a point on the disk to its corresponding position on the east or west limb, or vice versa, a chart was made, on the basis of Fox's velocities and gradient of polar retardation as derived from calcium flocculi (see *Report of the Seventeenth Annual Meeting of the American Astronomical Society, 1914*), which gave at a glance the distance of a point from the limbs of the sun in terms of days, for all except very high heliographic latitudes. This chart, with suitable lines for adjusting the inclination of the solar axis, was transferred photographically to a plate, which was mounted on a working desk of illuminated ground glass and was used for features on the solar disk not contiguous to the limbs.

Prominences occurring between ±45° of solar latitude and the poles have been counted separately from those lying within the usual range of spots and major flocculi. A center or base of eruption has been made the unit in obtaining numbers. Thus a group of connected prominences has a weight equal to the number of bases apparently contributing to it, and a fan-shaped bundle of jets emanating from one point has been counted as one prominence. Care has been taken not to count the same prominence on successive days, although a patently new eruption taking the place of one spent has been uniformly counted. The relatively small number of distinct typical forms of prominences enables the observer, in a surprisingly large number of cases, to tell from one plate whether a given prominence is actually on the limb. If photographs of a given prominence taken on successive days be available, it is easy to see whether it has passed the limb or is approaching it.

In tabulating the cases of heliographic connection between prominences and features on the disk the following rules have been observed: (*a*) A prominence anywhere within or contiguous to the field of flocculi surrounding a spot or group of spots has been counted separately as connected with both the spot and the flocculi. Hence, in Table I, if the number of connections of spot and prominence be subtracted from the corresponding number of connections of flocculus and prominence, the result is the number of prominences found to be related directly to flocculi in which no spot is visible. (*b*) A few strong filaments or dark flocculi have been credited with relation to a prominence, if such a one appeared at the position-angle of the former, even when the filament was visible as much as three to four days from the limb and could not be traced up to it because of cloudy weather or because the plates were of lower level, which of course do not show any except the strongest filaments. Such a filament has been counted as occurring on both disk and limb.

The present examination covers the plates taken between March 23, 1904, and January 24, 1917, a period of 4568 days. It covers almost completely the time of the last maximum of solar activity and runs well into the present one. The admirable records of frequencies and profile areas of prominences of the Kodaikanal Observatory since September 1, 1903, as well as the records maintained at various European observatories, give ample data for determining the maxima and minima of prominence activity. It was thought desirable, however, to derive the frequency-curves from the present material for the sake of uniformity of treatment. Probably no two observers would obtain quite the same counts without conference. The diagrams below give the mean frequencies by groups of 30 to 40 observing days for prominences occurring (*a*) between the ±45° parallels of heliographic latitude; (*b*) between these circles and the poles, called for convenience medial and polar prominences, respectively; (*c*) all prominences. With curve (*c*) in hand it is easy to divide the total time interval into two nearly equal parts so that the relations desired may be derived separately for times of smaller and greater prominence activity. The interval between January 1, 1909, and April 1, 1915, has been taken as comprising the former period; the time up to January 1, 1909, and that after April 1, 1915, taken together, as the latter.



Mean Daily Frequency of Prominences, 1904–1917
a medial prominences, *b* polar prominences, *c* all prominences

In Tables I and II are exhibited the principal results obtained in this investigation. With their headings and subheadings they are self-explanatory. In both Tables I and II the percentages given for prominences connected with filaments are referred to the total number of prominences, both medial and polar, that have been observed, inasmuch as filaments seem to occur indifferently on any part of the solar disk. The percentages given for spots and flocculi are each referred to the total number of medial prominences only, for the obvious reason mentioned above.

The dark absorption markings discovered by Hale and Ellerman[1] on their hydrogen and calcium plates and called by them simply dark flocculi[2] are visible only when the light from the central portions of the particular line employed is allowed to enter the second

TABLE I

FOR THE PERIOD OF MAXIMUM FREQUENCY

Total number of medial prominences	2867
Total number of polar prominences	1904
Filaments on solar limb or traceable to it	53
Filaments on solar disk	105

PROMINENCES	CONNECTED WITH					
	Spots		Flocculi		Filaments	
	No.	Percentage	No.	Percentage	No.	Percentage
Very small	20	0.7	76	2.6	10	0.2
Small and medium	103	3.6	243	8.5	18	0.4
Large	43	1.5	108	3.8	19	0.4
Total	166	5.8	427	14.9	47	1.0

TABLE II

FOR THE PERIOD OF MINIMUM FREQUENCY

Total number of medial prominences	1201
Total number of polar prominences	437
Filaments on solar limb or traceable to it	25
Filaments on solar disk	20

PROMINENCES	CONNECTED WITH					
	Spots		Flocculi		Filaments	
	No.	Percentage	No.	Percentage	No.	Percentage
Very small	21	1.7	45	3.7	6	0.4
Small and medium	37	3.1	67	5.6	7	0.4
Large	12	1.0	21	1.8	3	0.2
Total	70	5.8	133	11.1	16	1.0

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[1] *Publications of the Yerkes Observatory*, 3, Part I, 1903, p. 19.

[2] These absorption markings occur outside of, as well as within, the regions where the ordinary bright flocculi are seen. M. Deslandres calls them filaments and alignments, and distinguishes the two as follows: the former are sharp and well defined and on calcium plates appear chiefly when K_3 or H_3 is completely isolated from its

slit of the spectroheliograph. It is a matter of surprise that the relatively small dispersion ordinarily used with the Rumford should reveal them as often as it does. It has been evident from a number of disk plates of the present series that steady seeing is of secondary importance. One instance will suffice for illustration. On November 7, 1907, a strong filament appeared on the east limb of the sun in latitude +28° and extending for 15° or more in a southwesterly direction toward the equator. It remained visible on six observing days for a total interval of nine days. All the plates show that the second slit was set quite accurately on the center of the H line. Meanwhile the "seeing" ranged from 2 to 4 on the scale of 5 for perfect steadiness.

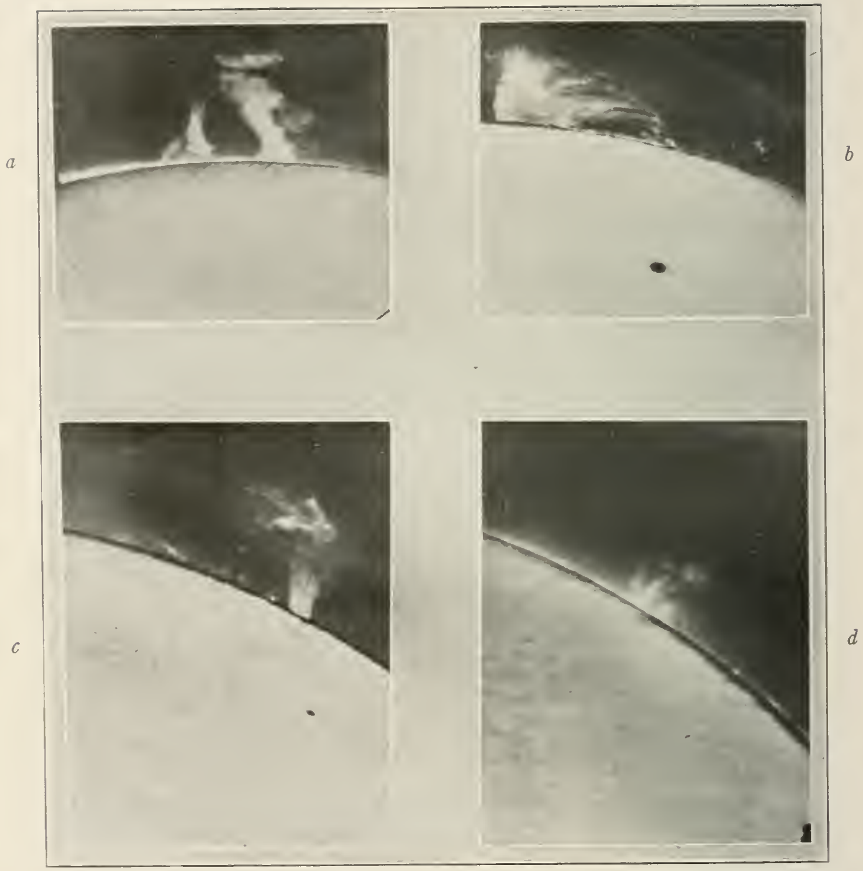
So far as the results for the filaments are concerned, it is significant to take the ratio of the number of filaments which are connected with prominences to the total number of filaments on the solar limb or traceable to it. The percentages taken in this sense are 89 and 64, respectively, for the two periods. On a great many high-level plates of the solar disk there are vast areas lying within and beyond (that is, poleward from) the regular belts in which spots and flocculi occur, which have a very rough appearance, although no groups of flocculi are visible in them. The calcium structure within such areas is very complex. Possibly we have here the first stages of flocculation combined with a veritable network of the Deslandres alignments of various forms and sizes. Large eruptive prominences have in many cases been observed to rise from such rough regions.

Intensely brilliant spots in areas of flocculi, when traced to the limb, usually show as jets and rarely as prominences of any size.

broader bases K_2 or H_2; the latter are broader and much more diffuse and on plates of high level may be seen as a dusky border along the length of a filament and sometimes as a prolongation of it. Alignments are visible in large numbers on plates taken with smaller dispersion, that is, when light from K_2 and H_2 as well as K_3 or H_3 is certainly entering the second slit, and according to Deslandres they form at times a complete network over large areas on the solar surface.

In this paper I have for convenience used the term filaments to designate both of these phenomena, although it is probable that a majority of the markings I have tabulated would be classed by Deslandres as alignments. Physically the two names perhaps designate the appearances of the same thing photographed at the highest and at an intermediate level, respectively.

PLATE III



Photographs of Prominences Combined with Those of Disk

a. No connection.
Prominence, 1916, September 9, $15^h\ 37^m$ G.M.T.
Disk, 1916, September 8, $14^h\ 25^m$.

b. Atmospheric connection.
Prominence, 1916, June 28, $16^h\ 9^m$ G.M.T.
Disk, 1916, June 27, $15^h\ 0^m$.

c. Direct connection.
Prominence, 1916, June 2, $15^h\ 50^m$.
Disk, 1916, June 1, $15^h\ 18^m$.

d. No connection.
Prominence, 1905, November 14, $16^h\ 58^m$.
Disk, 1905, November 13, $17^h\ 18^m$.

At the fifth conference of the International Union for Cooperation in Solar Research, held at Bonn in 1913, it was decided "that the limiting height for statistics of frequency be 30 seconds of arc." On a Rumford plate with the 40-inch telescope this corresponds to 2.8 mm, and since many intensely bright prominences show which are far below this limit, I early decided to include them in this count. In the tables under "very small" are included many such prominences whose height will vary from 30 to 12 seconds of arc. Mere deformations of the chromosphere, such as are frequently observed about spots seen on the limb, have not been counted.

It does not seem necessary to publish the rather voluminous references and notes made in this examination of 5519 plates. They contain the serial number of each prominence plate, the total number of new medial and polar prominences visible on it, the serial numbers of disk plates which show connections (for prominences on the east limb, these are in general plates taken on the same and following days; for prominences on the west limb, the same and preceding days), and finally full notes on the connection which is established.

Figs. *a*, *b*, *c*, and *d* of Plate III are introduced to illustrate the arbitrariness with which even the eruptive prominences do and do not occur where tradition has led us to expect them. From the data shown in the tables it is evident that cases of connection or non-connection between prominences and pronounced features on the solar disk, similar to those shown in the figures, can be multiplied in large numbers. All the prominences exhibited are of the eruptive type, although they differ in degree of activity. All four happened to occur on the west limb of the sun.

Each cut is made from two negatives, one showing the prominence, and a plate of the disk taken on the preceding day. This will enable the reader to see at a glance what is the appearance of the solar surface from which any one of the four groups of prominences rises. The large spot shown in the lower part of Fig. *b* is somewhat over two days from the limb. The large prominence in Fig. *a* appeared at solar latitude +63°, the smaller one at +57°. Several distinct eruptions took place between these two points on the limb within 2 to 3 days. The simultaneous pair shown in the figure changed markedly in form within two hours of observation. The larger one is 2′ in height. There was no spot or floccular area within 40° of this seat of eruptions.

The base of the large eruptions shown in Fig. *b* lies at latitude −30°. The center of the accompanying group of jets is at −20°, and probably coincides with the spot which at the time was less than 1 to $\frac{1}{2}$ days or about 0′.7 from the limb. The main prominence, 1′.4 in height, rises out of a clear field on the sun. Because of the apparent interaction between its streamers and the spot shown in the lower part of the figure it was counted as having connection, although the initial eruption probably was independent of the spot. The two prominences shown in Figs. *c* and *d* are of the same eruptive type. The former, a violent eruption, 2′ in height, appeared in solar latitude +20° and directly over a small spot just visible almost at the base of the prominence, where an hour earlier only a system of faint arching jets about 1′ high was visible. The latter occurred at −38° and had no connection with any object on the disk.

The results of this investigation may be summarized as follows:

1. Only 236 out of 4068 prominences of all sizes which have been observed between +45° and −45° of solar latitude, or 5.8 per cent, occur in the immediate vicinity of sun-spots.

2. In the same region only 324, or 8.0 per cent, of the prominences occur with flocculi in which no spot was visible during the days of observation.

3. This *infrequency* of connection between prominences and spots and flocculi exists practically to the same degree for times of greater and of less solar activity.

4. Of the 78 filaments which have been observed near the solar limb, 63, or 81 per cent, showed connection with prominences.

5. A considerable number of the large eruptive prominences occur either in unmarked regions of the solar surface or where the surface seems roughened.

There seems to be no reason for preserving the inherited idea that the prominences inevitably or even usually occur in close connection with sun-spots and flocculi.

Yerkes Observatory
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SPECTROSCOPIC FIELD-LIGHT

By F. E. FOWLE[1]

The intensity of energy observed in any part of a spectrum is due to the true intensity proper to the corresponding wave-length, plus that due to the always present energy from other sources in the never dark field, plus that scattered in from other regions of the spectrum. This intruding energy is generally known as field-light, or more properly field-energy. Naturally a portion of energy proper to the observed wave-length is lost by scattering into other regions of the spectrum.

The first part of the field-light, due to extraneous sources and present even with closed slit, is effective both when the intensity in the spectrum of the source of energy is measured and when the zero of energy with the slit closed is taken. Measuring the intensity as the difference between the corresponding intensities eliminates the effect of this disturbance.

The second and more troublesome portion is due to the energy scattered from other parts of the spectrum into that part under observation. It is probably due mostly to light scattered by the image-forming mirror. That some must be scattered is evident; for, on the best silvered mirror, the outline of the incident beam may be noted when one is looking at the mirror even at a considerable angle away from that in which the light is almost wholly reflected.

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[1] Published with the permission of the Secretary of the Smithsonian Institution.

In a well-designed spectroscope of moderate dispersion, producing a spectrum in which the intensity ranges a hundred fold or even a thousand fold, this scattered light may have little evil influence. In the research at present pursued by the writer on long-wave radiation, where the whole spectrum from the violet to 20 μ comprises a dispersion of only about 1°5 and the intensities range ten thousand fold, this source of disturbance becomes exceedingly troublesome in the fainter part of the spectrum. The most intense region lies at wave-lengths less than 4 μ, and from this part, which is transmissible by quartz, most of the scattered light comes. Special interest is centered in the wave-lengths longer than 4 μ and not transmissible by quartz. It has been customary under such circumstances, in the long-wave portion, to distinguish between the true energy and that scattered into the region by observing the intensity both without and with a plate of quartz placed between the source of energy and the slit of the spectroscope. With the quartz in place, the energy producing the deflections at wave-lengths greater than 4 μ, to which quartz is opaque, must be all false and due to wave-lengths less than 4 μ. Without the quartz, the deflection must be due to this false energy, plus the true long-wave radiation, plus certain other corrections presently to be considered.

Turning now to Fig. 1, there will be seen a reproduction of a record of the deflections of a galvanometer needle which measures the amount of energy absorbed by a bolometer as there passes over it, under varied circumstances, the 15° prismatic rock-salt spectrum of a lamp of 88 Nernst filaments. The record consists of three sets of curves: first, a lower, dotted, single-branched record indicating the zero of the galvanometer needle when a shutter at about 300° K. was inserted between the lamp and the slit; second, an upper, five-branched, dotted curve, *a*, *b*, *c*, *d*, *e*, the observed energy-spectrum of the lamp. The branch *a* shows the rapid rise of energy to a maximum at about 1.8 μ and was made with a slit 0.67 mm wide, the inflowing energy being farther reduced by a rotating sector. There is as rapid a fall of energy on the right-hand side of long wave-lengths, and the sector was removed to allow sufficient energy to flow through the slit to produce the branch *b;* then for the branch *c* the slit was increased to 2.36 mm, and for the branch *d* to 5.82 mm, and finally for the branch *e* to 10.69 mm.[1]

For the present, however, the central curve *a'b'c'd'e'* will be considered. This connects a series of points taken with the same conditions of slit as for the upper curve, but in front of the slit a quartz plate one-half a centimeter thick was inserted. This curve consists of two parts of different significance, namely, region *a'* and region *b'c'd'e'*.

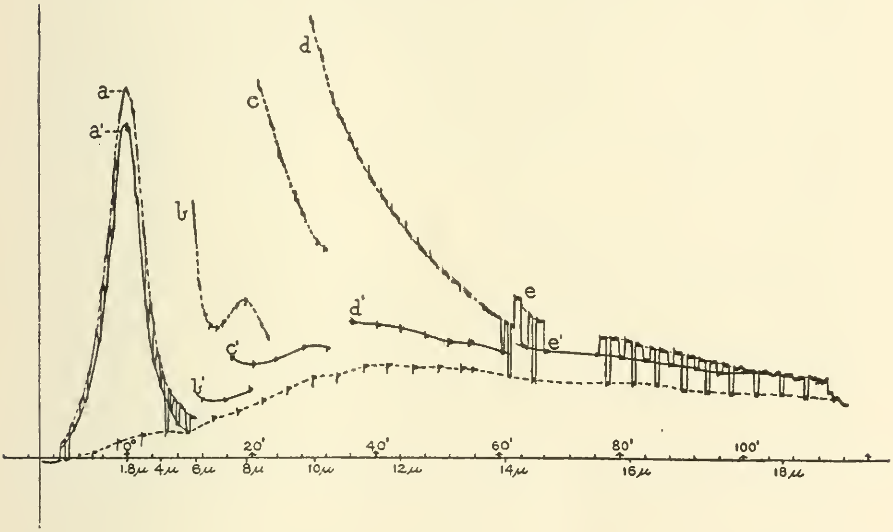

FIG. 1

Region a'.—Curve *a'* belongs to energy of wave-lengths between 0.6 and 4.0 μ to which quartz is nearly transparent. The difference in areas *a* and *a'* (with certain allowances for the absorption of the longer of these waves between 3 and 4 μ) measures the amount of energy of this region reflected from the quartz surfaces. This quantity can also be computed by Fresnel's and by Bouguer's formulae for reflection and absorption, respectively, from the known coefficients of refraction (Rubens) and transmission (Merritt). From several computations made before the idea of inserting the quartz in this branch had occurred, this loss was computed to be about 19 per cent. Considering the great impurity of the spectrum used for computing this loss, this is in sufficient agreement with the mean value of 15 per cent found from subsequent curves like a and a'.

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[1] These slit-widths were obtained either by the use of brass slits of fixed widths inserted in front of the regular spectroscope slit, or by closing the latter until its jaws just touched a measured cylindrical template.

Region $b'c'd'e'$.—These curves belong to the region for which quartz is opaque, and the energy recorded must come from the region for which the quartz is transparent, that is, region a'. Since there is some energy, to which quartz is transparent, reflected from the quartz when inserted here (namely, an amount corresponding to the difference between a and a'), the deflections in this region of long wave-lengths do not measure directly all the energy scattered from the region a, but they must be increased in the ratio a/a' or 1.18.

These two regions a' and $b'c'd'e'$, corrected as just described, are reproduced in Fig. 2 as a' and $b'c'$, the latter magnified a thousand fold and three thousand fold relative to the first. The first curve may be considered as the energy-curve of the region producing the scattering shown in the second. The energy-curve of the first region a', from which the energy is scattered, appears so symmetrical that it seems probable that the distribution of the scattered energy from it, $b'c'$, would not be materially altered (and the following computations will show that it practically is not) if all its energy were concentrated in a central strip of spectrum 4′ wide (1 cm of plate), as indicated by the dotted lines in Fig. 2. Let this central strip be joined to the curve $b'c'$ by what seems a plausible interpolation. This central strip, with its winglike appendage $b'c'$, represents the energy-curve of an approximately monochromatic line with the radiation scattered to its side of long wave-lengths. A symmetrical wing should stretch to its left to indicate the energy scattered to the side of short wave-length. It is proposed to use this latter curve to obtain the scattering taking place at each point in the spectrum from wave-lengths greater than 4 μ and not directly obtained by the insertion of the quartz plate.

This involves the assumption that the shape of this curve representing the scattering is the same for all wave-lengths coming into consideration. Gorton[1] has published some curves which show, for great angles of incidence and unpolished, comparatively rough surfaces, the variation of the scattering with the wave-lengths.

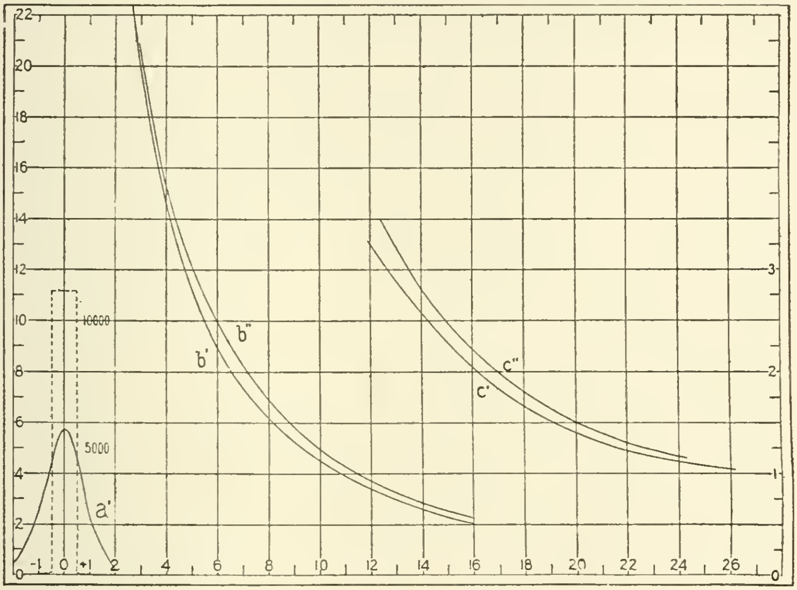

FIG. 2.—Abscissae are differences of deviation, zero at $1.8\,\mu$.

Ordinates are energy of Nernst lamps (2200° K.) transmitted by quartz plate, corrected for reflections from its surfaces.

Plot may be considered as nearly equivalent to the energy-curve of an approximately monochromatic line shown by the dotted lines with the energy $b'c'$ scattered to the side of longer wave-length.

$b''c''$ is the total field-light in the spectrum of the Nernst lamps computed by the process described in the text.

These indicate, as would be expected, less scattering for the longer wave-lengths. His smallest angle of incidence was 54°. His observations indicate, as the angle of incidence decreases, a much more uniform power of reflection for the different wave-lengths even with his rough surfaces, so that the assumption of a comparatively uniform scattering power at nearly perpendicular incidence (about 3° in the present case) on polished surfaces may not be unreasonable.

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[1] *Physical Review*, 7, 74, 1916.

Now to represent analytically the observed spectrum, there should be built up an expression the first term of which represents, for any deviation, the energy due to the ordinary dispersion of the prism, minus a term representing the amount of energy lost from this region by scattering, plus a third term representing the energy scattered into this deviation from other deviations.

Let θ be the deviation taken as zero at 1.8 μ. Let $S(\theta)$ represent the function which is the analytical expression of the complete, symmetrical curve of scattering of the approximately monochromatic line, the central and right-hand portion of which curve is shown in Fig. 2. Let the true energy-curve of the lamp be $E(\theta)$ and the observed curve $E_0(\theta)$.

The first term of the desired expression, the energy proper to the deviation θ, is evidently $E(\theta)$.

The second term, which represents the energy properly belonging to this deviation, but scattered elsewhere, may be obtained to a close approximation by taking the area of the curve of scattering, $\int_{-\infty}^{\infty} S(x)dx$, and multiplying it by the ratio of the ordinate of the true energy-curve, $E(\theta)$, at this deviation to that at 1.8 μ, $E(\theta_0)$, where θ_0 is used to signify $\theta =$ zero.

The third term, which represents energy belonging to other deviations, but scattered to this, is obtained to a close approximation by taking the amount of scattering, $S(x)dx$, given by the scattering curve for the deviation x, and multiplying it by the ratio of the energy ordinate in the true spectrum at that place, $E(\theta+x)$ to $E(\theta_0)$, and integrating over the whole spectrum.

Both integrals contain the term $S(x)\frac{E(\theta)}{E(\theta_0)}dx$, which is not desired, but fortunately occurs in the two integrals with opposite signs. In other words, the function used for the scattered energy includes the energy which should be observed at $\theta =$ zero and which is not to be considered as scattered energy. This energy is allowed for in $E(\theta)$.

Accordingly the following expression is obtained:

$$E_0(\theta)=E(\theta)-\frac{E(\theta)}{E(\theta_0)}\int_{-\infty}^{\infty}S(x)dx+\frac{1}{E(\theta_0)}\int_{-\infty}^{\infty}E(\theta+x)S(x)dx,$$

or, transposing,

$$E(\theta)=E_0(\theta)+\frac{E(\theta)}{E(\theta_0)}\int_{-\infty}^{\infty}S(x)dx-\frac{1}{E(\theta_0)}\int_{-\infty}^{\infty}E(\theta+x)S(x)dx.$$

All the functions on the right-hand side of the second equations are known except $E(\theta)$. But for this may be substituted the value obtained by subtracting 1.8 times the observed field-light, $S(\theta)$, from the observed full deflection of the lamp. The resulting values, which we call $E_a(\theta)$, will be found to differ at the greatest by only about 1 per cent from $E(\theta)$, and hence is sufficiently close an approximation to use in connection with the second and third terms.

Table I shows the process used to evaluate the terms of this expression. The numbers in the first column and first line indicate the deviations (θ) measured in centimeters on the plate (1 cm$=4'$ of deviation in the spectrum). Each other number is a measure of the energy under one of the curves just defined summed for a difference of deviation extending from -0.5 to $+0.5$ cm from the deviation θ.

$$E_a(\theta).$$

Consider first the numbers surrounded by the rectangles and looking like a flight of steps in the table. These represent $E_a(\theta)$, the first approximation to the amount of energy in the spectrum of the lamp for the corresponding deviations indicated in the first line or column. They represent also the terms common to both integrals.

$$\frac{E(\theta)}{E(\theta_0)}\int_{-\infty}^{\infty}S(x)dx.$$

Consider next the numbers in the other vertical columns. These give the amounts of energy scattered from the "step" values into the deviations indicated in the first column and are determined by the function S. The maximum in each case falls at the deviation corresponding to the "step" value, the other values falling off

TABLE I

Deviations ↓ θ →	-4	-3	-2	-1	0	+1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	A Quota Field Lt Σ	B Total Field Lt Σ	B/A		D/A ×1.18
-10	9	20	71	237	450	155	30	5	3	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	472	987			
-9	11	24	85	278	520	181	35	5	4	3	2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1134	1151			
-8	15	30	103	334	612	209	42	6	4	4	2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1345	1364			
-7	20	39	130	405	735	246	48	8	5	4	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	1623	1647			
-6	28	54	168	510	892	295	57	9	6	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	2004	2031			
-5	36	75	233	660	1120	358	69	10	7	6	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	2551	2581			
-4	$\boxed{11180}$	96	324	915	1450	450	83	13	8	7	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	3318	3354			
-3	36	$\boxed{29960}$	418	1274	2010	583	104	15	9	8	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	4425	4469			
-2	28	96	$\boxed{129700}$	1640	2800	808	134	19	11	10	5	3	2	1	1	1	0	0	0	0	0	0	0	0	0	5506	5559			
-1	20	75	418	$\boxed{508600}$	3600	1125	187	24	14	12	6	4	2	1	1	1	0	0	0	0	0	0	0	0	0	5425	5490			
0	15	54	324	1640	$\boxed{1118000}$	1450	260	34	18	15	8	4	3	2	1	1	0	0	0	0	0	0	0	0	0	3743	3829			
+1	11	39	233	1274	3600	$\boxed{449000}$	334	48	25	19	10	5	3	2	1	1	1	0	0	0	0	0	0	0	0	5491	5606			
2	9	30	168	915	2800	1450	$\boxed{104000}$	61	35	20	12	7	4	2	1	1	1	0	0	0	0	0	0	0	0	5372	5522			
3	7	24	130	660	2010	1125	334	$\boxed{19000}$	45	36	18	9	5	3	2	1	1	1	0	0	0	0	0	0	0	4240	4381	1.021	1.021	1.20
4	6	20	103	510	1450	808	260	61	$\boxed{14100}$	47	24	12	6	3	2	1	1	1	0	0	0	0	0	0	0	3157	3315	1.050	1.052	1.24
5	5	16	85	405	1120	583	187	48	45	$\boxed{14530}$	31	17	8	4	3	2	1	1	0	0	0	0	0	0	0	2401	2561	1.067	1.068	1.26
6	2	14	71	334	892	450	134	34	35	47	$\boxed{9780}$	22	12	6	3	2	1	1	1	0	0	0	0	0	0	1847	2061	1.086	1.088	1.29
7	4	12	60	278	735	358	104	24	25	36	31	$\boxed{6745}$	15	9	5	3	2	1	1	0	0	0	0	0	0	1551	1703	1.102	1.099	1.30
8	4	10	52	237	612	295	83	19	18	26	24	22	$\boxed{4740}$	11	7	4	2	1	1	0	0	0	0	0	0	1293	1428	1.105	1.106	1.31
9	3	9	45	205	520	246	69	15	14	19	18	17	15	$\boxed{3550}$	9	5	3	2	1	1	0	0	0	0	0	1097	1216	1.108	1.109	1.31
10	3	8	38	175	450	209	57	13	11	15	12	12	12	11	$\boxed{2740}$	7	4	2	1	1	0	0	0	0	0	940	1041	1.107	1.107	1.31
11	3	7	34	148	385	181	48	10	9	12	10	9	8	9	9	$\boxed{2124}$	5	3	2	1	0	0	0	0	0	806	893	1.108	1.108	1.31
12	2	6	30	132	325	155	42	9	8	10	8	7	6	6	7	7	$\boxed{1677}$	4	3	1	1	0	0	0	0	692	769	1.111	1.113	1.31
13	2	5	27	117	291	131	35	8	7	8	6	5	5	4	5	5	5	$\boxed{1374}$	3	2	1	0	0	0	0	608	672	1.105	1.105	1.30
14	2	5	23	104	258	117	30	6	6	7	5	4	4	3	3	4	4	4	$\boxed{1049}$	2	1	1	0	0	0	539	593	1.100	1.100	1.30
15	2	5	21	93	228	104	27	5	5	6	5	4	3	3	3	3	3	3	3	$\boxed{750}$	2	1	0	0	0	480	529	1.102	1.102	1.30
16	2	4	20	83	204	92	24	5	4	5	4	3	3	2	2	2	2	2	3	2	$\boxed{498}$	1	1	0	0	429	470	1.096	1.096	1.30
17	1	4	17	76	183	82	21	4	4	4	3	3	2	2	2	2	2	2	2	2	2	$\boxed{360}$	1	0	0	384	421	1.096	1.096	1.29
18	1	3	16	70	167	74	19	4	3	4	3	2	2	2	1	1	1	1	1	1	1	1	$\boxed{220}$	1	0	350	379	1.083	1.083	1.28
19	1	3	15	64	153	67	17	4	3	3	3	2	2	1	1	1	1	1	1	1	1	1	1	$\boxed{200}$	1	320	347	1.084	1.084	1.28
20	1	3	14	59	141	62	16	3	2	3	2	2	1	1	1	1	1	1	1	1	1	1	1	1	$\boxed{180}$	296	320	1.081	1.081	1.27
21	1	3	13	56	130	57	14	3	2	3	2	2	1	1	1	1	1	1	1	1	0	0	0	0	1	274	295	1.077	1.077	1.27
22	1	3	13	53	123	52	13	3	2	2	2	1	1	1	1	1	1	1	1	0	0	0	0	0	0	258	275	1.066	1.066	1.26
23	1	2	12	51	116	49	12	2	2	2	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	243	258	1.062	1.061	1.25
24	1	2	12	49	112	45	11	2	2	2	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	232	242	1.043	1.042	1.24
25	1	2	11	47	108	43	11	1	2	2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	223	232	1.040	1.040	1.23

Σ_{+1}^{+25} 17113

$2\Sigma_{+1}^{+25}$ 34226

$2\Sigma/(0)$ 0.030

similarly above and below. These values plotted with the deviations of the first column would make a curve similar to $S(\theta)$ but with the maximum ordinate equal to $E_a(\theta)$. These values, added together from the top to the bottom of the columns and including the "step" value, would give the value of the term containing the first integral or the amount of energy at and scattered away from the deviation indicated by the "step" value.[1] This sum, excluding the "step" value, and therefore equal to the amount lost from this "step" value by scattering, is expressed at the foot of the column of zero deviation and will be seen to amount to about 3 per cent of the intensity observed at the deviation to which the energy belongs. This correction of 3 per cent, in accordance with one of the assumptions, is independent of the wave-length and therefore does not alter the shape of the energy-curve.

$$\frac{1}{E(\theta_0)}\int_{-\infty}^{\infty} E(\theta+x)S(x)dx.$$

Consider now the horizontal lines. The numbers added in this direction, including the "step" value, give the values of the term containing the second integral. When added excluding the "step" number, they give the amount of light scattered into the spectrum at each region indicated by the "step" value from the regions whose proper deviations are indicated at the tops of the columns of the individual terms of the sum. Remembering that the first integral term does not affect the shape of the desired energy-curve, the values connected with the second integral are those in which interest will at present center.

In the fifth from the last column will be found the sums taken horizontally from the second to the eighth columns inclusive. These values give the energy scattered from the wave-lengths between the deviations -4 and $+2$ cm (i.e., wave-lengths transmissible by quartz) into the corresponding deviations of the horizontal lines. These values are proportional to the field-energy obtained by the use of the quartz plate. The next column gives the sums of the numbers of the horizontal lines taken completely across, but omitting the "step" values. They represent the total field-light due to scattering from all wave-lengths. The third from the last column gives the division of the second of these sums by the first, or is the ratio by which the total field-light exceeds that contributed from the region for which quartz is transparent. The next column represents the same factor except as determined by using the actually observed field-light in place of the first of these sums. The last two described columns are practically identical, indicating at least for the observed field-light that the assumed distribution, $S(\theta)$, produces practically the same result as the real distribution. The last column includes also the factor 1.18 to allow for the reflection from the quartz surfaces.

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[1] The columns to the left are not carried upward to their full extent.

SUMMARY

When a spectrum is formed, the energy observed at any wave-length is not simply what is proper to that wave-length, but it is diminished by a portion scattered into other parts of the spectrum and increased by a portion scattered in from other regions. Energy-curves for a rock-salt prism have been made of the radiation from a Nernst lamp with a quartz plate 0.5 cm thick inserted between the source of energy and the slit of the spectroscope. The quartz plate is nearly transparent to a wave-length of 4 μ and opaque beyond. The resulting energy-curve therefore consists of two parts of quite different significance: first, a nearly symmetrical, sharp maximum, the energy-curve of the source to the wave-length 4 μ; second, at longer wave-lengths, the energy-curve of the scattered light from this region into that for which quartz is opaque. The central maximum with the winglike appendage toward the long wave-lengths and a similar one drawn toward the short wave-lengths has been assumed to represent closely the energy-curve of a monochromatic line with the radiation scattered to either side in the spectrum. This curve was used to compute the energy scattered into and away from each region of the spectrum of the Nernst lamp. Table II gives a summary of the results.

The intensity of the light scattered from any region has been shown to amount to about 3 per cent of the true intensity of that region. At an angle 10′ from the direction in which the beam is regularly reflected the intensity of the scattered energy is only about two-tenths of 1 per cent of that of the main image, and at an angle of 100′ it is one one-hundredth of 1 per cent. In a 15° prismatic rock-salt spectrum of a body 2200° K. for these two deviations this field-light amounts to 5 and 500 per cent respectively of the true radiation.

TABLE II

Wave-length in (μ)	1.8	5.7	7.0	8.0	10.0	12.0	14.0	17.5
Deviation in (′)	0	10	15	20	30	45	60	100
Quartz transmits	85,000	206	117	85	50	27	17	8
Ditto corrected	100,000	242	141	100	59	30	20	10
Total field-light		244	146	107	66	33	22	11
Black-body radiation	100,000	5000	1800	1000	400	140	65	20
Nernst-lamp radiation	100,000	5000	1800	1000	400	140	50	2

Line 1: Wave-lengths in millionths of a meter (μ).

Line 2: Differences of deviation in minutes of arc (′), 15° prismatic rock-salt spectrum, zero deviation at 1.8 μ.

Line 3: Nernst-lamp spectrum observed through quartz plate ½ cm thick.

Line 4: The same corrected for the reflections from the surfaces of the quartz plate; nearly equivalent to points on the energy-curve of a monochromatic line with the scattered energy to the side of long wave-lengths.

Line 5: Total field-light which would be expected in the spectrum of a Nernst lamp when the intensity of the latter is 100,000 at 1.8 μ; see line 7.

Line 6: Computed relative intensities in black-body spectrum of a body at 2200° K. radiating to a similar one at 300° K.

Line 7: Corrected observed intensities of Nernst lamp approximating the conditions of line 6, but differing for wave-lengths longer than 12 μ because of the increasing absorption of energy in the rock-salt plate closing the vacuum bolometer case and the prism, and the decreasing absorption of energy by the lamp-blacked surface of the bolometer strip.

The foregoing data were obtained with silver-on-glass mirrors the surfaces of which were in excellent condition, freshly silvered, and polished to hard, compact surfaces. Table III shows a comparison with results obtained several months later after the mirrors were so badly tarnished as to be unfit for work in the visible spectrum. It will be noted that the scattering has increased relatively more for the greater deviations from the central image.

TABLE III

INCREASE OF SCATTERING WITH TARNISHING

Deviation in (′)	16	20	24	32	40	48	60	80	100
Wave-length in (μ)	4.0	8.2	8.9	10.3	11.4	12.5	13.8	15.8	17.5
Increase	1.22	1.26	1.27	1.34	1.42	1.40	1.47	1.66	1.70

It would be of interest to know whether the curves of distribution of the scattered energy were always of the same contour. The foregoing results were a by-product of a more extensive investigation to which this point was not pertinent and so was not investigated. However, part of the research involved the use of a different spectroscope with a rock-salt prism of 60°. In this spectroscope at 10 μ with a deviation of 150′, instead of the 30′ with the 15° prism, practically no field-light was observed upon the insertion of the quartz plate one-half a centimeter thick. Now Table II shows that at 10 μ the deflection which, with a rock-salt prism in both cases, should be dependent only on the wave-length, would be 400/100,000, whereas the field-light if dependent only on the deviation would perhaps be 4/100,000, or only 1 per cent of the true energy at that place. The deflection in the spectrum was so small at this wave-length that a deflection of 1 per cent of it would escape notice.

Before closing I wish to acknowledge my indebtedness to Dr. Abbot for his suggestions and criticisms at every step of the research of which the results just given are a part.

Astrophysical Observatory
Smithsonian Institution, Washington, D.C.
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ON THE DISTRIBUTION OF STARS IN TWELVE GLOBULAR CLUSTERS[1]

By FRANCIS G. PEASE and HARLOW SHAPLEY

All quantitative studies of the distribution in clusters have been concerned with the relation of the number of stars to distance from the center, and, with the exception of some results for the brighter stars recently published by Bailey,[2] no special attention has been paid to direction from center. It has been remarked in earlier papers that the conclusions relative to cluster structure, which are based on observations of star-density as a function solely of apparent distance, are of limited meaning and application. The brightest stars of each cluster, which are those mainly involved in previous investigations, are not typical in color or luminosity; and, in addition, the possibility of serious systematic error in photographing the highly concentrated interior is so great that little confidence can be placed upon observed distribution laws, except when they are derived from and are supposed to interpret only the outer part of the cluster. The difficulties inherent in stellar concentration are of much less importance, however, when the relative distribution is analyzed on the basis of direction from the center; and this method of study, moreover, is capable of bringing to light a property of clusters that the usual method has failed to reveal.

In the following pages are given the results of star-counts on a series of cluster photographs made with the 60-inch reflector at Mount Wilson. Later, with the accumulation of data bearing on the Eberhard effect and on the various relations of magnitudes and colors, it may be feasible to inquire more closely into the star-density with respect to distance from the center; but for the present it is practicable to discuss the results only as far as they are related to sphericity of cluster-form.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 129.

[2] *Harvard Annals*, **76**, No. 4, 1915.

The stars were originally counted for a rectangular system of co-ordinates,[1] a reseau on glass being superposed directly upon the original negative and the number of stars being tabulated for each small square, which is 31″.4 on a side. The total area counted on each plate is 675 square minutes of arc, comprising 2464 small squares. In all cases a second count was made with the plate reversed, and frequently several counts were made, particularly at the beginning of the work and for those plates which appeared to give discordant results. Each count was recorded on cross-section paper, and over these records was drawn a system of rings and 30° sectors, as illustrated for one quadrant in Fig. 1. The width of each annular region is four of the small divisions, that is, approximately two minutes of arc; the radius of the central area varies from two to six divisions, depending on the condensation of stars toward the center. Miss Van Deusen and Miss Richmond, of the Computing Division, have made the counts for the rectangular and polar systems, respectively, and have given much service in the statistical discussion of the results.

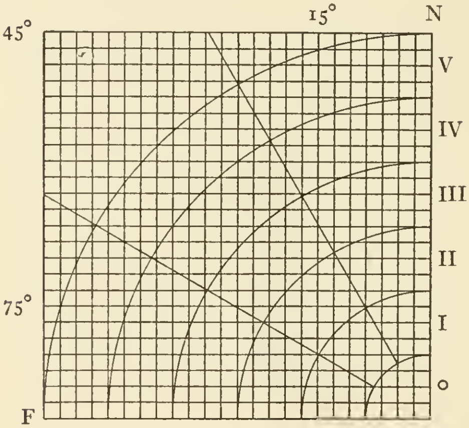

Fig. 1.—Scheme of rectangular and polar divisions (for the first quadrant) used in counting stars in globular clusters.

The photographs for which the counts have been completed are described in Table I. Nos. 64, 65, and 2456P are Seed 27 plates; all others are Seed 23. The total number of stars on each plate, seventh column, includes the count (or, when burned out, the estimate) of stars in the center, and also of those farther from the center than the limiting distance used in Tables II and III. This total number attempts to include all stars on each plate within

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[1] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year-book of the Carnegie Institution of Washington*, 13, 258, 1914.

TABLE I

Description of Plates

Cluster		Plate No.	Date		Exp. Time	No. of Counts	Total No. of Stars	Remarks
N.G.C.	Messier							
5024....	53	102	1912	May 13	180^m	2	10,100	Sky thick
5272....	3	64		Mar. 17	5	2	2,970	
		65		Mar. 17	15	2	6,440	
		89		May 11	20	2	5,230	
		2456P	1915	June 7	60	2	8,000†	
6093....	80	206	1913	July 2	64	2	4,050	Mirror not uniform
6205....	13	213		Sept. 1	1	2	820	
		220		Oct. 27	1	2	1,090	
		212		Sept. 1	3	2	1,700	
		13	1911	July 24	6	2	5,810	High wind
		18*		July 25	6	2	7,870	Troubled by clouds
		211*	1913	Sept. 1	6	2	2,160	
		218*		Oct. 27	6	2	3,140	
		14	1911	July 24	15	4	7,780	
		134*	1912	June 15	15	3	3,830	Very bad seeing
		19	1911	July 25	22	4	14,700	
		15		July 24	37·5	4	18,000	Sky white
		16		July 24	94	2	25,500	Sky white
		133	1912	June 15	300	2	35,000	Strong wind; mirror edge distorted
6218....	12	100		May 12	90	2	6,060	Windy
6229....	......	8	1911	July 23	69	2	1,540	Windy
6254....	10	124	1912	June 12	6	2	1,270	
		123		June 12	20	2	3,260	
		117		June 11	60	3	5,860	High wind
		114		June 10	180	3	12,200	
6341....	2	5	1911	July 2	100	2	6,800	Seeing poor
6402....	14	103	1912	May 13	120	2	4,920	
6779....	56	3	1911	July 2	60	2	6,600	High wind
6934....	......	2		July 1	105	2	3,370	
7078....	15	224*	1913	Oct. 27	1	2	1,100	
		223		Oct. 27	2	2	1,460	
		222		Oct. 27	6	2	2,340	
		21	1911	Aug. 22	6	2	2,030	Good seeing
		22*		Aug. 22	15	2	3,790	
		26		Aug. 25	15	2	3,320	
		225*	1913	Oct. 27	15	2	3,520	
		23	1911	Aug. 22	37·5	2	9,000	
		24		Aug. 22	94	2	21,000	
		25		Aug. 22	312	2	29,000	Plate completed on August 25, 1911

* Detailed counts are not given in Tables II or III.

† The area counted on plate 2456P is only 121 square minutes of arc.

the limits of the reseau, and it shows that conspicuously different totals for a cluster may be obtained from exposures of the same length. Apparently the seeing, transparency, quality of plate, development, and other factors may so influence the outcome that only for the mean of many plates can definite extensions of the magnitude-interval be assumed to result from corresponding increases in the exposure time.

Table II contains the star-counts for the eight clusters for which fairly definite results have been obtained. The clusters are in order of right ascension; the plates for each cluster are in order of increasing exposure time; the rings for each plate are in order of increasing distance from the center, and the sectors are indicated by the position-angle of the line that bisects them.[1] Unless there is a footnote to the contrary, the radius of the central area and the width of the rings are each four divisions (2′.09). The tabulated quantities are mean numbers for two counts. With the aid of plots of the sums of the horizontal and vertical columns in the original rectangular system, independent determinations of the position of the cluster's center were made for each count on every plate, thus giving an origin for the ring and sector system. That the adopted mean center for any cluster may vary slightly from plate to plate should be remembered in discussing the distribution for a given magnitude interval on the basis of differences. For this purpose, however, the combination of results for opposite sectors should largely eliminate the error of centering.

The count of stars in the central region for all but the shortest exposures is uncertain, if not completely impossible. Hence, these results are tabulated for quadrants only, and in general are omitted from the discussion. When the count is incomplete for any sector of the outermost ring, no number is given in the table, and the mean value in the last column refers only to the sectors for which counts are tabulated.

As it will be some years before a systematic continuation of this work can be undertaken, preliminary results are now given in

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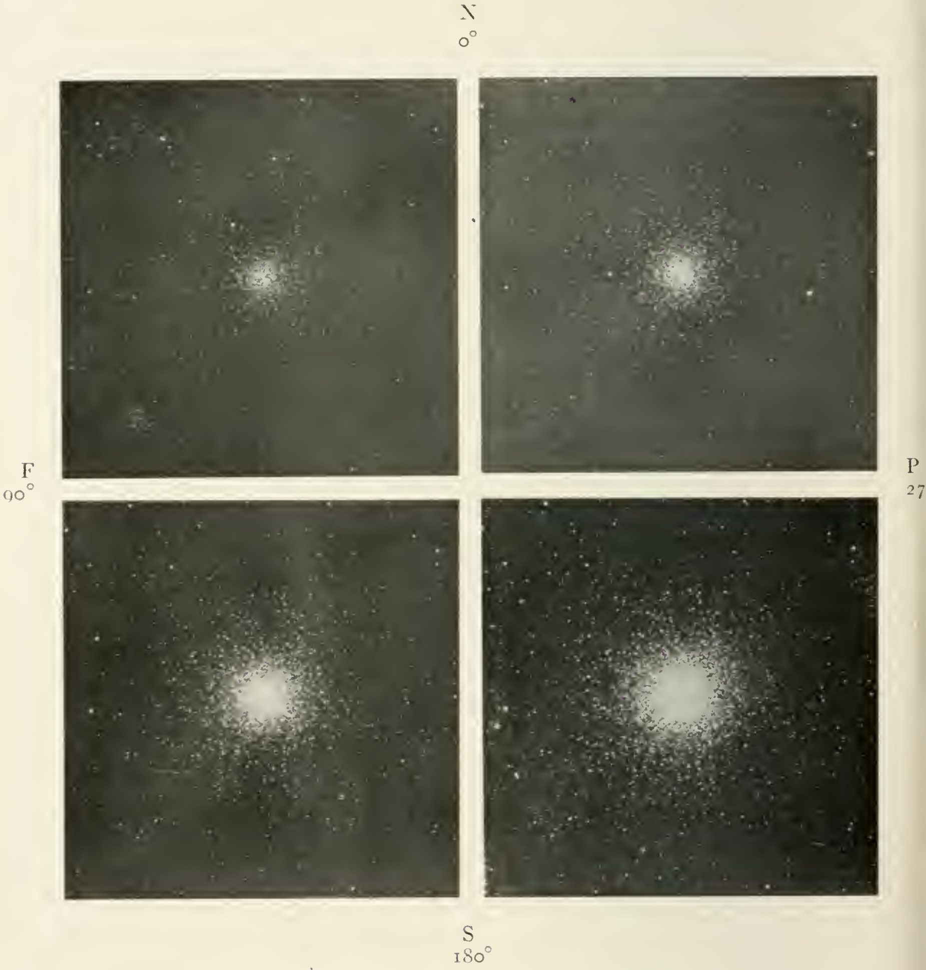
[1] In a communication to the *Proceedings of the National Academy of Sciences*, 3, 96, 1917, the angles of direction are counted in the opposite sense and from another origin.

PLATE IV



Four Successive Exposures on Messier 13

The position-angle of the major axis is 120°. The photographs reproduced are Nos. 13, 14, 15, and 16, described in Table I.

Table III for four clusters that later may be studied definitively. The future work will also include a further investigation of some of the clusters in Table II, in particular Messier 3 and Messier 56.

The foregoing tabular material can be discussed to advantage with the aid of diagrams. The data for Messier 13 are particularly complete and may be closely examined, leaving the results on other clusters for a briefer statement. Fig. 2 shows for several plates of Messier 13 the plot of the number of stars in successive sectors. That the cluster is symmetrically elongated is at once evident. The number of stars in two directions at right angles to each other differs by about 25 per cent. The plates of less than four minutes' exposure, however, dealing as they do with only one or two thousand stars which are peculiar in color and luminosity, do not show definitely this elliptical form. Among these brighter stars the axis of symmetry is not revealed until the distribution is studied on the basis of color-index or variable stars.[1]

At first thought it may be surprising that in a much-studied cluster such a pronounced elliptical distribution could exist and escape earlier detection. The reason must be that to the casual observer the brightest stars are conspicuous to the point of causing prejudice in the estimation of numbers, whereas, to one who actually counts the stars, the eighteenth-magnitude objects are the equal of those five hundred times more luminous. The irregular or sensibly spherical distribution of the giants hides the underlying ellipticity. In spite of the preponderance of the large images, however, the elongation of Messier 13 can be seen on some of the longer exposures, and, to a lesser extent, in the reproductions of the photographs in the accompanying plate.

That the elliptical form of Messier 13 is more or less independent of distance from the center is illustrated in Fig. 3, where, for plate 16, the data appear separately for successive regions, the ordinates, for the sake of intercomparison, being percentage deviations for the mean for each ring. Since in Fig. 1 little asymmetry is apparent that cannot be attributed to accidental groupings and errors of centering, the plots in Fig. 3 and in the following analogous diagrams represent the means for opposite sectors. The relation

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[1] *Proceedings of the National Academy of Sciences*, **3**, 276, 1917.

TABLE II

Distribution of Stars in Eight Clusters

N.G.C. 5024, MESSIER 53

Plate (Exp.)	Ring	Sectors												Mean
		15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°	
102 (180^m)	o*	......	344	......	......	336	......	......	314	......	......	306	......	325
	I	385	384	373	359	402	362	376	362	344	340	378	373	370
	II	200	182	160	187	208	189	200	184	204	204	212	217	196
	III	100	92	85	76	92	94	106	100	98	108	106	102	97
	IV	44	28	37	34	28	34	42	46	40	47	42	38	38
	V	Inc.	14	16	11	16	20	25	24	20	16	24	Inc.	19
	I–II	585	566	533	546	610	551	576	546	548	544	590	590	565
	III–IV	144	120	122	110	120	128	148	146	138	155	148	140	135
	I–IV	729	686	655	656	730	679	724	692	686	699	738	730	700

N.G.C. 5272, MESSIER 3

Plate (Exp.)	Ring	15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°	Mean
64 (5^m)	o*	......	459	......	......	514	......	......	474	......	......	418	......	466
	I	48	56	58	50	45	42	60	68	56	56	60	55	54
	II	20	14	20	10	12	16	17	24	16	25	26	16	18
	III	8	10	6	8	6	12	6	10	8	9	10	5	8
	IV	3	10	3	6	3	6	6	4	10	7	10	4	6
	V	Inc.	2	4	3	2	Inc.	Inc.	4	2	2	4	Inc.	3
	I–IV	79	90	87	74	66	76	89	106	90	97	106	80	87
65 (15^m)	o	......	669	......	......	628	......	......	698	......	......	726	......	680
	I	168	153	194	156	150	137	174	196	201	214	192	185	177
	II	70	68	50	54	52	69	54	98	102	121	100	64	75
	III	26	22	21	12	18	29	20	35	50	46	38	40	30
	IV	6	14	8	12	4	16	10	16	26	26	28	14	15
	V	Inc.	8	6	4	6	Inc.	Inc.	8	5	8	12	Inc.	7
	I–II	238	221	244	210	202	206	228	294	303	335	292	249	252
	III–IV	32	36	29	24	22	45	30	51	76	72	66	54	45
	I–IV	270	257	273	234	224	251	258	345	379	407	358	303	297

89 (20^{m})....	0........	....	632	....	....	628	....	....	644	....	....	635	....	635
	I........	150	135	152	138	110	128	115	135	132	152	144	149	137
	II........	46	43	42	42	43	68	45	53	42	54	68	48	50
	III.......	17	17	18	14	15	21	14	21	16	19	20	18	18
	IV........	6	16	6	12	4	17	11	9	14	10	16	8	11
	V........	Inc.	7	5	4	6	Inc.	Inc.	6	4	4	6	Inc.	5
	I–II......	196	178	194	180	153	196	160	188	174	206	212	197	186
	III–IV....	23	33	24	26	19	38	25	30	30	29	36	26	28
	I–IV......	219	211	218	206	172	234	185	218	204	235	248	223	214
2456P (60^{m})	0†........	....	233	....	....	320	....	....	259	....	....	156	....	242
	I........	233	213	220	208	242	246	240	268	229	248	253	242	237
	II........	177	210	197	195	185	195	197	201	197	202	216	174	196
	I–II......	410	423	417	403	427	441	437	469	426	450	469	416	432

N.G.C. 6205, MESSIER 13

213 (1^{m})....	0........	....	124	....	....	140	....	....	140	....	....	114	....	130
	I........	9	11	13	8	18	18	12	16	12	18	12	12	13
	II........	4	5	4	2	4	8	4	5	3	4	4	2	4
	III.......	0	2	2	4	2	2	1	2	1	4	1	2	2
	IV........	2	2	0	4	0	2	1	2	0	0	0	2	1
	V........	1	1	4	2	3	3	1	3	0	0	0	0	2
	I–V......	16	21	23	20	27	33	19	28	16	26	17	18	22
220 (1^{m})....	0........	....	156	....	....	163	....	....	170	....	....	176	....	166
	I–IV‡.....	....	74	....	....	90	....	....	92	....	....	97	....	88
212 (3^{m})....	0........	....	237	....	....	257	....	....	236	....	....	258	....	247
	I........	32	26	30	22	36	35	30	29	32	32	27	28	30
	II........	12	6	12	6	12	16	10	11	10	14	19	8	11
	III.......	3	4	8	7	2	4	4	6	4	6	6	3	5
	IV........	2	6	5	6	0	4	4	7	3	2	2	2	4
	V........	Inc.	6	4	2	4	3	2	6	1	4	2	Inc.	3
	I–IV......	49	42	55	41	50	59	48	53	49	54	54	41	50

* Radius of central area 1′.05.

† Intervals of distance from center for successive rings are 0′.0–1′.75, 1′.75–3′.25, and 3′.25–4′.75.

‡ Counted in quadrants only.

TABLE II—*Continued*

N.G.C. 6205, MESSIER 13—*Continued*

Plate (Exp.)	Ring	Sectors												Mean
		15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°	
13 (6^m)	0	......	792	......	......	858	......	......	786	......	......	840	......	819
	I	100	106	136	162	168	158	104	96	106	170	162	150	135
	II	31	24	46	45	26	29	36	32	38	42	40	23	34
	III	22	11	20	16	14	14	6	16	22	28	20	18	17
	IV	10	8	9	12	6	12	8	10	8	16	13	7	10
	V	Inc.	8	10	7	6	Inc.	Inc.	6	5	9	4	Inc.	7
	I–IV	163	149	211	235	214	213	154	154	174	256	235	198	196
14 (15^m)	0	......	852	......	......	980	......	......	922	......	......	866	......	905
	I	190	182	249	249	280	222	196	162	190	288	198	196	217
	II	61	50	92	78	84	62	64	36	47	70	71	62	65
	III	26	18	69	51	36	20	14	22	18	24	28	31	30
	IV	19	14	23	18	20	10	10	10	4	19	12	16	15
	V	Inc.	12	8	6	10	Inc.	Inc.	7	6	12	8	Inc.	9
	I–IV	296	264	433	396	420	314	284	230	259	401	309	305	326
19 (22^m)	0	......	1273	......	......	1231	......	......	1202	......	......	1320	......	1256
	I	414	429	449	483	462	386	341	352	418	474	478	430	426
	II	208	146	182	230	220	144	130	129	164	212	226	186	181
	III	82	65	86	103	92	72	62	64	76	98	111	88	83
	IV	30	32	27	43	40	36	36	38	26	41	37	34	35
	V	Inc.	17	18	12	Inc.	Inc.	Inc.	Inc.	23	16	14	Inc.	17
	I–II	622	575	631	713	682	530	471	481	582	686	704	616	608
	III–IV	112	97	113	146	132	108	98	102	102	139	148	122	118
	I–IV	734	672	744	859	814	638	569	583	684	825	852	738	726
15 (37^{m}5)	0	......	1224	......	......	1223	......	......	1260	......	......	1328	......	1259
	I	404	430	512	554	559	498	462	462	566	568	502	406	494
	II	212	204	254	282	282	204	190	202	260	282	272	208	238
	III	90	88	107	117	133	97	82	90	112	110	139	98	105
	IV	43	48	40	55	52	54	45	50	36	51	50	51	48
	V	Inc.	32	28	22	25	Inc.	Inc.	27	25	25	28	Inc.	22
	I–II	616	634	766	836	841	702	652	664	826	850	774	614	731
	III–IV	133	136	147	172	185	151	127	140	148	161	189	149	153
	I–IV	749	770	913	1008	1026	853	779	804	974	1011	963	763	884

16 (94^m)....	o........	......	990	......	......	760	......	......	626	......	......	768	......	786
	I........	623	668	750	762	764	728	712	683	758	778	718	670	718
	II........	358	361	423	464	476	386	330	352	402	438	479	394	405
	III.......	168	207	212	236	226	202	188	194	214	248	249	198	212
	IV........	112	104	90	128	114	115	108	114	112	126	134	99	113
	V........	Inc.	58	58	50	64	Inc.	Inc.	55	63	70	64	Inc.	60
	I–II......	981	1029	1173	1226	1240	1114	1042	1035	1160	1216	1197	1064	1123
	III–IV....	280	311	302	364	340	317	296	308	326	374	383	297	325
	I–IV......	1261	1340	1475	1590	1580	1431	1338	1343	1486	1590	1580	1361	1448
133 (300^m)..	o*........	......	......	......	......	......	3627	......	......	......	......	......	......	3627
	I........	560	640	624	684	788	642	586	597	629	658	712	662	648
	II........	362	374	410	471	431	360	302	292	358	396	424	394	381
	III.......	204	220	220	261	244	230	191	196	200	246	232	202	220
	IV........	141	136	116	118	130	Inc.	Inc.	124	130	157	134	100	129
	I–III.....	1126	1234	1254	1416	1463	1232	1079	1085	1187	1300	1368	1258	1250

N.G.C. 6218, MESSIER 12

100 (90^m)...	o........	......	386	......	......	386	......	......	378	......	......	365	......	379
	I........	149	168	164	164	178	162	166	174	164	171	176	174	168
	II........	90	100	97	98	108	103	98	106	92	100	93	95	98
	III.......	42	44	48	50	47	39	63	52	40	36	34	50	45
	IV........	22	22	26	24	20	27	30	24	34	30	28	30	26
	V........	Inc.	23	26	21	22	Inc.	Inc.	22	22	26	15	Inc.	22
	I–IV......	303	334	335	336	353	331	357	356	330	337	331	349	338

N.G.C. 6254, MESSIER 10

124 (6^m)....	o†........	......	68	......	......	85	......	......	92	......	......	81	......	82
	I........	41	42	34	36	45	50	46	42	42	44	45	37	42
	II........	12	8	10	16	20	14	20	16	15	18	18	8	15
	III.......	4	6	8	8	6	8	7	7	8	6	10	8	7
	IV........	4	4	4	5	2	4	5	3	8	6	4	3	4
	V........	2	4	7	7	4	5	7	6	6	8	5	4	5
	I–V.......	63	64	63	72	77	81	85	74	79	82	82	60	74

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* Because of the larger area of the burned-out center, the central region has a radius of 3′.14 instead of the usual 2′.09. The succeeding rings are 2′.09 in width.

† Radius of central area is 1′.05.

TABLE II—*Continued*

N.G.C. 6254, MESSIER 10—*Continued*

Plate (Exp.)	Ring	Sectors												Mean
		15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°	
123 (20^m)	o*	......	144	......	......	140	......	......	126	......	......	134	......	136
	I	114	131	133	116	110	100	105	108	106	114	126	136	117
	II	42	42	65	62	48	60	48	44	38	42	46	50	49
	III	18	23	22	32	26	20	27	22	19	17	21	24	23
	IV	8	10	8	17	11	14	12	11	12	12	6	9	11
	V	6	6	12	16	11	11	14	14	8	14	6	10	11
	I–V	188	212	240	243	206	205	206	199	183	199	205	229	210
117 (60^m)	o	......	432	......	......	454	......	......	424	......	......	439	......	437
	I	156	148	154	176	166	156	170	137	156	154	164	174	159
	II	70	69	86	96	77	70	80	96	66	63	86	84	79
	III	40	29	26	38	42	32	46	24	38	48	44	50	38
	IV	24	15	22	24	12	20	20	18	24	34	30	19	22
	V	Inc.	18	20	18	16	Inc.	Inc.	22	22	14	18	Inc.	18
	I–IV	290	261	288	334	297	278	316	275	284	299	324	327	298
114 (180^m)	o*	......	146	......	......	92	......	......	86	......	......	143	......	117
	I	271	304	292	306	314	268	296	290	290	282	293	286	291
	II	274	262	264	270	250	283	268	255	242	276	266	254	264
	III	161	169	150	173	170	162	159	161	164	152	151	154	160
	IV	95	104	104	112	96	103	113	86	102	104	108	83	101
	V	69	62	76	64	48	64	67	54	50	61	68	69	63
	I–III	706	735	706	749	734	713	723	706	696	710	710	694	715
	IV–V	163	166	180	176	144	167	180	140	152	165	176	152	163
	I–V	869	901	886	925	878	880	903	846	842	875	886	846	878

N.G.C. 6341, MESSIER 2

Plate (Exp.)	Ring	15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°	Mean
5 (100^m)	o*	......	107	......	......	112	......	......	136	......	......	142	......	124
	I	252	226	237	258	303	242	240	228	246	271	268	274	254
	II	154	118	132	148	153	140	124	114	146	171	170	164	144
	III	40	40	38	46	42	52	46	56	49	53	60	43	47
	IV	17	20	14	12	13	24	19	18	17	26	30	27	20
	V	Inc.	10	20	16	18	13	18	14	18	8	15	Inc.	15
	I–II	406	344	369	406	456	382	364	342	392	442	438	438	398
	III–IV	57	60	52	58	55	76	65	74	66	79	90	70	58
	I–IV	463	404	421	464	511	458	429	416	458	521	528	508	465

N.G.C. 6779, MESSIER 56

3 (60^m).....	o*........	......	151	......	......	158	......	......	132	......	......	128	......	142
	I.........	88	92	94	98	101	82	83	87	90	88	98	94	91
	II........	62	54	56	70	68	62	62	62	60	50	54	50	59
	III.......	84	63	62	67	62	73	74	58	66	66	54	68	66
	IV........	84	52	60	70	88	74	79	74	72	66	74	69	72
	V.........	76	66	82	84	83	84	98	83	90	86	74	81	82
	I–V.......	394	327	354	389	402	375	396	364	378	356	354	362	371

N.G.C. 7078, MESSIER 15

223 (2^m)....	o.........	......	256	......	......	216	......	......	256	......	......	268	......	249
	I.........	26	18	20	20	13	20	12	17	23	20	18	18	19
	II........	2	9	6	6	0	8	6	4	8	5	4	6	5
	III.......	4	1	6	8	1	5	6	8	4	4	6	2	5
	IV........	4	6	2	5	2	4	2	3	2	3	2	3	3
	V.........	Inc.	2	2	4	2	Inc.	Inc.	4	4	8	4	Inc.	4
	I–IV......	36	34	34	39	16	37	26	32	37	32	30	29	32
21 (6^m).....	o.........	......	394	......	......	426	......	......	395	......	......	392	......	402
	I.........	32	31	36	32	24	27	20	29	29	25	26	26	28
	II........	8	17	12	10	4	10	11	9	9	8	6	13	10
	III.......	6	5	8	10	5	6	6	9	4	5	6	9	7
	IV........	6	10	6	6	10	4	2	4	3	4	3	7	5
	V.........	Inc.	4	2	9	4	Inc.	Inc.	6	4	9	4	Inc.	5
	I–IV......	52	63	62	58	43	47	39	51	45	42	41	55	50
222 (6^m)....	o.........	......	433	......	......	416	......	......	462	......	......	463	......	444
	I.........	34	34	34	29	26	34	18	33	36	34	30	34	31
	II........	14	17	10	12	4	12	12	13	14	10	10	14	12
	III.......	6	2	8	13	2	5	8	10	6	6	8	8	7
	IV........	5	8	6	6	10	4	5	6	4	6	6	8	6
	V.........	Inc.	3	4	6	3	5	9	12	6	8	4	Inc.	6
	I–IV......	59	61	58	60	42	55	43	62	60	56	54	64	56
26 (15^m)....	o.........	......	488	......	......	456	......	......	558	......	......	556	......	514
	I.........	61	51	48	55	46	50	44	64	64	56	47	47	55
	II........	18	26	10	16	8	18	24	20	20	20	12	20	16
	III.......	10	3	11	14	2	8	10	16	12	10	8	9	14
	IV........	6	10	6	6	11	8	10	12	5	8	10	10	6
	V.........	Inc.	4	4	10	8	Inc.	Inc.	12	6	12	11	Inc.	8
	I–IV......	95	90	75	91	77	84	88	112	101	94	77	86	89

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* Radius of central area is 1'.05.

TABLE II—*Continued*

N.G.C. 7078, MESSIER 15—*Continued*

Plate (Exp.)	Ring	Sectors												Mean
		15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°	
23 ($37^{\rm m}5$)...	o.........	......	1028	......	......	965	......	......	1028	......	......	1104	......	1031
	I.........	242	232	240	210	212	202	248	272	220	213	238	221	229
	II.........	60	64	54	66	71	58	78	78	76	56	46	62	64
	III.......	28	30	27	32	18	31	28	39	32	29	30	19	29
	IV........	20	18	22	21	19	22	20	21	8	20	24	20	20
	V.........	Inc.	12	14	18	21	Inc.	Inc.	22	9	18	22	Inc.	17
	I–II.......	302	296	294	276	283	260	326	350	296	269	284	283	293
	III–IV....	48	48	49	53	37	53	48	60	40	49	54	39	48
	I–IV......	350	344	343	329	320	313	374	410	336	318	338	322	341
24 ($94^{\rm m}$)....	o.........	......	1499	......	......	1408	......	......	1396	......	......	1499	......	1450
	I.........	658	620	606	600	570	597	634	638	610	560	614	620	611
	II.........	264	266	269	250	274	280	320	334	293	262	248	283	279
	III.......	133	132	112	146	122	128	120	138	146	106	130	114	127
	IV........	71	50	70	80	78	72	84	84	54	54	64	53	68
	V.........	Inc.	35	30	52	40	Inc.	Inc.	60	30	54	39	Inc.	42
	I–II......	922	886	875	850	844	877	954	972	903	822	862	903	889
	III–IV....	204	182	182	226	200	200	204	222	200	160	194	167	195
	I–IV......	1126	1068	1057	1076	1044	1077	1158	1194	1103	982	1056	1070	1084
25 ($312^{\rm m}$)...	o.........	......	......	......	......	......	1972	......	......	......	......	......	......	1972
	I.........	682	662	624	609	610	593	634	680	635	604	604	674	634
	II.........	386	372	346	348	385	402	364	414	418	390	412	384	385
	III.......	218	221	180	200	194	203	174	212	226	192	223	199	204
	IV........	144	102	134	121	122	124	134	140	112	130	115	109	124
	V.........	Inc.	72	62	76	66	Inc.	Inc.	88	68	85	80	Inc.	76
	I–II......	1068	1034	970	957	995	995	998	1094	1053	994	1016	1058	1019
	III–IV....	362	323	314	321	316	327	308	352	338	322	338	308	327
	I–IV......	1430	1357	1284	1278	1311	1322	1306	1446	1391	1316	1354	1366	1347

TABLE III

Preliminary Results for Four Clusters

N.G.C.	Distance from Center	Sectors												Mean
		15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°	
6093	1′.0–11′.5	216	217	222	226	248	246	246	228	221	237	209	185	225
6229	1.0–11.5	45	46	52	43	47	43	44	42	44	52	49	50	46
6402	2.1–10.5	198	192	178	188	189	168	193	203	193	182	200	235	193
6934	1.0–11.5	154	179	170	164	157	180	195	194	195	195	193	188	180

TABLE IV

Percentage Deviations from the Mean for Messier 12 (Plate 100) and Messier 13 (Plate 15)

Cluster	Sectors												Average Deviations
	15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°	
Messier 12	−10	−1	−1	−1	+4	−2	+6	+5	−2	0	−2	+3	±3
Messier 13	−15	−13	+3	+14	+16	−4	−12	−9	+10	+14	+9	−14	±11

of ellipticity to magnitude-interval is shown by Fig. 4. The diagram at the top is based on plate 13, the bottom one on plate 16; the values for the second from the top are differences between plates 15 and 13, and the remaining curve represents plate 16 *minus* plate 15. These curves agree in showing ellipticity with approximately the same major axis for all intervals of magnitude; but their differences and irregularities should not be considered too seriously.

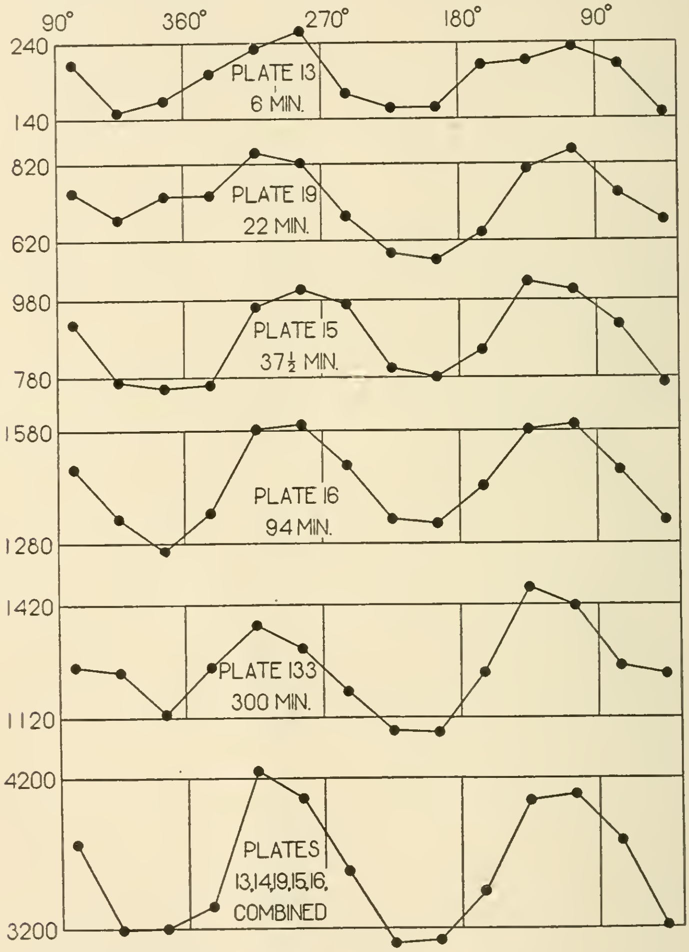

FIG. 2.—The axis of symmetry in Messier 13. Ordinates are numbers of stars. Abscissae are position-angles.

Reverting to the data in Table II, attention should be called to a few points concerning the other clusters.

The tabulated counts show distinctly for Messier 53 two maxima about 180° apart. At position-angle 165°, however, at what should be the highest point of one maximum, the numbers are peculiarly low in Rings I and II, and the resulting depression in the curve persists even when opposite sectors are combined, as in Fig. 5. If, as suggested in earlier communications, the axes of symmetry in these clusters are projections of galactic planes, this phenomenon, which can hardly be attributed to accidental irregularity, would have a close analogue in the great cleft extending south from Cygnus in our Milky Way.

The counts of Messier 3 are not very satisfactory, and data from more plates must be studied. As the results now stand, this cluster is the only one for which the departure from apparent sphericity appears to be non-symmetrical. A pronounced one-sidedness is shown by the results for plate 65; but plate 89, with almost the same number of stars, fails to verify the asymmetry, and the longer exposure, plate 2456P, is not decisive.

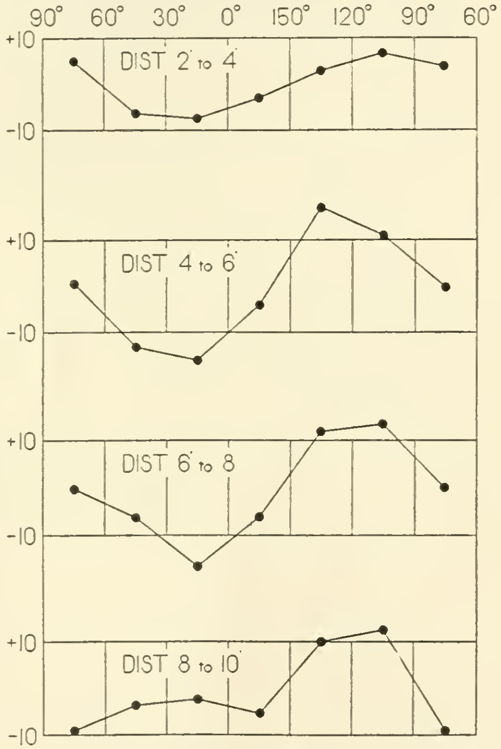

Fig. 3.—Ellipticity of Messier 13 for different distances from the center. Ordinates are percentage derivations. Abscissae are position-angles.

Messier 12, Messier 10, and Messier 56 are three clusters for which little or no deviation from apparent sphericity is shown by the counts in Table II, although the exposures for each have been long enough to include many faint stars. This does not mean necessarily that these systems are not flattened. It may indicate that their equatorial planes are inclined at a high angle to the line of sight. A random distribution of galactic planes should make at least one-third of them impossible of direct detection with our present methods; but, as an indirect test, the clusters seen more or less edgewise should show a greater average condensation toward the center than those with high inclinations. This is exactly what we find in the case of each of the three clusters mentioned above, as may be seen from the numbers in Table II, most conspicuously for Messier 56.

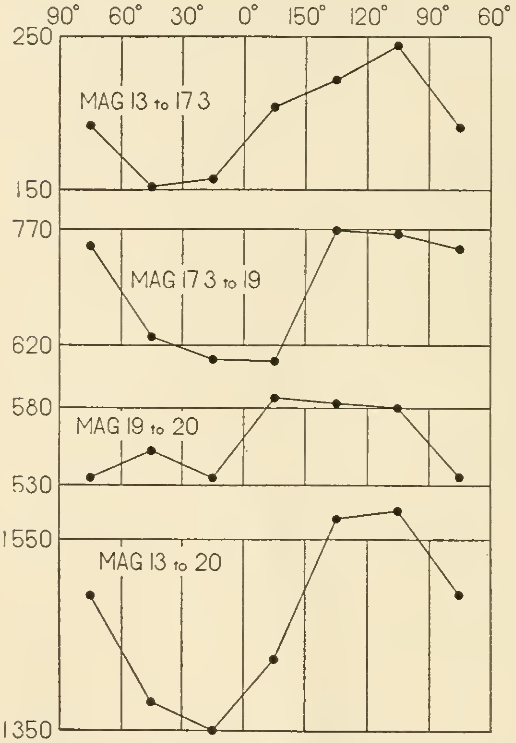

Fig. 4.—Ellipticity of Messier 13 for different intervals of magnitude. Ordinates are numbers of stars. Abscissae are position-angles.

To illustrate these points further, the small, probably accidental, deviation from perfectly circular form for Messier 12 is shown in Table IV, in contrast to the pronounced ellipticity of Messier 13; and the curves in Fig. 6 exhibit the relatively smaller central concentration of the former. The ordinates in this diagram, obtained from the values in the last column of Table II for plates 100 and 15, are made comparable by dividing the mean number for successive rings by the mean for Ring I.[1] It is doubtful, however, if the relative concentration can generally be used as an index of inclination, for uncertainties due to Eberhard effect, to differences in laws of density, and particularly to foreground stars must surely enter to some extent. A curve of the distribution for Messier 10 is shown in Fig. 5.

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[1] A small correction is made to allow for two squares in the center of plate 15 that are burned out.

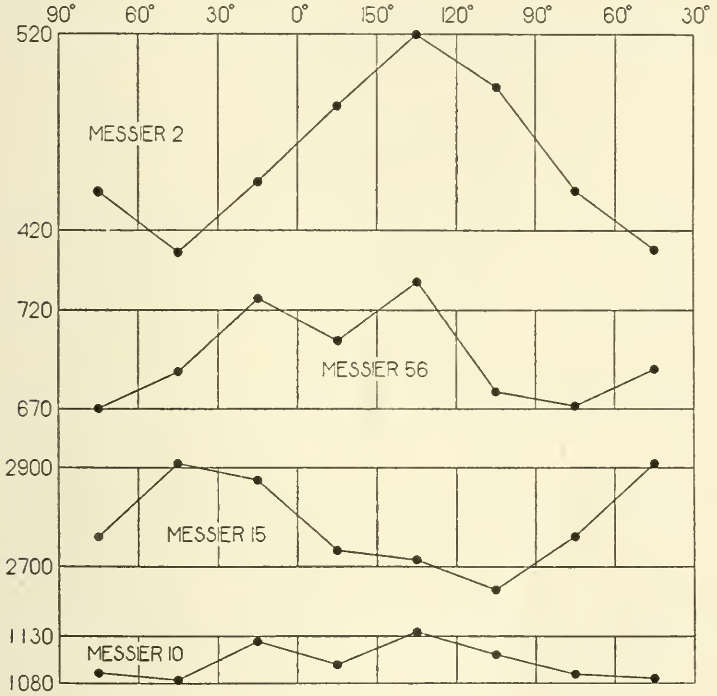

Fig. 5.—Axes of symmetry in globular clusters. Ordinates are numbers of stars. Abscissae are position-angles.

Curves are also given in Fig. 5 for Messier 2 and Messier 15. Both show the elongated distribution conspicuously, but for the latter the maxima in the unreflected distribution-curve are not exactly opposite and are of unequal height. This lack of symmetry tends to disappear with increasing exposure and may be only superficial, though the possibility must not be overlooked that the supposed galaxy in this system may be along a small circle of low latitude.

In order to bring together all the available evidence bearing on the forms of globular clusters, Fig. 7 is reproduced here from a communication to the *Proceedings of the National Academy of Sciences*.[1] The concentration of certain types of variables toward the Milky Way is a recognized phenomenon of our galactic system. The diagram shows that in ω Centauri also the ellipticity is much greater for the variables than for the stars in general. This interesting analogy further supports the hypothesis that the globular clusters are flattened systems of stars.

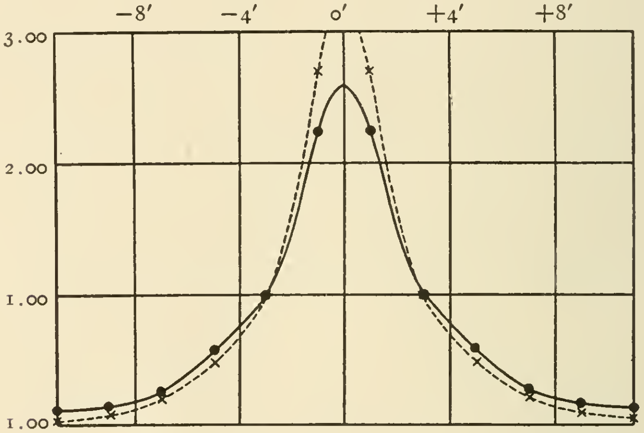

Fig. 6.—Comparison of the central concentration for Messier 12 (full line) and Messier 13, illustrating (with Table IV) the probably great difference in the inclinations of their galactic planes. Ordinates are relative numbers of stars. Abscissae are distances from the center.

SUMMARY

1. The present study of the distribution of stars in globular clusters is based upon counts of more than half a million star images on a series of plates made at the principal focus of the 60-inch reflector at Mount Wilson.

2. The material is used mainly as a test for the presence in these stellar systems of galactic planes, which should reveal themselves through an elliptical distribution of the stars.

3. Of the nine clusters for which fairly definite counts are available, five show elliptical form, three appear sensibly circular, and one may be peculiar. The three clusters with similar density

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[1] Vol. 3, 276, 1917.

in all directions from the center are noticeably less condensed, a condition that follows naturally if the poles of their galactic planes are assumed to be approximately in the line of sight.

4. The proportion of clusters with measurable elongation here found is about what should be expected on the galactic hypothesis if the inclinations are distributed at random.

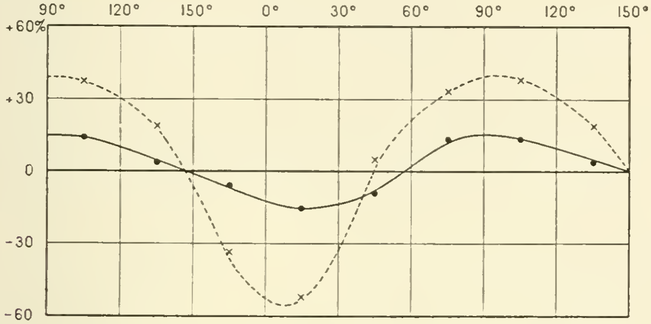

Fig. 7.—Galactic plane in ω Centauri; full line, all stars; dotted line, variables. Ordinates are deviations from mean. Abscissae are position-angles.

5. Axes of symmetry in these clusters do not appear until the arrangement of several thousand stars is analyzed. The failure previously to detect the ellipticity, which in some cases amounts to nearly 30 per cent, must be attributed to an influence of the brighter stars, which, in general, do not show elliptical distribution. The ellipticity appears at all distances from the center, and also for all magnitudes, after the giant red stars have been excluded.

Mount Wilson Solar Observatory
March 1917

GENERALIZED CO-ORDINATES, RELATIVITY, AND GRAVITATION

By EDWIN BIDWELL WILSON

Beginning some years ago with the suggestive *Aequivalenz-Hypothese* that an observer in an isolated system could not tell whether the system were at rest in a gravitational field or undergoing a uniform acceleration in a space free from such a field, Einstein[1] has by numerous successive steps built up a theory of gravitation and of "generalized relativity," as he calls it, in which he has recently been joined by Lorentz,[2] Hilbert,[3] and others. The possibly observable consequences of the theory are:

a) A ray of light from a star in occultation with the sun would be bent in by the amount $1''.75$.

b) The lines in the solar or stellar spectra would be shifted toward the red by an amount corresponding to a radial velocity of 0.634 km for the sun or $0.634\ M^{\frac{2}{3}}\rho^{\frac{1}{3}}$ for a star of mass M and density ρ relative to the sun.[4]

c) The perihelion of Mercury would shift by $43''$ per century as observed.

This last consequence is a striking confirmation and has attracted even wider interest in the theory than had previously been shown.[5]

I have recently given a few lectures on Einstein's work to the Harvard physicists at the invitation of Director Theodore Lyman, of the Jefferson Physical Laboratory, and although all of us in this country are in a somewhat precarious situation relative to new German scientific investigations, by virtue of the British blockade, which prevents our receiving German periodical literature, I venture to offer a few comments from my lectures.[1]

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[1] *Annalen der Physik* (4), **35**, 898–908, 1911; **38**, 355–369, 443–458, 1912; *Zeitschrift für Mathematik und Physik*, **62**, January 1914 (with M. Grossmann, reprinted as *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation*, 38 pp., B. G. Teubner); *Sitzungsberichte der K. Akad. Berlin*, p. 1030, November 1914; pp. 778, 831, 844, November 1915; *Annalen der Physik*, **49**, 769, 1916 (reprinted as *Die Grundlagen der allgemeinen Relativitätstheorie*, J. A. Barth); *Sitzungsberichte der K. Akad. Berlin*, p. 688, June 1916. (Some of these references are cited, not from the unobtainable originals, but from other citations, and are not verified.)

[2] *Proceedings of Amsterdam Academy of Science*, February 1916.

[3] *Göttinger Nachrichten*, p. 395, November 1915.

[4] *Monthly Notices*, **76**, 699–728, 1916; *The Observatory*, **39**, 412–419, 1916.

[5] *Nature*, **98**, 328–330, 1916.

1. *Generalized co-ordinates.*—The fundamental properties of our Euclidean space are expressed in the formula for length of arc:

$$ds^2 = dx^2 + dy^2 + dz^2.$$

It is ordinarily with x, y, z as variables that we express our laws of physics. We are, however, at liberty to use any system of curvilinear or generalized co-ordinates x_1, x_2, x_3, where

$$x = x(x_1, x_2, x_3), \qquad y = y(x_1, x_2, x_3), \qquad z = z(x_1, x_2, x_3). \tag{1}$$

The differential of arc then becomes a quadratic differential form,

$$ds^2 = \sum_{i,j} a_{ij} dx_i dx_j, \quad i, j = 1, 2, 3,$$

$$a_{ij} = S \frac{\delta x}{\delta x_i} \frac{\delta x}{\delta x_j} \tag{2}$$

where the summation S extends over x, y, z.

The expressions for the laws of physics are more complicated in the new co-ordinates, but the theory of differential forms provides a method of writing the equations. For example, if U be the ordinary gravitational potential, we have outside the gravitating mass

$$\frac{\delta^2 U}{\delta x^2} + \frac{\delta^2 U}{\delta y^2} + \frac{\delta^2 U}{\delta z^2} = 0.$$

The expression of Laplace's equation in general co-ordinates is

$$\sum_{i,j} a^{(ij)} \left\{ \frac{\delta^2 U}{\delta x_i \delta x_j} - \sum_{k,l} a^{(kl)} \left[{}^{i\,j}_{\,l}\right] \frac{\delta U}{\delta x_k} \right\} = 0,$$

where

$$\left[{}^{i\,j}_{\,l}\right] = \frac{1}{2} \left[\frac{\delta a_{il}}{\delta x_j} + \frac{\delta a_{jl}}{\delta x_i} - \frac{\delta a_{ij}}{\delta x_l} \right]$$

---

[1] This note is in fact an amplification of certain parts of my closing lecture.

and

$$a^{(ij)}=A_{ij}/a,$$

A_{ij} being the cofactor of a_{ij} in the determinant

$$a=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

of the coefficients of the differential form.

The element of volume is expressed with the aid of the Jacobian J of x, y, z, relative to x_1, x_2, x_3 as

$$dx\,dy\,dz=\frac{\delta(x, y, z)}{\delta(x_1, x_2, x_3,)}dx_1\,dx_2\,dx_3,$$

and the value of the determinant a is

$$\sqrt{a}=\frac{\delta(x, y, z)}{\delta(x_1, x_2, x_3,)}=J.$$

If the amount of matter in a volume dV is dM, the density is

$$\rho=\frac{dV}{dM}=\frac{J\,dx_1\,dx_2\,dx_3}{dM}=\frac{J\,dV'}{dM}=J\rho'.$$

We may then write Poisson's equation as

$$\sum_{i,j}a^{(ij)}\left\{\frac{\delta^2 U}{\delta x_i\delta x_j}-\sum_{k,l}a^{(kl)}\left[{}^{i\,j}_{\,l}\right]\frac{\delta U}{dx_k}\right\}=-4\pi\sqrt{a}\rho$$

if ρ be the value (ρ') of the density as measured by the quotient dV'/dM.

Such equations are familiar to physicists in essence, though not in notation, from using polar or other systems of co-ordinates—albeit physicists are not likely to redefine density as dV'/dM, but rather to use the original definition dV/dM and thus to consider the density as invariant of the co-ordinate system. The equations are perhaps familiar in their general form only to mathematicians who have busied themselves with such notations as those of Ricci in his absolute differential calculus,[1] or of Maschke in his symbolic theory of differential forms. In the dynamics of a particle, similarly complicated expressions occur[2] if we desire to use in place of x, y, z any set of generalized co-ordinates x_1, x_2, x_3.

I desire particularly to point out the obvious fact that, no matter what system of co-ordinates is used, the physical phenomena must necessarily remain the same. It would, for instance, be impossible to make Mercury's perihelion advance by using an unfamiliar and difficult system of co-ordinates.

2. *Relativity.*—The essential fact in the principle of relativity—the older familiar relativity—lies in Minkowski's dictum that space and time must henceforth be considered together as forming a four-dimensional manifold[3] in which the differential of arc is

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2. \tag{3}$$

This means with respect to generalized co-ordinates that we are at liberty to use any transformations,

$$\begin{aligned} x &= x\,(x_1, x_2, x_3, x_4), & y &= y\,(x_1, x_2, x_3, x_4), \\ z &= z\,(x_1, x_2, x_3, x_4), & t &= t\,(x_1, x_2, x_3, x_4), \end{aligned} \tag{4}$$

in place of (1), and that the form

$$ds^2 = \sum_{i,j} a_{ij}\, dx_i\, dx_j, \qquad i, j = 1, \ldots, 4, \tag{5}$$

$$a_{ij} = S \frac{\delta x}{\delta x_i} \frac{\delta x}{\delta x_j},$$

where the summation S extends over x, y, z, t, replaces (2). The equations in the new co-ordinates remain of a form similar to that arising before when only the space variables are changed. And just as before, the physical phenomena must be the same in the general variables as in the special x, y, z, t of ordinary relativity.

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[1] G. Ricci, *Theoria delle Superficie* (Padua, 1898) (lithographed); *Math. Ann.*, **54**, 125–201, 1900 (with T. Levi-Civita). My colleague C. L. E. Moore and I have just published an account of Ricci's method in connection with other investigations in *Proceedings of the American Academy of Arts and Sciences*, **52**, 269–368, 1916.

[2] See E. T. Whittaker, *Analytical Dynamics* (Cambridge, 1904), p. 38.

[3] See E. B. Wilson and G. N. Lewis, *Proceedings of the American Academy of Arts and Sciences*, **48**, 389–507, 1912.

3. *Einstein's theory.*—Now Einstein proposes to take as fundamental, not the special form (3) or any form (5) into which it may be transformed by a change of variables, but a different form,

$$ds^2 = \sum_{i,j} g_{ij}\, dx_i\, dx_j, \qquad i, j = 1, \ldots, 4, \tag{6}$$

in four variables. The space-time manifold of Minkowski defined by (3) is a flat non-Euclidean space of four dimensions; that of Einstein defined by (6) must be "curved," owing to the explicit assumption that (6) cannot be transformed into (3)—the amount of the "curvature" being, in fact, connected with the density of matter (or energy). This sort of suggestion was made by W. K. Clifford; but he, of course, did not speak of the four-dimensional space-time manifold, but merely of the three-dimensional manifold of spatial points.

Although it is not necessary to regard a curved space as imbedded in a flat space of higher dimensionality, it is often convenient to do so. There is a fundamental theorem which states that any differential form in n variables may be written as a sum of squares in a certain number $m \geqq n$ of variables. Thus

$$\sum_{i,j} g_{ij}\, dx_i\, dx_j = \sum_k \pm dy_k^2, \qquad \begin{array}{l} i, j = 1, \ldots, n, \\ k = 1, \ldots, m. \end{array}$$

The least number $m-n$ of extra variables necessary for this resolution is called the *class* of the given form. It is known that

$$m \leqq \tfrac{1}{2}n(n+1), \qquad m-n \leqq \tfrac{1}{2}n(n-1).$$

The class of a form in four variables like Einstein's may therefore be either 1, 2, 3, 4, 5, or 6. The class 0 is excluded, because the reduction would then be to ordinary relativity; but what the class of Einstein's form is seems as yet undetermined. Nordström has proposed a gravitational theory in which the space-time manifold is considered as imbedded in a flat space of five dimensions;[1] this would mean that the differential form must be of class 1.

4. *What is relativity?*—If the fundamental equations for the ultimate particles of matter in ordinary mechanics be taken in the

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[1] G. Nordström, *Physikalische Zeitschrift*, 15, 504, 1914; *Öfversigt Finska Vet.-Soc. Förhand.*, 57, Nos. 4 and 28, 1914–15.

form suggested by Boussinesq and others[1] who study the formulation of the foundations of mechanics, we have for each particle

$$m_i \frac{d^2(x_i, y_i, z_i)}{dt^2} = \sum_j f(r_{ij})(x_i - x_j, y_i - y_j, z_i - z_j),$$

$$r_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2]^{\frac{1}{2}}, \qquad j \neq i.$$

These equations are invariant with respect to the following transformations of x, y, z, t:

$$\begin{aligned} x' &= c_{11}x + c_{12}y + c_{13}z + ut + x_0, \\ y' &= c_{21}x + c_{22}y + c_{23}z + vt + y_0, \\ z' &= c_{31}x + c_{32}y + c_{33}z + wt + z_0, \\ t' &= t + t_0, \end{aligned} \tag{7}$$

where x_0, y_0, z_0, t_0, u, v, w are any constants and where the 9 constants c_{ij} are such as will determine a rotation of the axes of x, y, z, and hence are expressible in terms of 3 independent parameters. These transformations form a group of 10 parameters. For ordinary mechanics relativity means precisely this: that the equations of motion are unchanged by the ∞^{10} transformations of this group. If the equations of motion and of the group were expressed in terms of other variables x_1, x_2, x_3, t, the equations both of motion and of the group of transformations would have a more complicated form—and that would be all.

In examining the equations for electromagnetic phenomena, it is found that they are not unchanged by the group of transformations (7). They are, however, unchanged by the ∞^{10} transformations of another group—the so-called Lorentz group—namely,

$$\begin{aligned} x' &= c_{11}x + c_{12}y + c_{13}z + c_{14}t + x_0, \\ y' &= c_{21}x + c_{22}y + c_{23}z + c_{24}t + y_0, \\ z' &= c_{31}x + c_{32}y + c_{33}z + c_{34}t + z_0, \\ t' &= c_{41}x + c_{42}y + c_{43}z + c_{44}t + t_0, \end{aligned} \tag{8}$$

where the 16 constants c_{ij} are now subject to such relations as make the equations satisfy the identity

$$c^2dt'^2 - dx'^2 - dy'^2 - dz'^2 = c^2dt^2 - dx^2 - dy^2 - dz^2;$$

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[1] J. Boussinesq, *Mécanique générale* (Paris, 1889), p. 25; L. Boltzmann, *Principe der Mechanik*, I Th. (Leipzig, 1897), p. 25.

that is, (8) are the transformations of the group of motions in the non-Euclidean four-dimensional manifold of Minkowski. For electromagnetic theory relativity means precisely this: that the equations of electromagnetic phenomena are unchanged by the ∞^{10} transformations of the Lorentz group.[1]

In the early days, when it was the object of physicists to give a dynamical explanation for the universe of phenomena, consistency would have required that the equations of electromagnetic phenomena be so modified that they be invariant under the group (7) of mechanics. In latter times, when physicists have been striving to explain all phenomena in terms of the electromagnetic, consistency has required that mechanical laws be so formulated as to be invariant under the Lorentz group (8). From either point of view generalized co-ordinates might be employed.

5. *The abolition of relativity.*—Einstein's theory of gravitation depends upon a space-time manifold of which the element of arc,

$$ds^2=\sum_{i,j} g_{ij}\, dx_i\, dx_j, \qquad i, j=1, \ldots, 4, \tag{9}$$

is not reducible to $c^2dt^2-dx^2-dy^2-dz^2$. This at once abolishes relativity in either of the older senses without replacing it by any similar form of relativity. For if there were to be relativity in Einstein's new system, we should have to have some theorem like this: There is a group of ∞^k transformations

$$x'_i=f_i(x_1, x_2, x_3, x_4;\ c_1, c_2, \ldots, c_k), \qquad i=1, \ldots, 4, \tag{10}$$

dependent on k parameters, such that (a) the equations of physical phenomena are absolutely unaltered by the transformations of the group, and in particular (b) the transformations (10) applied to (9) give identically

$$\sum_{i,j} g_{ij}\, dx_i\, dx_j=\sum_{i,j} g'_{ij}\, dx'_i\, dy'_j,$$

the g''s upon the right being respectively the same functions of the x''s as the g's upon the left are of the x's.

A point which I wish particularly to make clear is that, although Einstein calls his new principle one of generalized relativity, it is not one of relativity at all, but of generalized co-ordinates. He is entirely right in laying down the principle that the equations of physics must be co-variant under any change of variables—that is, assumed under any circumstances—and he takes perhaps the best means of insuring the co-variance when he follows the methods of Ricci's absolute differential calculus; but relativity has nothing to do with this universal principle of co-variance under changes of variable, it has solely to do with absolute invariance under the transformations of some group.

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[1] See H. Minkowski, *Gesammelte Abhandlungen*, **2**, 431, 1911; F. Klein, *Jahresber. Deutsch. Math.-Ver.*, **19**, 281–300, 1910.

As a matter of fact, there may be a principle of relativity developable from Einstein's theory. That depends on the character of the fundamental form (9). We know that in the theory of surfaces in ordinary space there exists a group of "motions," i.e., a group of transformations which leaves ds^2 absolutely unchanged, on surfaces of constant (total) curvature like the sphere or pseudo-sphere; there are ∞^3 "motions" in the group just as in the case of the plane where the curvature is zero. We know also that for surfaces of which the curvature is not constant there is no group of ∞^3 motions—for some surfaces, like those of revolution, there is a group of motions of one parameter. In the case of Einstein's four-dimensional manifold the curvature is proportional to the density of matter (or energy),[1] and hence cannot be constant. This would appear to restrict the possible group of "motions" far below one of ten parameters, so that any such extensive group as that of Lorentz would be out of the question. The actual conditions for the existence of any group of "motions" are complicated and I cannot give them here—I could not in any case apply them unless the form of the 16 coefficients g_{ij} were known for some particular set of variables. So far as can be seen, however, Einstein in giving us his *Verallgemeinerte Relativitätstheorie* has taken us back to absolute space and time, perhaps more absolute than we have ever had—for it does not appear that even so simple a group of transformations as

$$x'=x+x_0, \quad y'=y+y_0, \quad z'=z+z_0, \quad t'=t+t_0, \tag{11}$$

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[1] See de Sitter, *op. cit.*, p. 706; $G=kT$, $T=\rho$.

which is common to the older relativities of mechanics and of electromagnetics, transforms his fundamental form (9) identically into itself.[1]

Nothing that I have here set forth should be interpreted as in any way detracting from the merits of Einstein's theory. It is exceedingly interesting and important to have Clifford's suggestion as to the possible meaning of "material particle" taken up, extended to the space-time manifold, and developed in such a way that when the actions and reactions of matter alone are considered each particle in the universe, including particles of light, moves along a geodesic line, i.e., along the straightest path, in the space-time manifold. As there are ∞^3 geodesics through each point (x, y, z, t), a particle is free to start from any position (x_0, y_0, z_0, t_0) with any component initial velocities (u_0, v_0, w_0) as in ordinary mechanics. If in formulating the theory such values are found for the 16 coefficients g_{ij} as will account for the advance in the perihelion of Mercury and predict other physical phenomena which shall turn out to be verified by observation, a brilliant advance will have been made in mathematical physics.

In closing I cannot refrain from expressing my dissent from the widely circulated remarks of Eddington in *Nature* (*loc. cit.*). He begins with a statement of the meaning of relativity which I have shown above has to do, not with relativity, but with generally admitted conceptions connected with generalized co-ordinates—this, however, may be regarded merely as a matter of definition, and is of no importance except as it may cause confusion by clashing with an already accepted definition. He states, however, at the end of his article:

> It is rather difficult to grasp the fact that the same laws of Nature may hold when some bizarre system of co-ordinates is chosen. Suppose an observer A uses rectangular co-ordinates, and B, through some kink in his mind, uses polar co-ordinates, without realizing that he is doing anything unusual. For A a ray of light can travel along the straight line x=constant; but evidently it cannot travel along the circle r=constant, which is B's idea of a straight line. The answer is that B through his peculiar system of measurement will suppose that he is in an intense gravitational field; he will calculate the curva-

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[1] M. Abraham's relatively simple theory of gravitation also gives up the Lorentz group, *Arch. f. Math. u. Phys.* (3), **20**, Heft 3, 193–209.

ture of the ray of light produced by this field; and, making allowance for it, he will find that the light actually travels along its theoretical curve (i.e., curve for B, but straight line for A).

Now as I understand the theory, B would not consider r=constant as a straight line. Both A and B would absolutely agree as to what they considered straight lines, i.e., geodesics; these would be the integrals of the differential equations.

$$\frac{d^2x_i}{ds^2}+\sum_{m,n,p} g^{(pi)}\left[{}^{m}{}_{p}{}^{n}\right]\frac{dx_m}{ds}\frac{dx_n}{ds}=0. \qquad (12)$$

The generalized co-variant form of these equations is the same for both A and B, but the actual forms, which would be obtained before integration was attempted, would be not invariant but very different because, using different systems of co-ordinates, A and B would have different analytic expressions for the symbols $g^{(pi)}$ and $[{}^{m}{}_{p}{}^{n}]$. When A got his equations he would perceive that $x=C$ was an integral, and hence a straight line; B would find that $r=C$ was not an integral, and hence not a straight line—he might perceive that $r \cos \theta=C$ was an integral, or he might not.

It could not be denied that if A and B used variables which involved the "time" in very different fashions they might differ as to the meaning of "gravitational field." They would still agree as to geodesics in their space-time manifolds. But what either would choose to call a straight line in space is difficult to say. The very possibility of the conception of space and time as different manifolds seems to require that the characteristics of space remain unaltered in time and that the characteristics of time remain unaltered in space, i.e., that phenomena admit the group (11). I should say that in a curved space-time manifold it might be, and in general would be, impossible to separate space and time—so the statement that a ray of light travels along a straight line in space becomes meaningless.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, March 13, 1917

A STUDY WITH THE ELECTRIC FURNACE OF THE ANOMALOUS DISPERSION OF METALLIC VAPORS[1]

By ARTHUR S. KING

The phenomenon of anomalous dispersion consists of a sudden change in the index of refraction of an absorbing medium for wave-lengths in the neighborhood of those which are strongly absorbed. It has been observed for various transparent media which show selective absorption. Metallic vapors, in many cases, show the effect close to their absorption lines when white light is passed through the vapor, and by suitable optical arrangements the change of refractive index may be shown. The principle of crossed prisms used by Kundt is generally employed, a prism of the absorbing vapor being formed with its refracting edge at right angles to the slit of the analyzing spectroscope.

The chief problem in such experiments is the production of a mass of vapor which will act as a prism. Wood,[2] working with sodium, used a cylinder of the vapor inclosed in a horizontal steel tube, heated below by a row of flames and cooled above. For the study of substances of higher melting-points Puccianti,[3] Schön,[4] and Geisler,[5] used an arrangement of the electric arc giving a non-uniform distribution of metallic vapor which acted as a prism. In the case of the arc, the possibilities of varying the state of the absorbing vapor are limited, and it seemed that an adaptation of the electric furnace offered a means of producing vapors of the more refractory elements under conditions which would give anomalous dispersion and, at the same time, permit a control of the temperature and distribution of the vapor by which variations of the phenomena could be studied. The use of mixed vapors in the furnace is a simple matter, so that for a given set of conditions the relative anomalous dispersion shown by the lines of different elements can be observed.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 130.

[2] *Phil. Mag.*, (6), **3**, 128, 1902; *Physical Optics*, p. 419.

[3] *Memorie della Società degli Spettroscopisti Italiani*, **33**, 133, 1904.

[4] *Zeitschrift für wiss. Photographie*, **5**, 349, 397, 1907.

[5] *Ibid.*, **7**, 89, 1909.

Assuming that the desired non-uniform distribution of vapor is available, a beam of white light, limited by a horizontal slit, is passed through the vapor and, being focused on the slit of the spectrograph, gives a strip of continuous spectrum in which the absorption lines of the vapor appear. The flexure of the continuous spectrum in opposite directions on the two sides of an absorption line registers the amount of anomalous dispersion produced by the vapor for the given line.

Experimental method.—The resistance tubes used in the electric furnace are of Acheson graphite, 30.5 cm long and of 12.5 mm bore, and are heated by the passage of the electric current from end to end. To adapt tubes for the present investigation, a slot about 6 mm wide was filed longitudinally between the end portions, which are held in contact blocks 20 cm apart. When placed in position, with the slot above and fragments of the substance to be examined on the bottom of the tube, a cylinder of vapor was obtained upon heating, having a strong density-gradient in the vapor between the molten mass and the open slot. The water-cooled copper pipe passing about 5 cm above the slot, by which the current was led to one end of the tube, accentuated the temperature difference. The vapor cylinder of varying density was thus equivalent in its dispersive effect to a prism with its refracting edge upward.

Certain features of the arrangement are important in interpreting the results. The most effective condition for producing strong anomalous dispersion of regular type corresponds to a temperature high enough to give broad absorption lines for the element studied, but not too far above the vaporizing-point, since a high density of the vapor in the lower part of the tube is required. If the temperature is such that the substance is fully vaporized, the rapid diffusion upward destroys the density-gradient, the prismatic action is very slight, and wide absorption lines showing little anomalous dispersion are obtained. With a mixture of substances requiring different temperatures for their vaporization, it is obvious that at a given stage one element may form an effective prism, while another has already reached a nearly uniform distribution, the anomalous dispersion being strong for one and weak, or absent, for the other. Also, for a given element, some lines may require a low temperature for their greatest strength, others a much higher temperature, and the same vapor prism will not be effective for both. In fact, a stage may be reached such that the conditions near the bottom of the tube, where high-temperature lines are produced, become positively unfavorable for low-temperature lines, so that the vapor absorbing the latter is denser above than below, and an inverted prism for lines of this character results. It is thus possible for regular and inverted anomalous dispersion to appear simultaneously for lines belonging to two different elements and for lines of different character belonging to the same element. Again, the same spectrum line can be made to show regular anomalous dispersion at low temperature and the inverted effect at higher temperature. All of these differences were observed in the experiments to be described.

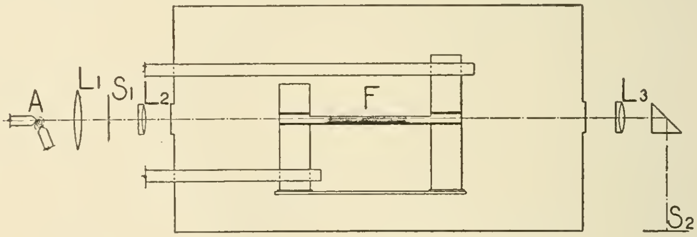


FIG. 1.—Arrangement of furnace and optical train

The arrangement of the optical parts and a vertical section of the furnace interior are shown in Fig. 1. Light from the positive pole of a carbon arc *A*, operated with 25–30 amperes at 110 volts, was focused on a horizontal slit S_1. The beam, rendered parallel by the lens L_2, passed through the vapor in the furnace tube *F*, and the image of S_1 was focused sharply on the slit S_2 of the vertical plane-grating spectrograph, the narrow band of white light passing across the slit. Use was made of both the 13- and 30-foot arrangements of the spectrograph, which give dispersions, in the second order, of approximately 2.1 and 0.89 A per mm respectively. Exposures with the 13-foot focus ranged from one-half to one minute.

The longer exposures with the 30-foot focus were used chiefly for large-scale photographs to test for shifts of lines occurring close to strong lines of pronounced anomalous dispersion.

For the production of well-defined anomalous dispersion it was found advantageous to operate the furnace *in vacuo*. At higher pressures the unsteady condition of the gas in the beam of light passing through the chamber is likely to impair the sharpness of the strip of continuous spectrum whose curvature measures the effect.

RESULTS

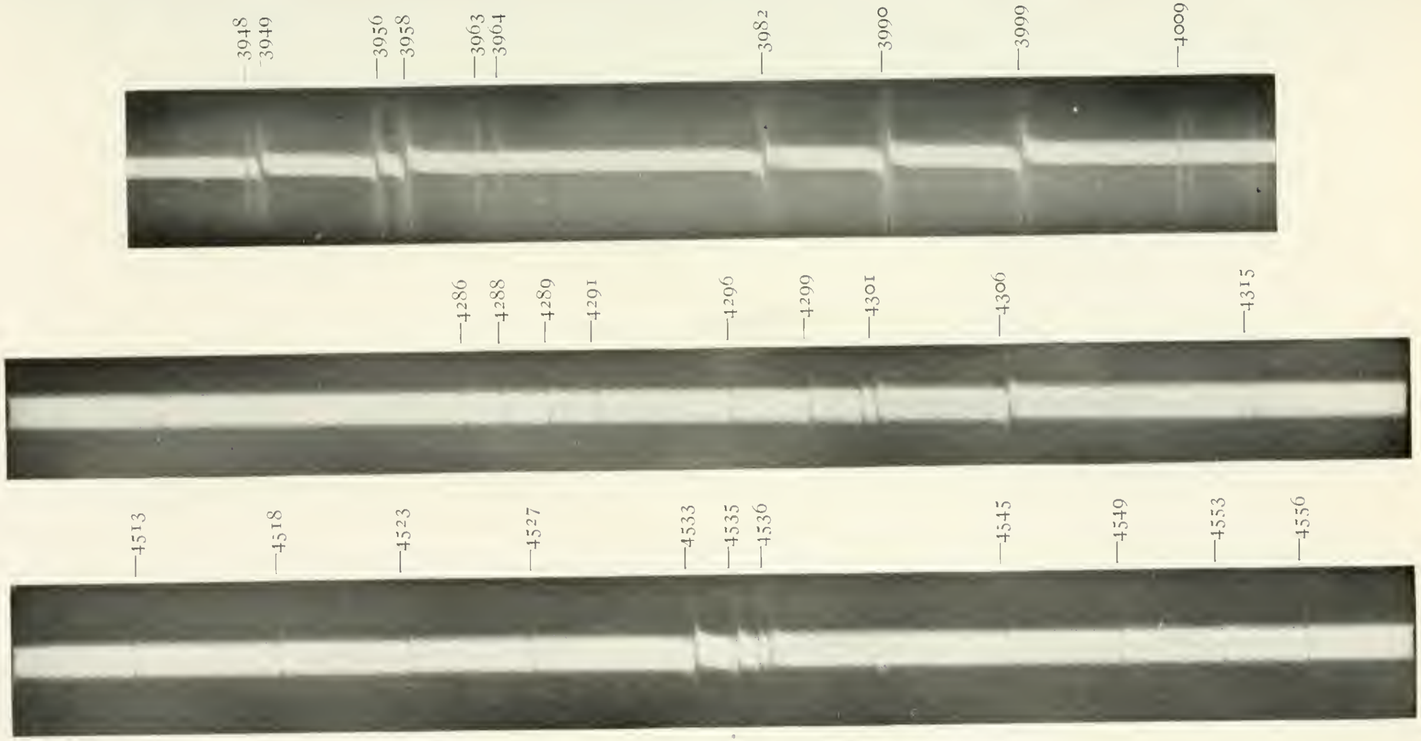
a) *Conditions for the production of anomalous dispersion.*—When white light from the pole of a carbon arc is passed through the furnace vapor, the absorption lines obtained are those observed in the emission spectrum of the furnace, and their widths in absorption are closely proportional to their emission intensities. The prominent lines of the absorption spectrum are thus the stronger furnace lines, belonging to Classes I and II of the furnace classification, while the faint absorption lines are those of Class III, requiring higher temperature. The furnace lines of higher classes have negligible absorption effect.

In discussions of anomalous dispersion the view that a spectrum line shows large or small dispersion as an inherent characteristic finds frequent expression, much as we say that a line shows large or small pressure displacement. The present experiments give no exception to the rule that the anomalous dispersion corresponds to the absorptive power shown by the line in question, *provided a proper prismatic distribution is present in the absorbing vapor.* This relation is very evident when a stretch of spectrum containing a number of lines of different intensities is examined, for the interval over which the curvature of the continuous spectrum on either side of a line is perceptible increases with the line's strength. The three sections of the titanium spectrum in Plate V illustrate this fact and show clearly the practicability of the furnace method for the study of substances of high melting-point. In the upper strip the emission lines of the furnace spectrum, in most cases reversed, appear as continuations of the absorption lines. It is in harmony with the proportionality of anomalous dispersion to the strength of the line that, throughout this investigation, no case was found for which a strong furnace line could not be made to show a correspondingly large anomalous dispersion, though for some lines it was necessary to determine by trial the proper conditions for an effective vapor prism. Of course, it cannot be stated now that this relation is quite general, but the furnace results furnish a high probability that anomalous dispersion is conditioned only by the presence of a vapor prism which strongly absorbs the line in question.

b) *Variability of anomalous dispersion with the condition of the absorbing vapor.*—We may now discuss examples of the condition already noted that, when mixed vapors are used in the furnace tube, a state of the vapor producing anomalous dispersion for one constituent may give a uniform distribution or even invert the prism for the second element, thus causing for this vapor either no curvature of the continuous spectrum or a flexure opposite to that of the first element. Instances in which one element showed strong anomalous dispersion and in which another, at the same time, gave only wide absorption lines occurred constantly during the experiments. Thus the three strong manganese lines λλ 4031, 4033, 4035, though often very wide, gave uniformly no anomalous dispersion at temperatures of the furnace such that neighboring lines of iron, chromium, and titanium showed the effect strongly. At a much lower temperature, however, before the other elements were fully vaporized, they showed very strong anomalous dispersion. The effect is shown in Plate VII*a*.

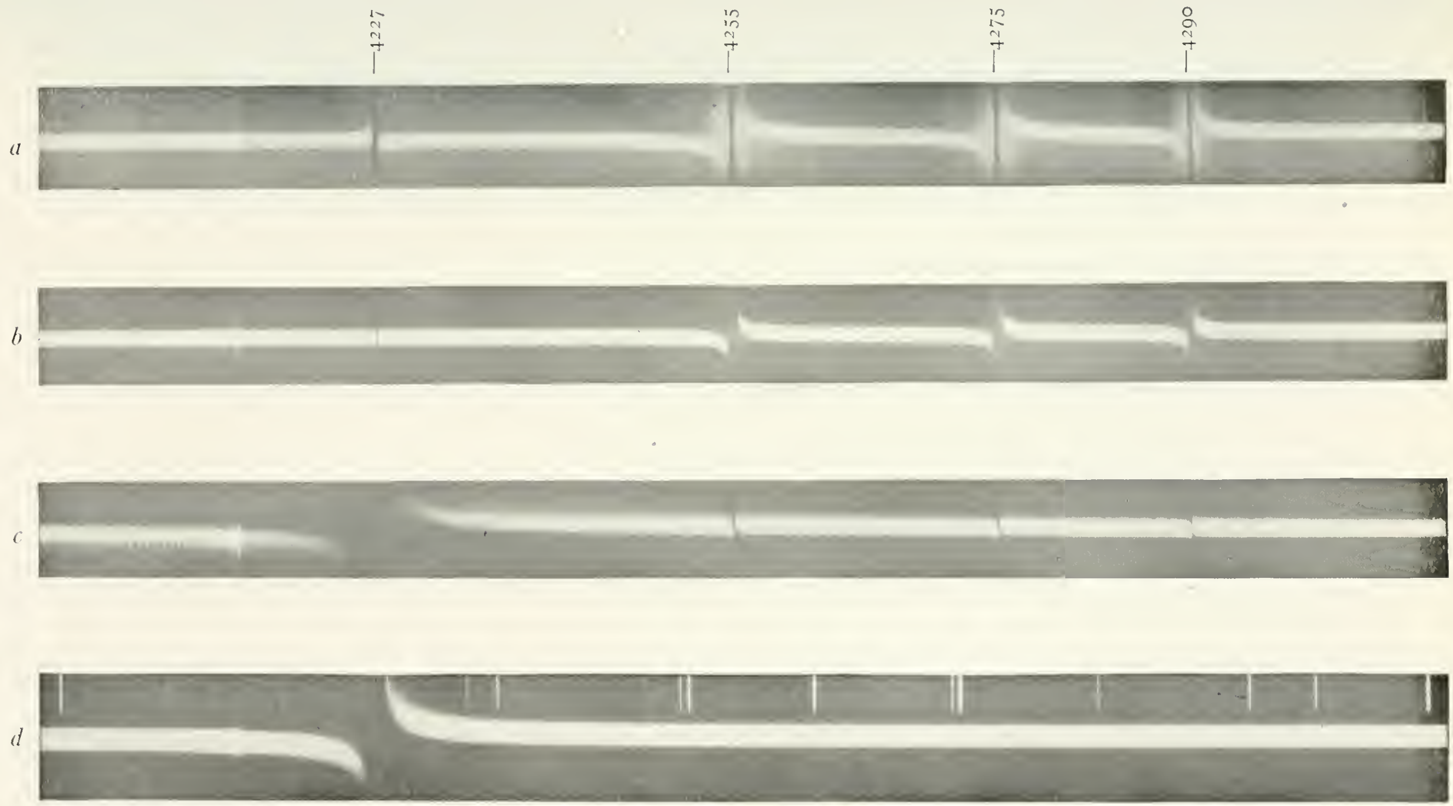
Experiments in which calcium and chromium were vaporized together gave an interesting contrast in the behavior of λ 4227 of the first element to that of λλ 4255, 4275, 4290 of chromium. In Plate VI are reproduced four spectrograms taken at different temperatures; VI*d*, at low temperature (with iron-arc comparison), shows a very large effect for λ 4227, the chromium lines being scarcely visible; VI*c* and VI*b* show flexure in the same direction for both calcium and chromium, though to very different degrees; VI*a*, at the highest temperature of all, gives the maximum effect for the chromium lines, while for λ 4227 the curvature is reversed, thus indicating that the prism of calcium vapor has become in-

PLATE V



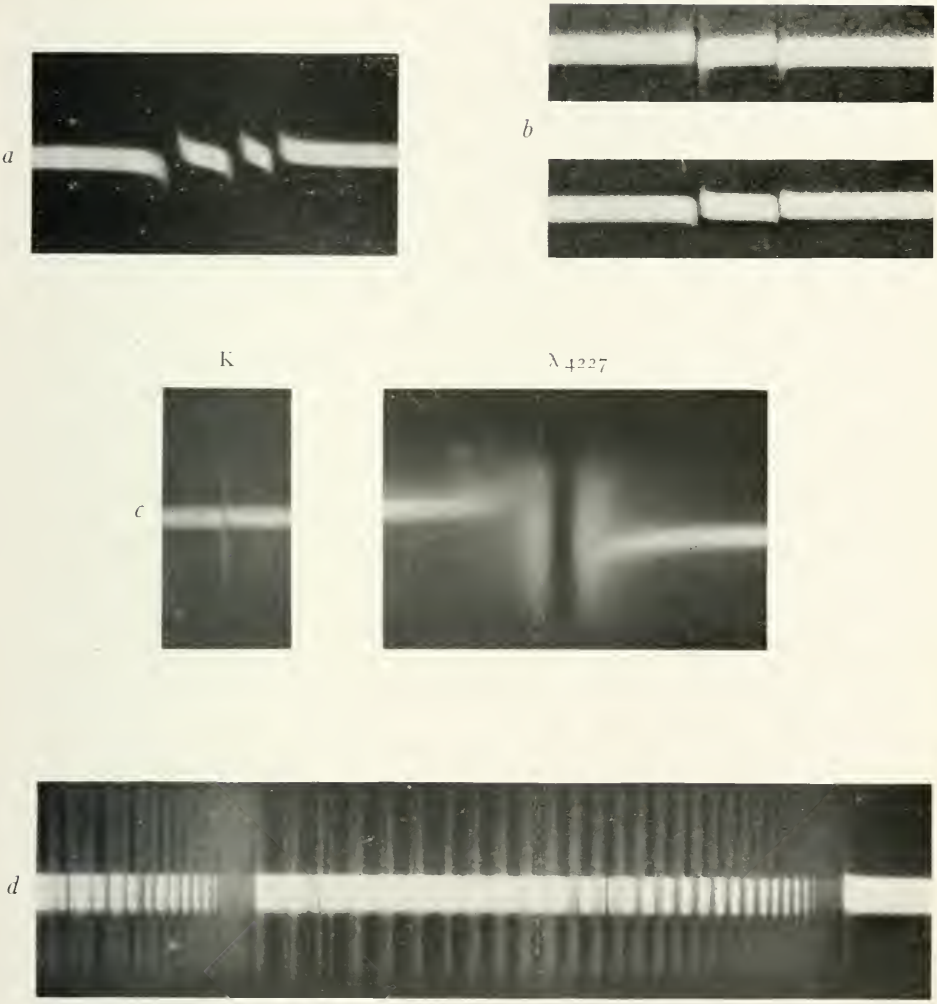
Anomalous Dispersion of Titanium Lines

PLATE VI



Anomalous Dispersion of λ 4227 of Calcium and of Three Chromium Lines at Four Stages of Decreasing Temperatures, Showing Inverted Effect for λ 4227 at Highest Temperature (*a*)

PLATE VII



Anomalous Dispersion in Line and Band Spectra

a Manganese lines $\lambda\lambda$ 4031–35

b D lines (low vapor-density), showing inversion of effect due to higher temperature

c Opposite effects produced simultaneously for calcium lines K and λ 4227

d Anomalous dispersion for "cyanogen" band with principal head at λ 3883

verted, so that the vapor producing λ 4227 is denser above than below, while the chromium prism remains with the denser region below.

Probably the dispersion of all low-temperature flame lines can be inverted in this way by properly chosen temperatures. In addition to the inversion of λ 4227, which occurred on many spectrograms, the effect was obtained for the manganese triplet already mentioned, for the D lines of sodium, for the aluminium pair λλ 3944–62, and for the potassium pair λλ 4044–47. The D lines, made narrow by a small amount of salt in the tube, and showing respectively for low and high temperatures the usual anomalous dispersion (below) and the inverted effect (above), are illustrated in Plate VII*b*.

By close adjustment of conditions numerous relative inversions were produced for different elements even of higher melting-points. Thus iron and chromium lines were inverted with respect to titanium when the three were vaporized together, and lines of manganese were inverted with respect to those of iron.

These effects are fully explained on the assumption that each kind of vapor forms an independent absorbing medium which, if conditions are such as to give prismatic distribution, shows its own anomalous dispersion without regard to what may be shown by other vapors in the mixture. By an extension of this principle, lines belonging to the same element, but emitted by different centers, should show simultaneously, under proper conditions, direct and inverted anomalous dispersion; for one set of particles, radiating freely at relatively high temperature, would have the denser part of their prism below, while the other set, emitting lines of lower temperature, would have their prism inverted. To test this possibility, the lines must be in the same region of the spectrum and radically different in their response to temperature; further, both must be radiated by the furnace with sufficient intensity to produce good absorption lines. In the calcium spectrum, λ 4227 and H and K seemed favorable for a comparison. At temperatures high enough to make H and K strong, λ 4227 is uniformly inverted in its anomalous dispersion, so that the main problem was to produce a distinct effect for H and K. This was done by suspending, a few millimeters above the slot in the tube, on insulated supports, a thick rod of graphite, so that when the tube was heated the cold rod increased the temperature-gradient in the calcium vapor. Later a water-cooled copper tube passing 1 cm above the slot was used for the same purpose. A number of spectrograms with these arrangements gave anomalous dispersion for all three lines, the result for H and K being that of a prism with denser vapor below, while λ 4227 showed very large anomalous dispersion in the opposite direction. The effect for the K line and for λ 4227, enlarged from the same negative, is shown in Plate VII*e*.

The experiments of Puccianti[1] have shown that this is not an invariable result for these lines, since, with the arc containing calcium as the absorbing source, he obtained anomalous dispersion in the same direction for all three lines, though very different in amount. With the conditions adopted for the furnace it has been possible to make the particles absorbing the H and K lines act as a body of vapor distinct from that absorbing λ 4227.

c) The anomalous dispersion of iron, chromium, and titanium.— The results of observations over an extended range of wave-length on the relative amount of anomalous dispersion shown by different lines and its relation to other spectral characteristics will now be given for iron, chromium, and titanium. For iron and chromium a portion of the range here covered has been studied by Geisler[2] with the interferometer, using the arc to supply the absorbing vapor. Of the effect for titanium no previous investigation is known to the writer.

Tables I, II, and III give the lines of iron, chromium, and titanium, respectively, for which distinct anomalous dispersion has appeared. An estimate of the amount of the effect is given for each line, "1" denoting that the curvature of the continuous spectrum on opposite sides of the line is just perceptible. The wave-lengths for iron are those of Burns[3] on the International System, abbreviated to two decimal places, while the values of Exner and Haschek are used for the chromium and titanium lists.

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[1] *Memorie della Società degli Spettroscopisti Italiani*, **33**, 133, 1904.

[2] *Zeitschrift für wiss. Photographie*, **7**, 89, 1909.

[3] *Lick Observatory Bulletin*, No. 247, 1913.

TABLE I

IRON

λ	Degree of Anomalous Dispersion	λ	Degree of Anomalous Dispersion	λ	Degree of Anomalous Dispersion
3581.20	8	3795.00	1	3930.30	6
3605.45	5	3798.51	1	4005.25	1
3608.86	2	3799.55	1	4045.82	3
3618.77	2	3812.97	1	4063.60	2
3631.46	2	3815.84	2	4071.75	2
3647.85	2	3820.43	6	4132.06	1
3679.92	3	3824.44	4	4143.87	1
3687.46	2	3825.89	5	4202.03	1
3705.57	3	3827.83	2	4250.79	1
3707.92	1	3834.23	3	4271.76	2
3709.25	1	3840.44	2	4294.13	1
3719.94	20	3841.05	1	4307.91	3
3722.57	4	3856.37	6	4325.77	2
3727.62	2	3859.91	15	4375.93	1*
3733.32	3	3872.51	1	4383.55	4
3734.87	8	3878.58	4	4404.75	2
3737.14	12	3886.29	8	4415.13	1
3745.56	10	3887.05	1	4427.31	1*
3745.90		3895.66	3	5269.54	1
3748.26	6	3899.71	4	5328.04	1
3749.49	5	3902.95	2	5371.50	1
3758.23	4	3906.48	2	5397.14	1
3763.79	3	3920.26	4	5405.78	1
3767.19	2	3922.92	5		
3787.88	1	3927.93	5		

* Subject to inversion of effect at high temperature.

TABLE II

CHROMIUM

λ	Degree of Anomalous Dispersion	λ	Degree of Anomalous Dispersion	λ	Degree of Anomalous Dispersion
3578.81	150	4339.62	4	4626.35	1
3593.64	100	4339.90	2	4646.35	2
3605.49	75	4344.68	4	4651.49	1
3885.36	1	4351.22	1	4652.38	2
3886.92	1	4351.98	5	5204.71	20
3908.91	2	4359.82	2	5206.24	30
3916.41	1	4371.48	2	5208.60	40
3919.32	3	4373.42	1	5247.72	2
3921.21	1	4385.15	2	5264.35	4
3928.82	1	4497.03	2	5265.90	2
3941.67	1	4546.10	2	5296.86	2
3963.85	1	4580.26	1	5298.46	3
4254.51	100	4591.61	1	5345.99	4
4275.00	80	4600.90	1	5348.50	2
4289.90	70	4613.53	1	5410.01	6
4337.74	3	4616.28	1		

TABLE III

Titanium

λ	Degree of Anomalous Dispersion	λ	Degree of Anomalous Dispersion	λ	Degree of Anomalous Dispersion
3635.61	40	4024.76	5	4536.25	3
3642.81	45	4055.17	1	4544.88	2
3653.66	50	4060.42	1	4548.93	3
3654.74	2	4064.39	1	4552.70	3
3658.26	4	4065.24	1	4555.70	2
3660.80	1	4078.62	1	4562.80	1
3669.15	3	4112.87	1	4656.63	5
3671.85	4	4286.19	3	4667.77	6
3690.09	2	4287.59	3	4682.10	6
3717.52	2	4289.26	3	4691.50	1
3722.73	2	4291.19	3	4693.88	1
3729.97	10	4295.93	3	4841.08	2
3741.25	15	4298.89	4	4981.93	6
3753.00	20	4299.40	1	4991.24	6
3753.77	3	4299.81	1	4999.68	5
3771.84	4	4300.73	5	5007.35	4
3901.14	2	4301.24	5	5009.79	1
3904.99	4	4306.09	7	5014.39	6
3914.50	3	4314.96	3	5016.29	1
3921.61	2	4326.54	1	5020.20	1
3924.71	6	4427.28	1	5023.02	1
3930.04	5	4449.35	1	5025.01	1
3947.98	6	4451.13	1	5036.08	1
3948.87	10	4453.52	1	5036.65	1
3956.50	10	4455.50	2	5038.59	1
3958.39	15	4457.61	2	5040.14	5
3963.05	4	4512.90	2	5064.79	4
3964.48	4	4518.19	3	5147.63	1
3981.95	10	4522.98	3	5152.35	1
3982.63	3	4527.47	2	5173.92	4
3989.94	12	4533.40	8	5193.12	4
3998.80	15	4534.95	6	5210.59	5
4009.12	5	4535.71	5	5219.86	1
4009.82	1	4536.12	3	5252.23	1

In comparing the material in Tables I, II, and III with the line-intensities in the emission spectra of iron,[1] chromium,[2] and titanium[3] at various temperatures of the furnace, the degree of anomalous dispersion was found without exception to be roughly proportional to the intensity of the line in question at the temperature (about 2500° C.) to which the vapor was heated in these experiments. All lines of iron and chromium included in Tables I and II have been placed in Class I or Class II of the furnace classification, being strong through a long range of temperatures. The same is true for titanium, except that ten lines of Class III, out of the total of 102, show perceptible anomalous dispersion. The fact that a line is relatively strong at low temperature (Class I) seems to have no bearing, except that pronounced lines of this type, such as λ 4376 of iron, sometimes attain considerable width as absorption lines without showing anomalous dispersion, and in some cases reveal an incipient inversion of the effect when other iron lines show regular anomalous dispersion. It is probably the same effect as was observed distinctly for the calcium lines H, K, and λ 4227.

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[1] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

[2] *Mt. Wilson Contr.*, No. 94; *Astrophysical Journal*, **41**, 86, 1915.

[3] *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

The anomalous dispersion shown by iron lines is much smaller than that for lines of chromium and titanium when the three elements are mixed in the tube. This is especially noticeable in the green and yellow, where the low-temperature iron lines remain narrow and show very small effects. The same relation exists, however, with regard to the widening and reversal of lines in the regular furnace spectra of these elements, and the anomalous dispersion seems to follow closely the ability of a line to reverse.

At the red end of the spectrum the lines of these three elements remain narrow in the furnace, and no anomalous dispersion was obtained. The effect clearly becomes stronger and more general as the wave-length decreases, but this also probably results from the greater absorptive power and the resulting ease of reversal shown by the lines of these elements in the region of shorter wave-length. When strong low-temperature lines, easily reversible in arc and furnace, occur in the region of greater wave-length, they readily show anomalous dispersion. Examples are the D lines, several barium lines in the red, and λ 6708 of lithium. The degree of anomalous dispersion thus appears to have no direct dependence on the wave-length.

d) Anomalous dispersion for a banded spectrum.—It is known that absorption bands may show anomalous dispersion, such effects having been observed by Geisler[1] for the bands of copper, calcium, strontium, barium, and calcium fluoride. On several plates taken in the present experiments a perceptible anomalous dispersion was observed for the "cyanogen" band λ 3883, which the experiments of Grotrian and Runge[1] indicate is due to nitrogen alone. The effect could be increased when the furnace was filled with air at one-half atmosphere, thus giving a larger supply of nitrogen, the appearance under these conditions being shown in Plate VII*d*. Not only the dense heads, but the stronger component lines of the band are produced in absorption, with the corresponding parts of the emission band showing faintly above and below. The direction of curvature of the continuous spectrum, forming parallelograms near the head of the band, indicates a greater density in the lower part of the absorbing cylinder. This must result from the presence of more radiating (and absorbing) particles in the region of highest temperature, since, if the band is due to nitrogen alone, the absolute density of the gas should be less in this region where the cylinder absorbs most strongly. The gradation of intensity in the emission lines bears out this conclusion, the portion of the line emitted by the lower layer of gas being strongest.

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[1] *Zeitschrift für wiss. Photographie*, **7**, 89, 1909.

MEASUREMENTS TO DETECT A MUTUAL DISPLACEMENT OF CLOSE LINES DUE TO ANOMALOUS DISPERSION

The theory advanced by Julius,[2] according to which lines very close together in the spectrum should undergo a mutual repulsion when anomalous dispersion is active, has been fully discussed in relation to the lines of the solar spectrum by St. John[3] and Albrecht,[4] and also in a later paper by Julius.[5] An attempt has been made recently by the writer to detect such an effect in the case of electric-furnace lines occurring close together. Lines suitable for this test should be separated by but a few tenths of an angstrom, and, since the theory of the repulsive action is based on a superposition of the changes in refractive index due to the two lines, both should show large anomalous dispersion. As this would require two wide lines, neither of which could be measured with accuracy, cases were looked for in which a narrow absorption line occurred close to a line showing large anomalous dispersion, the wave-length of the narrow line being measured from a neighboring standard line both when anomalous dispersion was active and when it was not. Very few pairs which satisfied these requirements could be found, but they were fulfilled in a fair degree by titanium and calcium lines occurring close to the chromium lines λ 4275 and λ 4290. The dispersion used was 1 mm = 0.89 A.

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[1] *Physikalische Zeitschrift*, 15, 545, 1914.

[2] *Astrophysical Journal*, 40, 1, 1914.

[3] *Mt. Wilson Contr.*, Nos. 93, 123; *Astrophysical Journal*, 41, 28, 1915; 44, 311, 1916.

[4] *Astrophysical Journal*, 41, 333, 1915.

[5] *Ibid.*, 43, 53, 1916.

Measurements were made on 27 spectrograms which showed varying degrees of anomalous dispersion. The measures for plates in which the titanium or calcium line was superposed on the curved spectrum near the strong chromium line were compared with those for which the anomalous dispersion was very slight and also with those for the emission lines given by the furnace and in the arc. (The test line was known to have the same pressure displacement as the standard line, so that measurements for the arc in air showed no sensible difference from those for the vacuum furnace.)

The following example shows the treatment of this material:

Measurements of λ 4289.237, Ti. (Rowland scale), separated 0.65 A from λ 4289.885, Cr. Standard line: λ 4287.566, Ti.

	Interval when Anomalous Dispersion Is Absent or Negligible	Interval for Large Anomalous Dispersion
	1.670	1.667
	1.674	1.667
	1.665	1.671
	1.670	1.671
	1.669	1.670
	1.668	1.668
	.....	1.670
	.....	1.668
Means...	1.669	1.669

Measures of λ 4289.237 referred to a more distant standard line, λ 4286.168 of titanium, also gave exactly the same result when anomalous dispersion was absent as when present in large degree.

The interval between λ 4289.237 and λ 4291.114 of titanium was also measured, these lines being on opposite sides of the strong chromium line and distant from it 0.65 and 1.23 A, respectively. While not very close to the chromium line, they were within the curved spectrum due to the latter, and any attraction or repulsion exerted on the titanium lines should be additive. The mean measurements with and without anomalous dispersion differed by only 0.001 A.

The calcium line λ 4289.525, distant 0.36 A from λ 4289.885 of chromium, was measured from a neighboring calcium line. A difference with and without anomalous dispersion of 0.002 A was found in the direction indicating an approach to the chromium line, though the difference is probably too small to be significant.

The only difference of appreciable size obtained was for λ 4274.746 of titanium, distant 0.21 A from λ 4274.958 of chromium, the measurements being made from the titanium line λ 4272.590. The difference found was 0.006 A in the direction giving an *approach* to the chromium line. The measurements were very difficult, however, and probably systematically in error, owing to the wide chromium line weakening the continuous spectrum on that side of the titanium line, which would have resulted in a difference of the kind observed. The use of very close pairs, which should be most favorable for the test, thus unavoidably introduces a source of error.

It is thus seen that no evidence whatever of repulsion between close lines is found, so that the effect required by theory must be very small. This conclusion is borne out in a recent deduction by Sir Joseph Larmor,[1] to whom a summary of the foregoing results was submitted, according to which the repulsion to be expected should be too small to be detected by the laboratory methods now available. As long as the effect is small and the lines tested necessarily very close, a serious obstacle arises from the fact that an effort to magnify the effect through stronger anomalous dispersion causes the lines to blend or at least to become unsuitable for measurement.

A further test with these lines was made in the regular furnace to see if a displacement could be found when a line occurs very close to a strong line known to give a high anomalous-dispersion effect.

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[1] *Astrophysical Journal*, 44, 265, 1916.

For this it was possible to use a larger scale, a dispersion of 1 mm = 0.6 A being employed. The titanium and calcium lines were photographed with and without the mixture of chromium vapor. Under these conditions the calcium line and one of those of titanium gave differences of 0.002 A, but these were opposite in direction and probably within the errors of measurement, while the other titanium line, closest of the three to a chromium line, agreed within 0.001 A when titanium was used alone and when it was mixed with chromium. For this test the result may therefore be considered negative.

SUMMARY

1. A method has been developed for the study of anomalous dispersion for metallic vapors of substances of high melting-point which allows close control of the conditions of the absorbing vapor.
2. The lines in the spectra of iron, chromium, and titanium, and also lines of special interest in other spectra, have been examined with reference to the degree of anomalous dispersion shown.
3. The rule was found to hold that the anomalous dispersion shown by a line is in proportion to its strength in absorption, provided the vapor absorbing the line in question has a non-uniform distribution equivalent to a prism.
4. Lines showing strong anomalous dispersion at low temperature can often be made, at higher temperature, to show the phenomenon with refraction in the opposite direction, thus indicating that the vapor prism absorbing such lines has been inverted.
5. Under this condition another element, requiring still higher temperature for vaporization, may show anomalous dispersion of regular type, so that for the two elements opposite effects occur simultaneously.
6. The same condition may be brought about for lines belonging to the same element but requiring very different temperatures for their production, λ 4227 of calcium thus showing an effect opposite to that simultaneously given by the H and K lines.
7. The experiments have thus demonstrated the capacity of each element to give its own anomalous dispersion independently of other vapors which may be present, a similar relation holding for

the particles of the same element emitting groups of lines of different character.

8. The greater prevalence of effects of anomalous dispersion in the region of shorter wave-length appears to result from the greater absorptive power generally found for lines in this region. When lines of suitable character occur in the red, correspondingly large effects can be obtained for them.

9. Distinct anomalous dispersion was obtained for the band at λ 3883.

10. Tests for a mutual repulsion between close lines, one of which is in a condition to show large anomalous dispersion, give no evidence of such an effect.

11. No change of wave-length is found when a line in the regular furnace spectrum occurs very close to a strong line known to show large anomalous dispersion.

MOUNT WILSON SOLAR OBSERVATORY
March 1917

REMARKS ON THE TEMPERATURE OF SPACE

By CH. FABRY

I. At a point of free space the notion of temperature does not have any meaning in itself. To make the notion precise it is necessary to imagine a small test body introduced into the region to be studied: under the influence of the radiations which are traversing space this body will assume a certain temperature of equilibrium which we may be tempted to call the temperature of space at that point, in the same way that we take the indication of the thermometer as the measure of the temperature of a region of uniform temperature in which it is placed.

But unfortunately we see, on examining the matter a little more closely, that the problem is much less simple than it appears and that the temperature of the testing body may vary between almost infinite limits, according to the properties of the surface of the body. We are able at the start to eliminate every complication due to the form of the testing body, by supposing it to be spherical and by admitting that its temperature becomes uniform by conductivity, which will take place almost necessarily if the body is small. The only possible influence that remains, then, is that of the properties of the surface, and it is there that we find the difficulty. According as its surface is more or less absorbing, the energy transformed into heat in a unit of time is greater or less. A *black* surface is by definition that which absorbs the most; we might think in consequence that the black surface is the one which gives the highest temperature. If it were so, a black testing body would have great interest as giving the maximum temperature of equilibrium at the point considered, and this temperature of equilibrium would be an important element in characterizing the state of the region in space being studied. Unfortunately, that view is totally incorrect. Bodies endowed with an absorption, partial but selective, may assume temperatures of equilibrium enormously higher (or much lower) than that of the black body; this may be of some interest in a practical way, but from the point of view of theory it is only one of the many possible temperatures of equilibrium, and it does not seem to merit any special consideration in defining the state of free space.

The temperature of equilibrium is defined by the fact that the body loses just as much energy as it absorbs in every second. For a given body isolated in space the loss of energy takes place solely by radiation and depends only upon the temperature T of the body. Let $f(T)$ be the energy radiated per second; if B is the energy absorbed per second, the equation of equilibrium may be written

$$B=f(T).$$

The black body is that for which the first member has the largest value, but it is also that which radiates the most for the same temperature. If the absorption is partial but non-selective (a *gray* body), the two members are multiplied by the same factor, less than unity; the temperature of equilibrium remains the same as for the black body. If the absorption is selective, B is less than for the black body, but the $f(T)$ is completely changed; according to circumstances, the temperature of equilibrium may be lower or higher than that of the black body.

II. Let us show this by a concrete example. Let us expose the spherical testing body (isolated in space and not at the surface of the earth) to the solar radiation such as it is at the distance from the sun to the earth. This radiation is almost entirely comprised in the spectral region between $0.3\ \mu$ and $2\ \mu$, which we shall call the region of short wave-lengths in comparison with that which includes the rays emitted at low temperature.

If the surface of the testing body is black, that body will radiate according to Stefan's law; its temperature of equilibrium is 280° Abs., as may be readily calculated.

Suppose now that the surface has a selective absorption. If it absorbs the short wave-lengths only slightly, but is very absorbent for the long wave-lengths (beyond $6\ \mu$, for example), it will radiate like the black body, but will absorb hardly anything; its temperature of equilibrium is very low. The contrary will be the case for a body which strongly absorbs the short wave-lengths, but for which its absorptive power (and by consequence its emissive power) is zero for the long wave-lengths. Such a body, at temperatures of 300° to 400° Abs., radiates *practically nothing.* It absorbs only one portion of the solar radiation, but its temperature rises up to that point at which it would be able to radiate enough to come to equilibrium with that which it absorbs. But the body is not capable of radiating anything except the short wave-lengths, which cannot appear except at very high temperature; it will therefore heat up very much more than the black body.[1]

III. In order to evaluate the foregoing numerically, I proceed to write the equation of thermal equilibrium in the case of any radiation whatever.

We assume that the testing body is spherical and at a uniform temperature. Let S be its surface, s the surface of a great circle; whence $S/s=4$. The absorbing properties will be defined by the absorptive power α, which is a function of the wave-length λ, so that $\alpha=\phi(\lambda)$. The radiation received is defined by its energy-curve as a function of λ,

$$E=\psi(\lambda),$$

which is equivalent to saying that $E\,s\,d\lambda$ represents the energy received per second by the sphere under the form of radiations comprised between λ and $\lambda+d\lambda$. This radiation may come from a single direction or from any direction, which is of no importance because of the spherical form of the receiver.

It is finally necessary to introduce the *function of radiation* of the black body, which gives the energy-curve radiated by 1 sq. cm of the black body at the absolute temperature T, namely,

$$R=F(\lambda,\ T).$$

With this notation the energy which the testing body absorbs per second is

$$s\int_0^\infty \alpha\,E\,d\lambda.$$

The energy which is radiated (in view of the fact that the emissive power is equal to the absorbing power) is

$$S\int_0^\infty \alpha\,R\,d\lambda.$$

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[1] This is a purely theoretical point of view, without any reference to experiments possible on the earth. There will be some experimental applications in an article which will soon appear in the *Journal de physique.*

On equating these two quantities, we obtain the equation of equilibrium

$$4\int_0^\infty \phi(\lambda)\, F(\lambda, T)\, d\lambda = \int_0^\infty \phi(\lambda)\, \psi(\lambda)\, d\lambda. \tag{1}$$

This equation does not contain any unknown but T, and always gives for that quantity one and only one value. Numerical computation will be always facilitated if the different functions which enter into the equation are given by numerical tables. The radiation function $F(\lambda, T)$ is otherwise known by its algebraic expression.

IV. To define the energy-curve of the radiation received, I proceed to assume that the radiation is entirely emitted by black bodies, all at the same temperature Θ. Now, let Ω be the sum of the solid angles under which these radiating bodies are seen by the receiving body. This determines completely the energy-curve of the radiation received and we have

$$\psi(\lambda) = \frac{\Omega}{\pi} F(\lambda, \Theta).$$

If we designate by $1/M$ the fraction of the celestial sphere occupied by the radiating bodies, defined by

$$M = \frac{4\pi}{\Omega},$$

the equation of equilibrium of the receiving body becomes

$$M\int_0^\infty \phi(\lambda)\, F(\lambda, T)\, d\lambda = \int_0^\infty \phi(\lambda)\, F(\lambda, \Theta)\, d\lambda. \tag{2}$$

It only remains to make different hypotheses as to the absorbent properties of the surface of the receiving body—that is to say, as to $\phi(\lambda)$.

1. *Receiving body, black or gray.*—$\phi(\lambda)$ is here a constant and disappears from the equation. We have obtained for T the following value, which we should have been able to write immediately by applying Stefan's law:

$$T = \Theta M^{-\frac{1}{4}}. \tag{3}$$

2. *Receiving body having a single absorption band.*—Let us suppose that the receiver has a single absorption band of wave-length λ_1. The law of absorption at the interior of the band remains absolutely

arbitrary, and the absorption may be only partial. The only hypothesis is that the size of the band is small in comparison to its wave-length.

Under these conditions the integral figuring in the first member of equation (2) may be replaced by

$$F(\lambda_1, T)\int_0^\infty \phi(\lambda)\, d\lambda.$$

The second member receives an analogous transformation, so that the equation of equilibrium becomes

$$MF(\lambda_1, T)=F(\lambda_1, \Theta). \tag{4}$$

In order to get the equation which gives T, it remains to determine the function F. Employing Planck's equation, we find[1]

$$\frac{1}{T}=\frac{\lambda_1}{c}\left[ln\ M+ln\left(e^{\frac{c}{\lambda_1\Theta}}-1+\frac{1}{M}\right)\right]. \tag{5}$$

In many cases $\frac{c}{\lambda_1\Theta}$ is large; we may then simplify the formula (which amounts to employing the formula of Wien in place of that of Planck) and write

$$\frac{1}{T}=\frac{1}{\Theta}+\frac{\lambda_1}{c}\ ln\ M. \tag{6}$$

For a given radiation (M and Θ given) the temperature of equilibrium T, which of course may not exceed Θ, varies enormously with the wave-length of the absorption band. If λ_1 is large, the temperature of equilibrium is very low. The temperature rises accordingly as we consider the absorption bands situated more and more toward the shorter wave-lengths. If the hypotheses which are made remain true for all the values of λ_1 the temperature of equilibrium tends toward that of the radiating bodies, while λ_1 tends toward zero, and this remains the case even if the radiating bodies are exceedingly distant.

Numerical example.—Let us suppose that the radiation is produced solely by the sun, and let us examine what will happen for different testing bodies at various distances. The solar radiation will resemble that of a black body at 6000° Abs. Although that hypothesis may be only roughly exact, it will be sufficient to fix clearly the order of magnitude of the phenomena. First let us place the testing body at a distance from the sun equal to that of the earth. The radiating body then has an apparent diameter of 32′ and we find $ln\ M = 12.1$. Taking $c = 14{,}350$ micron-degrees, and applying formulae (3), (5), and (6), we get the results shown in Table I.[1]

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[1] The symbol *ln* here designates the Napierian logarithm. All the temperatures are measured on the absolute scale.

TABLE I

λ_1	T
0.4 micron	1980°
0.5	1700
1.0	1000
2.0	550
5.0	250
10.0	130
Black body	280

It will be seen that, if the receiving body has only one absorption band at the violet end of the spectrum, it will nearly reach the temperature of the melting-point of platinum by simple exposure to a beam of solar rays before it has penetrated into the earth's atmosphere. This strange result is readily explained if it is noticed that the body considered is not able to exchange its energy except in the form of violet radiation; for the radiations of that sort do not commence to be emitted to an appreciable degree except at a very high temperature. So long as such a temperature has not been reached, the body continues to warm up.

Suppose now that we move out from the sun to a distance D times the distance from the earth to the sun. M varies with D^2; if the testing body is black, its temperature varies as $D^{\frac{1}{2}}$; if its absorption is selective, the law of decrease is entirely different and may be much more gentle. If we take formula (6), which is applicable for small values of λ_1, and if we call T_1 the temperature of equilibrium at the distance of the earth ($D=1$), the temperature T at the distance D will be given by

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[1] The table has not been pushed below $\lambda_1 = 0.4\ \mu$, because we do not know what is the solar radiation for the very short wave-lengths. The radiation for long wave-lengths (5 or 10 μ) is also unknown; the corresponding figures are given only as an indication of the values, but it is certain that the corresponding temperatures will be very low.

$$\frac{1}{T}=\frac{1}{T_1}+\frac{\lambda_1}{c}\cdot 2\log D.$$

Table II gives some results for a black receiver and for a body having a single absorption band at $\lambda_1=0.4\,\mu$. The distance 360,000 (5 light-years) is that at which the sun would appear as a star of the first magnitude; at the distance 3,600,000 it would become a star of the sixth magnitude. The radiation of that star alone would maintain at 750° the temperature of the testing body endowed with selective absorption.

TABLE II

D	TEMPERATURE	
	Black Body	$\lambda_1=0.4\mu$
1	280°0	1980°
30	50.0	1450
360,000	0.4	830
3,600,000	0.1	750

V. I will give a final example. Let us suppose that the sun should disappear or that we remove so far from it that it becomes any one of the stars. There will remain the radiation of the *ensemble* of the sidereal universe. We may calculate the amount that will arrive, if we make the following hypotheses, which are of course very uncertain.

1. The intrinsic brightness of the nocturnal, nongalactic sky is equivalent to a star of magnitude 5 per square degree.[1] This corresponds to the magnitude -6.5 for the whole celestial sphere. In the more brilliant portions of the Milky Way this brightness would be about double. To take this into account, let us call the magnitude -7 for the *ensemble* of the heavens.

2. We will admit that all of that light comes from the sidereal universe and that it contains no portion of diffused solar light—a hypothesis which is very uncertain.

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[1] Ch. Fabry, *Astrophysical Journal*, 31, 394, 1910.

3. We will suppose that the composition of the light is identical with that of the sun, which itself is similar to that of a black body at 6000°. With these assumptions it is easy to calculate that the black testing body will be brought up to 3° Abs. by the radiation of the stars. Should we take that figure as the temperature of interstellar space in the region that we inhabit? That would not have much meaning, for a testing body which absorbed only in the region of $0.4\,\mu$ would reach 1000° Abs.

VI. Is it possible that there exist celestial bodies endowed with a selective absorption such that they are able to realize the very high temperatures predicted by the theory? This seems to me entirely improbable for solid bodies, but not for gaseous bodies. Gases exercise upon light an absorption eminently selective. Let us assume a gaseous mass which would produce an absorption, even if very weak, upon certain radiations of the visible spectrum or of the ultra-violet. If the absorbing power of the gas is zero for the infra-red, and if there is no other cause for loss of energy than the thermal radiation, that gas will attain a temperature enormously higher than that of a black body in the same region of space. The gas will warm up to the point where it emits the only radiations which it is able to emit, that is to say, those which it absorbs; hence it possesses the property of emitting certain radiations under the influence of the same incident luminous radiations. The gas is endowed with a sort of *fluorescence*, or rather *re-emission*, analogous to that studied by R. W. Wood and by L. Dunoyer, but in reality it is nothing but purely thermal radiation.

A case in which one may suspect the existence of such a phenomenon is that of the tails of comets. The band spectrum of these bodies indicates emission by a gas, associated with the spectrum of the sun, since its intensity increases when the comet approaches the sun. It has been possible to reproduce in the laboratory the different bands of the cometary spectrum, but it yet remains to find the cause which produces the luminosity of comets. It has been sought to explain this by cathodic radiation of the sun, which has also been invoked to explain the aurora borealis; but this last phenomenon would indicate a very intermittent cathodic emission, which does not seem easy to reconcile with the phenomena observed in comets. In a manner similarly hypothetical, we may imagine a purely thermal radiation due to the very high temperature caused by a slight selective absorption exerted upon the solar rays. In a question so difficult we should not neglect any possible explanation.

SUMMARY

If we attempt to define precisely the temperature at a point of space by the introduction of a *testing body*, we find that the temperature of that body may vary enormously according to its absorptive properties.

A testing body with a *black* surface does not have any special interest from the theoretical point of view, but a body with selective absorption may take a temperature much lower or *much higher* than that of the black body.

When the absorption is limited to the short wave-lengths, a feeble radiation may develop a very elevated temperature.

It is not impossible that this last case occurs for gaseous masses, which would then have the property of emitting, under the influence of the incident radiation, the same radiations which they absorb.

One may thus attempt in a very hypothetical manner an explanation of the emission spectrum of comets by this process.

February 1917

THE ROTATION OF PRISMS OF CONSTANT DEVIATION

By W. E. FORSYTHE

In the *Physical Review* (**29**, 37) for July 1909 there appeared a paper by Mr. H. S. Uhler on the "Rotation of Constant Deviation Prisms." In this paper there is pointed out the necessity of having the prism so mounted and so rotated that the objective glass of the telescope is always illuminated symmetrically.

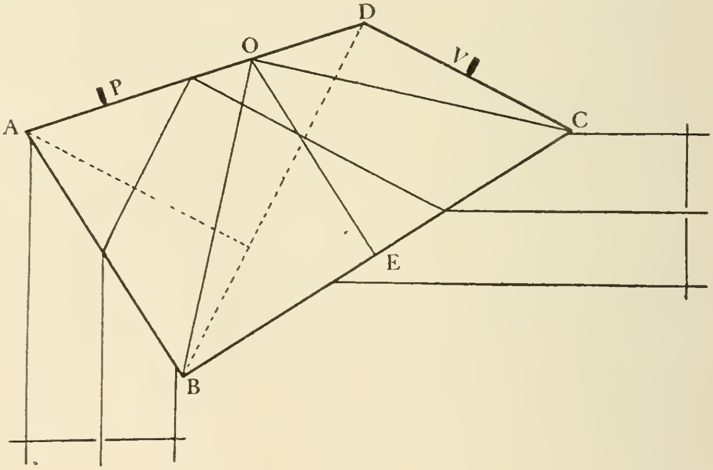

Fig. 1

In determining the point about which a quadrilateral prism of a constant deviation of 90° should be rotated, Mr. Uhler had in mind the necessity of using the maximum amount of the prism for each position. He states that the effective axis of the emergent beam from the prism should be kept as near as possible to the axis of the telescope. If the constant-deviation spectroscope is to be used in measuring radiation of different wave-lengths rather than in locating spectral lines, there must be a fixed limiting diaphragm, probably located either in the telescope or in the collimator of the spectroscope. If such a fixed diaphragm is used, the result will be that for one extreme of the spectrum under investigation the whole of the prism probably cannot be used. For this condition it would seem best so to mount and so to rotate the prism that it will always be at minimum deviation. In this paper it is shown that the prism may be so rotated as to remain always in the position of minimum deviation.

In Fig. 1 is shown a diagram of a quadrilateral prism[1] that gives a deviation of 90° for the condition ordinarily spoken of as minimum deviation.

Below, it is shown that, if the prism be rotated about the line of intersection of the plane bisecting the angle ABC and the reflecting surface of the prism, the prism will remain in the position of so-called minimum deviation.

In Fig. 1 $ABCD$ is a cross-section of a quadrilateral prism equivalent to a triangular prism with angle $2\alpha (\alpha < 45^\circ)$. This makes the angles at A, B, C, and D equal $45^\circ + \alpha$, 90°, 135°, and $90^\circ - \alpha$, respectively.

Take $\overline{OB}$ bisecting the angle ABC and draw $\overline{OE}$ perpendicular to $\overline{BC}$. To prove $\overline{OB} = \overline{OC}$, or $\overline{BE} = \overline{EC}$.

Thus

$$AO = \frac{AB}{\sin (90 - \alpha)} \cdot \sin 45^\circ.$$

$$BE = AO \cdot \cos (45 - \alpha) = \frac{AB}{\sin (90 - \alpha)} \cdot \sin 45^\circ \cdot \cos (45 - \alpha) = AB \frac{(\cos \alpha + \sin \alpha)}{\cos \alpha} \cdot \frac{1}{2}.$$

$$BD = \frac{AB}{\sin 45} \cdot \sin (45 + \alpha).$$

$$BC = \frac{BD}{\cos \alpha} = \frac{AB}{\sin 45^\circ} \cdot \frac{\sin (45 + \alpha)}{\cos \alpha}.$$

$$\therefore BE = 1/2 BC.$$

Thus

$$\overline{OB} = \overline{OC}.$$

The side AD of the quadrilateral prism is generally made longer in comparison with the side AB than would be given by making it equivalent to a particular triangular prism. However, for this discussion the prism is taken so that the angles and linear dimensions are such as would result by the construction from a triangular prism.

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[1] Pellin and Broca, *Journal de Physique*, (3), 8, 314, 1899.

It will now be shown that the quadrilateral prism outline in Fig. 1 will remain in the position of so-called minimum deviation for radiation of different wave-lengths if it is rotated about the line of intersection of the bisector of the angle *ABC* and the reflecting surface *AD*. The projection of the line of intersection is the point *O*. In order to prove this, let *EFGHI*, Fig. 2, be a ray of wave-length λ passing through the prism at minimum deviation for position I. (Position I is indicated by unprimed letters.) Also, let *EF′G′H′* be another ray of wave-length λ′ passing along *EF* and through the prism at minimum deviation to the point *H′*. This second ray of wave-length λ′ passes through the prism in position II. (Position II is indicated by primed letters.) This second position is obtained from position I by a rotation about the point *O*.

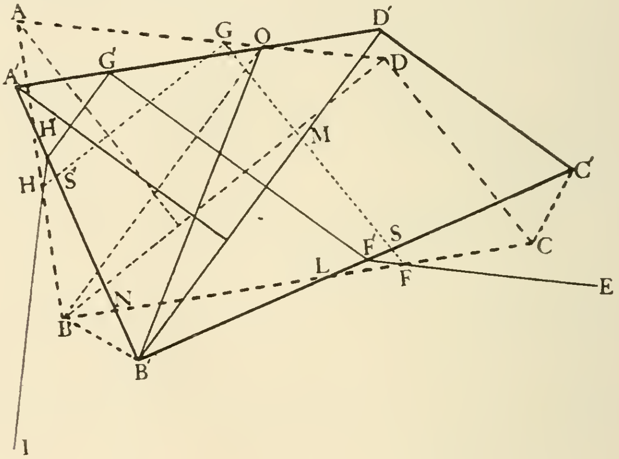

FIG. 2

For this second ray of wave-length λ′, which starts along the same path as the first ray to pass through the prism at minimum deviation and to continue along the same path as ray I after passing through the prism, the line *H′HI* must be a straight line and, also, the angle *HH′S′* must be equal to the angle *FF′S*.

To show that $H'HI$ is a straight line it is sufficient to show that the angle $H'HA'$ = the angle IHB. If the second ray is to pass through the prism at minimum deviation, it will leave the prism at the same angle that it entered. Thus the angle $HH'S'$ must be shown to be equal to the angle $FF'S$. To do this it will be sufficient to show that the polygon $BHH'B'$ = the polygon $CFF'C'$.

In the polygon $BHH'B'$ and $CFF'C'$

$$\overline{BH}=\overline{FC} \text{ for } \overline{MG}=\overline{MD} \begin{cases} \overline{MG}=\overline{HB}\cos\alpha \\ \overline{MD}=\overline{FC}\cos\alpha \end{cases}$$

$\overline{B'H'}=\overline{F'C'}$ for the same reason.

$\overline{BB'}=\overline{CC'}$, being chords of equal arcs.

The angle $BB'H'$ = the angle $CC'F'$, being measured by equal arcs.

The angle HBB' = the angle FCC', being measured by equal arcs.

Thus the polygon $BB'H'H$ = the polygon $CC'F'F$. From this it follows that the angle $HH'S'$ = the angle $FF'S$ and the line $H'HI$ is a straight line.

Hence it follows that if the prism is rotated about the line of intersection of the bisector of the angle ABC and the reflecting surface of the prism it will remain in the position of minimum deviation. With the prism rotated about the point O, at no time will there be a lateral shift of the beam. Thus the wave-length calibration will not depend upon the character of the lenses of the spectroscope.

Locating the quadrilateral prism on the prism-table.—If the prism is removed from the prism-table of a Hilger instrument, it is quite a tedious process to get it back in the proper place again. The author has not seen described any method of locating a quadrilateral prism in the proper place on the prism-table other than by the method of "cut and try." Now assume that the instrument is constructed so that the telescope is at right angles to the collimator and that the light entering the prism at the angle of minimum deviation leaves it at right angles to the direction at which it enters. Also assume that the wave-length calibration is accurate for some position of the prism. With reference now either to the mounting as used in the Hilger instrument or to the one suggested above, it is very easy, with the use of monochromatic light, to get the prism in such a position that the transmitted monochromatic radiation falls upon the cross-hairs or the eyepiece-slit of the telescope when the calibration on the drum indicates that it should do so. However, owing to the fact that the lenses are not perfectly achromatic, light of another wave-length will not come to a focus upon the cross-hair when the calibration on the drum indicates that it should do so, unless the telescope objective is illuminated symmetrically.

A method which will make sure that the light has passed axially through the collimator and the telescope lenses is to use over each lens a diaphragm with a small hole of such size that only a very small portion of the center of each lens is used. If such diaphragms are used, it is evident that, if monochromatic light from the collimator slit falls upon the cross-hairs in the telescope, this radiation has passed through the lens system axially.

Light comes from the collimator lens in a parallel beam. If only the center of each lens is used, the only light from the collimator slit that can come to a focus on the cross-hairs of the telescope is that light which was turned through 90° by the prism. Thus, if the prism is now placed on the table in such a position that light of any wave-length from the collimator comes to a focus on the cross-hairs of the eyepiece, it has passed through the prism in the position of so-called "minimum deviation."

By reference to Fig. 1 it can easily be seen that, if the radiation from the central part of the collimator lens does not pass through the central part of the face *AB* of the prism, a motion of the prism parallel to the axis of the telescope or of the collimator or of a combination of these two motions will be necessary in order to cause the radiation to pass through the central part of the prism. If now a screen, with small opening in the proper place, is constructed so as to fit over the prism, it will be seen that the prism could be placed in the symmetrical position by one operation. This, however, would not necessarily be the proper position unless the wave-length used was that for which the radiation was to pass through the central part of the face *AB* of the prism. With the wave-length known and with the use of the diaphragm, it would be a comparatively simple operation to get the prism properly located.

A very simple way to make such diaphragms is to have caps for the collimator and telescope lens. These caps can be made to fit over the ends of the telescope and collimator. The diaphragm could easily be made to fit over one end of the prism.

It is often convenient to be able to remove the prism and replace it on the prism-table. To do this requires some sort of mechanical holding device. A method that has been found very convenient is to follow the general method of having stops raised about 1 mm above the prism-table against which the prism can be placed. These stops, used in connection with the clamp usually furnished on the prism-table of the Hilger instrument, suffice to hold the prism exactly in position. The stops, placed on the prism-table at about the points *P* and *V*, as shown in Fig. 1, are so constructed that each can be moved by a screw motion. If the axis of rotation is made to coincide with the line of bisection of the angle of the prism and the reflection face, one of the stops should be located on the prism-table so as permanently to fix the point.

The principal advantage of having the prism rotated about the line of intersection of the bisectors of the angle *ABC* and the reflecting surface is, as was pointed out above, that the prism would always remain in the position of minimum deviation, and thus there would be no lateral shift of the beam. From this it follows that the calibration of the wave-length scale would not depend upon the character of the lenses used.

The position of rotation would depend only upon the size of the prism and the position of the telescope and collimator and not at all upon the index of refraction of the glass used. The only requirement is that the index of refraction be such that light can pass through the prism at minimum deviation. Thus prisms of the same size but of different kinds of glass could conveniently be used in the same spectroscope. Of course the same wave-length calibration would not do for the two prisms. With the use the foregoing method of mounting, it would be a simple matter to convert an ordinary spectroscope into a constant-deviation spectroscope. The mounting described follows the same general scheme as that used in the well-known Wadsworth mounting.

SUMMARY

It has been shown that a quadrilateral prism can be so mounted and so rotated as to remain always in the position of so-called minimum deviation. A method has also been outlined for locating the prism on the prism-table in approximately the proper position for either the mounting suggested in the paper or for the method used in the Hilger instruments.

Nela Research Laboratory
National Lamp Works of the General Electric Company
Nela Park, Cleveland, Ohio
April 1917

KARL SCHWARZSCHILD

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THE
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KARL SCHWARZSCHILD

By EJNAR HERTZSPRUNG

The death of Karl Schwarzschild[1] is a loss to science, all the harder to bear because he was taken away, not "full of years," but when at the summit of productivity.

Not long before his illness Schwarzschild said to the writer: "We have reached the height of life, before long we shall go downward." He did not live to see that time. He died standing. If he had known beforehand that his life would be so short, he could not have used it to better purpose.

Karl Schwarzschild was born at Frankfort-on-the-Main, the son of a Hebrew merchant. Like so many other prominent men, he was the only member of the family who took to science. He manifested his scientific abilities as a schoolboy, at the age of sixteen, when he published two advanced papers on the determination of orbits of double stars, which were printed in 1890 in the *Astronomische Nachrichten.* The next publication, in 1892, contains observations of variable stars. This shows how the direction of Schwarzschild's activity was already fixed in his student years—theory first, practice in the second place. He found his right place at once. "From the beginning," he once said to me, "I have been interested in all branches of astronomy." We may add now: "in all with success." In fact, the many-sidedness of Schwarzschild was astonishing. His high mathematical capacity, so uncommon in the practical astronomer, made him treat the most difficult problems of our science with startling virtuosity from a theoretical point of view. Still it was mathematical sense, not mathematical knowledge, that he most appreciated in others. He had nothing of the inapproachable authority. He was the born teacher and understood how to make the most complicated things clear—so clear that the students were even said to complain that they missed the sensation of dealing with abstruse problems. The readiness with which Schwarzschild grasped the essential point in everything was perhaps his strongest characteristic; the tedious details were left for others of less capacity.

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[1] Born October 9, 1873; died May 11, 1916.

It will be almost impossible for one man to give an adequate sketch of the whole of his scientific labor, because it covered such a wide field. I shall confine myself to some of his papers which are most nearly in the line of this *Journal*, of which he was a collaborator. I have lying before me a complete list of Schwarzschild's papers, carefully selected by Professor Ludendorff. I am sure that Schwarzschild himself would have been astonished to learn that this list contains about 120 numbers.

When we look at the more especially astrophysical work of Schwarzschild, it must be borne in mind that mathematical analysis was what chiefly interested him. An outline of his astrophysical publications alone will therefore cover only the smaller part of his work.

It is characteristic of his papers that they never contain simply bare data, but rather give observations as illustrations for some new method or other item which forms the gist of the communication. Many pearls are hidden in places where we should not naturally look for them. His original paper, of the year 1896, on the measurement of double stars with the aid of a variable objective-grating, in order to diminish the systematic errors of observation, contains also the first measure of an effective wave-length.

Schwarzschild took his degree at Munich in 1896 on a paper treating of Poincaré's theory of rotating liquid bodies. Afterward he was for three years assistant at the von Kuffner Observatory at

Ottakring, near Vienna. It is here that he created the efficient method of photographic stellar photometry with the aid of extra-focal images. In the elaborate paper written on the subject he also gave the well-known formula for a second approximation (the reciprocity law being the first) of the law of density of photographic films, viz., density $= f$ (intensity $\times$ exposure-time^q), where the exponent q is a constant. This second approximation has proved to be of good service in the case of long exposure-times used in stellar photometry. Besides photographic magnitudes of a few hundred stars in the Pleiades, Praesepe, and h, χ Persei, the paper contains photographic observations of the two variable stars η Aquilae and β Lyrae. These two last objects both yield a good example of how Schwarzschild's papers never missed the point. The observations of η Aquilae led up to the discovery of the color-variation in the Cepheid variables by showing that the photographic range of variation exceeds the visual range. The photographic observations of β Lyrae, on the other hand, compared with the visual observations of others, yielded a proof of the independence of the velocity of light of the wave-length.

The next two years, from 1899 to 1901, Schwarzschild was active as a privat-docent in the University of Munich.

In the year 1901, at the age of scarcely twenty-eight, he was appointed professor of astronomy and director of the observatory in the University of Göttingen. There he felt at home, not least on account of the especially mathematical sphere of this university. Göttingen he loved above all the other places where he worked. It was also there that he found his wife.

One of the best-known astrophysical works of Schwarzschild during that directorship is the "Göttingen Actinometry," which is to be considered as a continuation of his work on photographic stellar photometry started at Vienna. For the purpose he constructed the *Schraffierkassette.* Instead of using the disk of the intra- or extra-focal star-image for measurement in the microphotometer, the new method effects the necessary broadening of the image mainly through a small and regular shaking of the plate. The main advantage of this device is the considerable increase in the useful part of the field.

The Göttingen Actinometry, completed with this instrument, contains photographic magnitudes of the 3500 stars brighter than 7.5 in the *Bonner Durchmusterung* in the zone 0° to $+20^\circ$ declination. It thus comprises in this region the same stars as the visual Potsdam photometry. The material therefore yields the color-index of every star. Besides this, about 165 stars of the North Polar region were included. A single plate covers 1^h in α and 20° in δ, the focal length being 46 cm. The accuracy reached is considerable, the mean error being about $0\overset{\mathrm{M}}{.}03$ for the catalogue magnitude of a star between 5 and 8. It is an interesting fact in connection with the question about the general constancy of the light of the stars that, in spite of the increased accuracy of the single observation, only very few new variables were found or suspected. Among these few the most interesting is the bright variable SZ Tauri, with small range, of the type of ζ Geminorum.

The color-indices of the Göttingen Actinometry showed that the connection between color and spectrum of small dispersion was very intimate—twenty-five cases of considerable discrepancy could practically all be reduced to defective spectral classification. The statistics of color-indices brought to light among the stars between $4\overset{\mathrm{M}}{.}5$ and $7\overset{\mathrm{M}}{.}0$, visual Potsdam scale, a curious minimum of the number of stars of intermediate color. The fact is intimately related to the phenomenon of "giant" and "dwarf" stars. The predominance of white stars in the Milky Way is also clearly brought out.

In 1905 Schwarzschild went to Algiers to observe the total solar eclipse. Photographs of the flash spectrum were obtained with a spectrograph of "ultra-violet" glass. In the same year he investigated the distribution of ultra-violet light (λ about 320 $\mu\mu$) on the surface of the sun from photographs taken at Jena, using a layer of metallic silver as a color-filter.

Among the several popular papers by Schwarzschild we call attention to a lecture on celestial mechanics, delivered in 1903. It was printed at six different places and deserves special mention for astrophysicists, because it gives an unusually clear outline of theoretical astronomy. In that lecture a sentence occurs which throws a flashlight on his way of teaching. He says: "Poincaré's work is shrouded in an atmosphere of abstract reasoning which hides his conceptions as the veil which hid the statue at Sais. I have felt the more justified in attempting a simple presentation of Poincaré's work, as this veil has been more rarely lifted than looked upon with respectful awe."

Schwarzschild devoted considerable attention to the construction and the theory of instruments. Among his most valuable contributions on the subject belong his researches on geometrical optics, dating from 1905. He based them on the known theorem of the shortest light-path, the "Eikonal," and developed therefrom the theory of optical instruments. He showed the possibility of constructing a double mirror in the Cassegrain arrangement, which would give a field practically as perfect as that of a photographic refractor of the *carte du ciel* type, in spite of a ratio of aperture to focal length of 1 to 3.5.

Another instrumental invention dating from the time of his residence at Göttingen is the hanging zenith-camera for the determination of the latitude. The instrument yielded as a by-product the declination of 375 stars near the zenith. Furthermore, he constructed a special compass for the navigation of airships.

In 1909 Schwarzschild was appointed director of the Astrophysical Observatory at Potsdam. It was with a heavy heart that he left Göttingen. The new position as the head of an extensive institute claimed much time for non-scientific matter, and it is an evidence of Schwarzschild's indefatigable application that his productivity did not seem to suffer. Not only that; he took the keenest interest in the special work of every member of the staff. His invaluable help was never appealed to in vain; the readiness with which it was given was, for the writer, even a real danger to discretion.

During Schwarzschild's short directorship at Potsdam the most notable instrumental improvement was the regrinding of the two objectives of the great refractor. For the rest, his theoretical and practical work is so many-sided that it is difficult to give a coherent description of it all.

No wonder that the problems of stellar statistics, so accessible to mathematical treatment, especially appealed to his mind. His best-known achievement in this field is the ellipsoidal hypothesis of stellar movements. As the distribution of stellar velocities showed marked deviations from the Maxwellian law, Kapteyn advanced his hypothesis of two star-streams. But Schwarzschild showed that the unitary consideration would fit the observations nearly as well, if the Maxwellian velocity was only taken different in different directions. In other papers Schwarzschild gave a general solution of the integral equations of stellar statistics. In the continuation of these investigations he obtained among other things the interesting result that the stellar velocity, relative to the sun and projected on the sphere, cannot increase more than to double its amount, for 8 magnitudes decrease in absolute brightness.

In quite another field lie Schwarzschild's investigations relating to the theory of the tails of comets. From the year 1901 dates his well-known theoretical investigation on the pressure of light on small spherical particles. Under the action of the sun such a particle will be attracted proportionally to its volume and repelled by the pressure of light proportionally to its surface. Consequently there will be a determined size of the particle at which these two forces counterbalance each other. On still smaller particles the pressure of light will predominate, and they will hence be repelled from the sun. All this, however, is true only down to a size comparable to the wave-lengths of light, where diffraction begins to play a prominent part and the effect of repulsion is stopped. The maximum ratio between repulsion and attraction is of the order of 20 to 1.

When Halley's comet reappeared in 1910, photographs suitable for photometric use were obtained by the expedition to Teneriffe, equipped by the Potsdam Observatory. This material was discussed by Schwarzschild and showed the important result that the decrease in intensity of the comet's tail with the distance from the head is in the main explained by the corresponding decrease in density. In other words, the luminosity of the repelled particles is retained during their path along the tail, just as would be the case if it were due to resonance of the light of the sun.

At Potsdam, Schwarzschild developed an elaborate theory for the determination of radial velocities with the objective-prism, proposed by E. C. Pickering, and he tried the method practically on the group of the Hyades. In the course of similar work he discovered the strong bright K line in the second-type spectrum of the short-period spectroscopic binary σ Geminorum. The fact may prove to be of special interest in connection with the question about the constitution of the absolutely bright yellow stars.

To the theory of equilibrium of stellar atmospheres, more particularly that of the sun, Schwarzschild has made most valuable contributions. It is natural that this complicated question could be considered only in its main aspects, and his treatment of the subject is a testimony to his intuitive power of recognizing the essential points. In the comparison between the spectrum of the sun—especially the region of the calcium lines H and K—in the center and near the border of the disk he came to the conclusion that the diffusion of light in the sun's atmosphere probably substantially accounts for the observed facts.

Schwarzschild's last paper, written during his illness, bears on the application of the quantum-theory to spectral series. The ardent wish for scientific occupation did not leave him until the last. It seemed incredible to the visitor at his sickbed that he was a man so near death. At such an occasion one feels the mastery of a strong spirit over earthly misery.

Looking backward on his life, we can only preserve in our mind the rich memories he has left behind and try to understand what made him such an unusual man. If I were to name the most characteristic of Schwarzschild's qualities, I would not hesitate to say "elasticity." His mind was always susceptible to new impressions. He had a happy ability for forgetting all petty things—he never brooded over them. It was perhaps the secret of his freshness that he never loaded his mind with immaterial subjects. He was always on the lookout for new ideas. He was in every respect at full development, as if there were nothing more latent in him. Owing to this, he may perhaps not have had much appreciation of dormant capabilities in others—he wanted to see everything in action as it was in himself. He considered the theoretical mastery over the subject as the highest step on the ladder of the human mind; compared with this, the practical realization was an inferior matter. His judgment about persons and things was quick and sharp and nearly always to the point. At the same time he liked to be contradicted. It was no empty phrase he once used: "I cannot get on with a man who is respectful to me." He wanted "pure" science. A particularly fascinating charm surrounded Schwarzschild. At his loss we feel poorer.

Potsdam
1917

THE RELATIONSHIP OF STELLAR MOTIONS TO ABSOLUTE MAGNITUDE[1]

BY WALTER S. ADAMS AND GUSTAF STRÖMBERG

An important product of the radial-velocity observations of recent years is the clear evidence that the stars of large proper motion are characterized by high velocities in space. The fact that large proper motion is to some extent in itself an indication of high velocity must of course enter into consideration, but a computation of space velocities with the aid of mean parallax determinations shows that this cannot account for the results found unless a distribution of velocities is assumed quite other than that of the ordinary law of Maxwell. In a paper published in 1915 Kapteyn and Adams[2] found this result from a discussion of the data for a considerable number of stars, and suggested two alternative hypotheses: (1) that velocity is a function of absolute magnitude; that is, the intrinsically faint stars move more rapidly than the bright stars; (2) that the nearer stars move more rapidly than the distant stars. These conclusions are self-evident, since the stars of large proper motion in general have large parallaxes, and hence are relatively near and intrinsically faint. On the assumption of absolute magnitude as the effective agent, a change of about 1.1 km in radial velocity was found for each unit of magnitude in the case of the K stars.

The results were based wholly upon parallaxes derived from a formula connecting mean parallax with proper motion. The recent development of a method for deriving absolute magnitudes directly from stellar spectra has added greatly to the material available for such an investigation, and we have accordingly taken up the study of the question with a view to discriminating between the suggested explanations. In addition to those stars for which absolute magnitudes have been obtained directly, use has been made of a large number of stars with radial velocities determined at the Lick Observatory. For most of these the parallaxes have been calculated with the aid of a new formula connecting mean parallax with proper motion. The investigation is based upon about 1300 stars of the F, G, K, and M types of spectrum. In the successive portions of this paper we shall endeavor to indicate, first, the evidence afforded by a direct comparison of the arithmetical mean of the radial velocities with absolute magnitudes; secondly, the effect upon the results of the law of distribution of velocities, of stream-motion, and of errors in the parallax determinations; thirdly, the confirmatory evidence afforded by the linear components of the space-velocities.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 131.

[2] *Mount Wilson Communication*, No. 1; *Proceedings of the National Academy of Sciences*, 1, 14, 1915,

ARITHMETICAL MEAN OF RADIAL VELOCITIES[1]

As the first and simplest step in the research we have compared radial velocities directly with absolute magnitudes. For this purpose the stars have been divided into groups defined by certain limits of parallax, and within each group the stars have been arranged according to absolute magnitude. The average of the radial velocities freed from the sun's motion is then formed for each mean absolute magnitude within these groups of stars. The results accordingly show the relationship of radial velocity to absolute magnitude among groups of stars located in concentric shells of space around the sun. In this way the effect of magnitude is separated from a possible effect of distance from the sun. In order to secure the large numbers of stars which are necessary for such statistical comparisons, rather wide limits of parallax have been chosen. The results for the F and G stars and for the K and M stars are given separately in Table I. M is the absolute magnitude,[2] and V' the radial velocity corrected for the sun's motion. The number of stars is given in the second column.

A very few stars of quite abnormal velocity have been omitted in the case of the F and G types, but with hardly an exception they are stars of low absolute magnitude, and their inclusion would merely increase the change in V' with absolute magnitude.

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[1] In the following pages the arithmetical mean is designated by a bar above the symbol; the geometrical mean, by a bar below.

[2] Absolute magnitude is here defined as apparent magnitude reduced to the distance corresponding to a parallax of 0″.1.

The increase in the value of the radial velocity with the absolute magnitude is shown clearly for essentially all of the groups in Table I. It is most marked in the case of the stars of large parallax, since these show the largest range in absolute magnitude, and their parallaxes are most accurately determined. The smallest parallaxes, on the other hand, are subject to much larger percentage errors. The absence of intrinsically faint stars among the groups of small parallax is, of course, due to the fact that such stars are too faint to have been observed for radial velocity with existing instruments. Interpreted directly, therefore, two general conclusions may be drawn from these results: (1) Among stars at the same distance from the sun, as defined by these parallax limits, there is an increase of radial velocity with decrease in absolute brightness. (2) There is little evidence of a variation in radial velocity with distance from the sun.

TABLE I

π	Number	$\overline{M}$	$\overline{V}'$	π	Number	$\overline{M}$	$\overline{V}'$
F AND G STARS							
			km				km
$\geqq 0''.050$	42	3.0	15.4	$0''.024$ to $0''.013$	105	0.3	10.8
	57	4.8	16.5		79	1.4	11.7
	29	6.8	21.2		41	3.1	13.5
0.049 to 0.025	56	1.2	15.8	$\leqq 0.012$	83	0.3	11.6
	87	3.0	15.1		58	1.4	14.0
	27	5.6	19.1		19	2.5	15.4
K AND M STARS							
$\geqq 0.050$	29	2.4	20.7	0.024 to 0.017	48	0.6	14.8
	27	6.2	25.1		99	1.4	17.5
	38	7.7	27.2		18	2.7	19.7
	13	10.0	30.2	0.016 to 0.013	36	0.6	14.5
0.049 to 0.034	36	1.3	21.2		41	1.3	14.3
	28	2.7	24.3		10	2.2	17.2
	9	6.3	30.7	$\leqq 0.012$	18	0.5	16.3
0.033 to 0.025	23	0.3	13.0		33	1.4	15.6
	55	1.5	14.1		13	2.3	13.4
	26	2.4	22.0				

If we combine the results for all of the zones according to absolute magnitude we obtain the values given in Table II.

TABLE II

F and G Stars			K and M Stars		
Number	$\bar{M}$	$\bar{V}'$	Number	$\bar{M}$	$\bar{V}'$
		km			km
188	0.3	11.2	125	0.5	14.6
193	1.3	13.6	264	1.4	16.6
189	3.0	14.9	124	2.5	20.6
84	5.1	17.3	74	7.0	26.9
29	6.8	21.2	13	10.0	30.0

A plot of these results in Fig. 1 shows that they are represented with sufficient accuracy by two straight lines nearly, if not quite, parallel to one another. A linear relationship, therefore, is indicated between absolute magnitude and radial velocity, the change in velocity for each unit of magnitude amounting to 1.5 km for the spectral types considered. A value of 1.1 km was found for the stars of type K by Kapteyn and Adams in the paper to which reference has already been made.

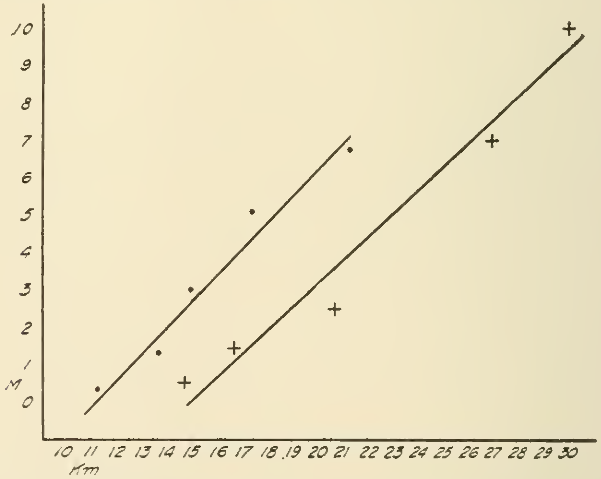

FIG. 1.—Variation of radial velocity with absolute magnitude: points, for F and G stars; crosses, for K and M stars.

DERIVATION OF THE PARALLAXES

The vital importance of accurate parallax determinations for such an investigation is obvious. Since spectroscopic parallaxes were available for only about 700 of the stars used, it became necessary to supplement these values with others derived by aid of a mean parallax formula. A brief description of this formula is therefore essential.

The measurements of van Maanen upon the parallaxes of stars of very small proper motion, together with spectroscopic determinations, have shown that the values for such stars are considerably larger than would be expected from their proper motions. Thus for stars with proper motions less than 0ʺ020 annually the parallaxes appear to be nearly independent of proper motion and dependent only on the magnitude. This is probably due to the fact that a list of stars of very small proper motion, but bright enough to be contained in a catalogue such as that of Boss, involves the selection, not only of distant stars of high intrinsic luminosity, but also of those whose velocity components at right angles to the line of sight are nearly parallel to the sun's velocity component in the same plane and are of the same order of magnitude. Since the intrinsic luminosity of a star cannot be infinitely great, we should expect that the selection of apparently bright stars of very small proper motion would involve a selection of *direction* of space-motion as well as a selection of distant stars. The values given by Kapteyn's mean parallax formula[1] are, therefore, probably too small for stars of very small proper motion.

An additional consideration is that among the distant stars we observe only those of high luminosity. Accordingly, if, as the results of this investigation indicate, the highly luminous stars have small radial velocities and hence small space-velocities, we should expect that the usual mean parallax formula, which is founded on the assumption that the velocity- and the luminosity-curves are independent of distance, would give too small parallaxes for the stars of small proper motion.

For these reasons we have not made use of Kapteyn's equation in its usual form, but have modified it in such a way as to obtain finite parallaxes for zero proper motion by the introduction of an additional constant. The form used is

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[1] *Publications of the Astronomical Laboratory of Groningen*, No. 8.

$$\pi = A(\mu + c)^b \epsilon^m \text{ or } \log \pi = \log A + b \log (\mu + c) + m \log \epsilon .$$

The spectroscopically observed parallaxes were used in the derivation of the constants in order to render the results homogeneous. Since the value of b was found nearly equal to 1, and it proved most difficult to determine b and c separately, the more simple form

$$\log \pi = \log A + \log (\mu + c) + m \log \epsilon$$

was used for stars with proper motions less than 1″, and gave results in good agreement with observation. Since proper motion is a different measure of parallax in the neighborhood of the sun's apices and apart from them, the constants have been determined separately for the stars near and distant from these points. The formula has been referred to the magnitude scale of the Harvard system.

The mean errors in $\log \pi$ derived from this equation are dependent upon the magnitude and proper motion. They are largest for the stars of types K and M, but are considerably smaller than those given by the formulae of Kapteyn and van Rhijn.[1] Further details of this formula and its use will be given in a future communication.

FREQUENCY DISTRIBUTION OF VELOCITIES

Reference has already been made to the suggestion in the paper by Kapteyn and Adams that the distribution of velocities in space might be quite other than that given by Maxwell's law, and that this fact might be used to explain the smaller radial velocities of the stars of small proper motion. It is evident that, if in any selected group of stars there is an excess of large velocities as compared with that given by Maxwell's law, we are inclined to assign such stars too large parallaxes owing to their large proper motions, and consequently to conclude that the mean intrinsic brightness of the stars in the group is too low. We might, therefore, arrive at false conclusions regarding a relationship between velocity and either distance or absolute magnitude. It is therefore most desirable that the effect of frequency distribution be eliminated so far as possible. The following method was devised by Strömberg for computing the parallaxes in a manner independent of the law of velocity-distribution.

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[1] *Mt. Wilson Contr.*, No. 110; *Astrophysical Journal*, 43, 36, 1916.

The star's velocity in space is projected upon three axes, one of which corresponds to radial velocity, and the other two to the components of proper motion which are denoted by ν and τ in Kapteyn's notation. The ν axis is directed toward the sun's apex, and the τ axis is at right angles to the ν axis. The three components of the star's velocity in space, corrected for the sun's motion, are as follows:

$$V'=V+V_0\cos d;\qquad V_\nu=\frac{K\nu}{\pi}+V_0\sin d;\qquad V_\tau=\frac{K\tau}{\pi},$$

in which V is the star's radial velocity, V_0 the sun's velocity, d the angle between the star and the sun's apex, K a constant[1] equal to 4.738, and ν and τ the components of the proper motion. If we assume that the frequency function F for these three components is the same, without any assumption as to its nature, we have

$$F(V')=F(V_\tau)\text{ or }F_1(\log V')=F_1(\log V_\tau)\text{ or }\overline{\log V'}=\overline{\log V_\tau}.$$

Denoting mean values by a bar above the symbol, we find

$$\overline{\log\pi}=\log K+\overline{\log\tau}-\overline{\log V'}. \tag{1}$$

If the *geometrical* means, denoted by a bar below the symbol, are then introduced, we have

$$\log\underline{\pi}=\overline{\log\pi};\qquad \log\underline{\tau}=\overline{\log\tau};\qquad \log\underline{V'}=\overline{\log V'}$$

or, finally,

$$\underline{\pi}=K\frac{\underline{\tau}}{\underline{V'}}. \tag{2}$$

This equation is of the same form as that derived by Campbell for the arithmetical means and valid for a small range in the parallaxes. Equation (2), however, is correct whatever the variation in π, τ, or V'. In equations (1) and (2) the numerical values of τ and V' are to be considered without regard to sign.

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[1] W. W. Campbell, *Stellar Motions*, p. 222.

A similar computation using the ν component gives as a final result

$$\underline{\pi}=\frac{K\underline{\nu}}{\underline{(V'-V_0\sin d)}}. \qquad (3)$$

The two equations (2) and (3) have been used to compute the parallaxes for the groups of stars used in Table I. These results are shown in Table III, as well as the geometrical means $\underline{\pi}$ of the parallaxes employed.

TABLE III

Number	π	$\underline{\pi}$	$\underline{\pi}(\tau)$	$\underline{\pi}(\nu)$
F and G Stars				
142..........	≧0″.050	0″.082	0″.078	0″.086
95..........	0.049–0.034	.041	.045	.044
91..........	.033– .025	.028	.023	.021
122..........	.024– .017	.020	.017	.013
110..........	.016– .013	.015	.006	.004
68..........	0.012– .010	.011	.004	.003
97..........	≦0.009	0.007	0.003	0.003
K and M Stars				
108..........	≧0″.050	0″.094	0″.075	0″.103
73..........	0.049– .034	.039	.021	.030
103..........	.033– .025	.028	.019	.022
162..........	.024– .017	.020	.010	.010
86..........	0.016– .013	.015	.004	.004
63..........	≦0.012	0.009	0.003	0.004

This comparison shows a good degree of accordance for the groups of large parallax between the values of the parallax actually employed and the computed values from the ν and τ components. For the groups of small parallax, however, the values used are appreciably larger than the computed ones. A part of this difference is due to the selection of stars with motions mainly in the line of sight and tangential components parallel and nearly equal to the sun's motion. This is seen from the results of Table IV where the radial velocities for the distant stars are considerably larger than the linear τ components. It is probable, however, that the parallax values employed are subject to a certain amount of systematic

TABLE IV

π	M	$\overline{M}$	$\underline{\pi}$	$\underline{V'}$	$\underline{V}_\tau$	$\frac{1}{2}(\underline{V'}+\underline{V}_\tau)$	No.	$\overline{V'}$	No.
				F and G Stars					
				km	km	km		km	
≧0″.050	≦3.9	2.9	0″.076	12.7	8.8	10.8	41	15.3	42
	4.0–4.9	4.47	.082	13.4	15.2	14.3	35	16.8	34
	5.0–5.9	5.35	.080	16.9	16.5	16.7	33	16.0	23
	≧6.0	6.92	.094	19.2	16.9	18.0	33	21.2	29
0.049–0.034	≦1.9	1.28	.040	9.3	7.9	8.6	19	13.9	19
	2.0–2.9	2.51	.039	14.6	9.6	12.1	24	15.0	19
	3.0–4.9	3.73	.042	12.9	21.1	17.0	26	15.4	24
	≧5.0	5.62	.042	23.2	36.9	30.0	26	20.0	21
0.033–0.025	≦1.9	1.09	.029	10.5	7.0	8.8	37	16.9	37
	2.0–2.9	2.32	.028	9.7	7.3	8.5	31	14.6	31
	≧3.0	4.61	.029	19.0	26.2	22.6	23	15.9	19
0.024–0.017	≦0.9	0.24	.020	6.2	2.9	4.6	43	8.6	42
	1.0–1.9	1.44	.020	7.6	8.6	8.1	50	13.6	49
	≧2.0	3.37	.020	11.5	16.8	14.1	29	11.9	25
0.016–0.013	≦1.9	0.73	.015	7.6	3.4	5.5	94	12.7	93
	≧2.0	3.10	.015	11.3	4.6	7.5	16	15.9	16
0.012–0.010	≦0.9	0.37	.011	7.1	3.4	5.3	36	11.4	35
	≧1.0	1.52	.011	11.0	4.0	7.5	32	14.1	31
≦0.009	≦0.9	0.21	.007	7.8	3.4	5.6	49	11.7	48
	≧1.0	1.73	0.007	14.0	6.7	10.4	48	14.5	46
				K and M Stars					
≧0″.050	≦2.9	1.95	0″.066	11.1	8.2	9.6	19	17.2	19
	3.0–6.9	5.38	.081	14.8	13.2	14.0	38	25.7	37
	7.0–7.9	7.27	.092	18.8	18.1	18.5	24	27.5	24
	≧8.0	9.34	.149	22.2	23.1	22.6	27	28.4	27
0.049–0.034	≦1.9	1.29	.038	16.9	6.2	11.6	36	21.2	36
	2.0–3.9	2.69	.040	18.1	12.1	15.1	27	24.3	28
	≧4.0	6.06	.041	24.3	23.1	23.7	9	30.7	9
0.033–0.025	≦0.9	0.26	.028	6.8	3.1	5.0	23	13.0	23
	1.0–1.9	1.49	.028	8.2	7.0	7.6	55	14.1	55
	≧2.0	2.46	.029	17.8	9.4	13.6	25	22.0	26
0.024–0.017	≦0.9	0.59	.020	9.8	3.3	6.6	46	14.8	48
	1.0–1.9	1.40	.020	12.2	6.7	9.5	98	17.5	99
	≧2.0	2.68	.021	14.7	8.3	11.5	18	19.7	18
0.016–0.013	≦0.9	0.61	.015	10.0	2.5	6.2	35	14.5	36
	≧1.0	1.48	.015	10.4	2.9	6.7	51	14.8	51
≦0.012	≦0.9	0.53	.008	9.1	3.4	6.3	18	16.3	18
	1.0–1.9	1.39	.009	11.8	4.9	8.4	32	15.6	33
	≧2.0	2.32	0.009	11.3	......	....	13	13.4	13

error, based as they are in the final analysis upon an extremely scanty amount of observational material.

EFFECT OF STREAM-MOTION

The stream-motion of the stars used in this investigation might affect to some extent the means of the radial velocities, although with the relatively large groups employed the influence would certainly be slight. As a means of reducing the effect, we have computed the geometrical means of the radial velocities $\underline{V'}$ and the linear τ components V_τ, and have formed the arithmetical means of these two values. If stream-motion is to be regarded as affecting mainly the direction of the space-velocities, the influence upon the mean of the two components should be greatly diminished. Table IV contains these results. The linear τ component $\underline{V_\tau}$ is calculated from the geometrical mean of the parallaxes of the stars in the group by the aid of the formula

$$\log \underline{V_\tau} = \log K + \overline{\log \tau} - \overline{\log \pi}.$$

For purposes of comparison the arithmetical mean $\overline{V'}$ is added for the different magnitude groups.

The results of Table IV agree with those of Table I in indicating a decided increase in velocity with decreasing absolute brightness among the stars of the several zones. This is true for both of the components used. There seems to be little evidence of a decrease in velocity with distance for a constant absolute magnitude, and it appears certain that the effect if present must be relatively slight.

As a final test of the results, we have used only the stars with parallaxes exceeding 0ʺ017. For these we have computed the geometrical means of the radial velocities (excluding a few stars with velocities smaller than 0.3 km), and the corresponding τ and ν parallaxes; also the absolute magnitudes and the linear τ components, using the means of $\underline{\pi}(\tau)$ and $\underline{\pi}(\nu)$. The absolute magnitudes and the τ components, therefore, are independent of measured parallaxes and of the frequency-distribution of the velocities. A single exception is found in the last group of the K and M stars for which the observed parallaxes are used. The results are given in Table V.

The increase of velocity with decrease of absolute brightness is shown very clearly in these results. The same result is true for the apparent brightness $\overline{m}$ of Table V. Under this heading are given the arithmetical means of the apparent magnitudes of the stars included within the groups *M*.

TABLE V

M	No.	$\underline{\pi}(\tau)$	$\underline{\pi}(\nu)$	$\frac{1}{2}(\underline{\pi}(\tau)+\underline{\pi}(\nu))$	$\overline{m}$	$\overline{M}$	$\underline{V}'$	$\underline{V}_{\tau}$	$\frac{1}{2}(\underline{V}'+\underline{V}_{\tau})$
F and G Stars									
							km	km	km
$\leqq$0.9....	60	0″.012	0″.010	0″.011	3.38	−1.43	6.33	6.69	6.51
1.0–1.9....	94	.023	.018	.020	4.44	+0.99	9.56	10.57	10.06
2.0–2.9....	78	.024	.026	.025	4.84	1.83	10.86	10.59	10.73
3.0–3.9....	52	.094	.073	.084	4.77	4.38	10.31	11.64	10.97
4.0–4.9....	62	.060	.059	.060	5.84	4.71	15.97	16.01	15.99
5.0–5.9....	64	.073	.057	.065	6.69	5.74	19.54	21.97	20.75
$\geqq$6.0....	40	0.077	0.095	0.086	7.32	6.99	20.96	18.78	19.98
	450	0.036	0.034	0.035	5.18	2.89	11.89	12.28	12.08
K and M Stars									
$\leqq$0.9....	77	0.009	0.007	0.008	3.54	−1.89	8.54	9.29	8.92
1.0–1.9....	188	.015	.018	.017	4.44	+0.54	11.65	10.42	11.04
2.0–2.9....	66	.022	.026	.024	4.64	1.57	14.23	13.14	13.69
3.0–5.9....	39	.035	.060	.048	5.55	3.95	20.59	15.20	17.90
6.0–7.9....	48	.081	.124	.102	7.33	7.38	17.97	14.17	16.07
$\geqq$8.0....	29	(0.137)	(0.203)	0.147	8.42	9.25	17.88	16.61	17.25
	447	0.022	0.026	0.024	4.98	1.88	12.88	11.64	12.26

RADIAL VELOCITY AND SPECTRAL TYPE

A comparison of the average radial velocity of the F and G stars with that of the K and M stars is complicated by the fact that the former show a much greater velocity-dispersion. Two stars with velocities of over 300 km belong to the F and G group, and there are several others of quite abnormal velocity. These stars have been excluded in the absolute-magnitude discussion, and it is for this reason that Table II cannot be regarded as giving the direct relationship between spectral type and velocity. For these reasons it seems probable to us that the use of the geometrical mean, which is not abnormally influenced by the inclusion of stars of exceptionally high velocity, is preferable for this purpose. Table VI gives the geometrical means of the radial and component velocities together with one-half their sum. The parallaxes have been corrected for systematic errors. All stars with parallaxes exceeding 0″.017 have been included.

TABLE VI

No.	M	$\underline{\pi}(\tau)$	$\underline{\pi}(\nu)$	π	$\overline{M}$	$\underline{V}'$	$\underline{V}_\tau$	$\frac{1}{2}(\underline{V}'+\underline{V}_\tau)$
F and G Stars								
						km	km	km
154....	$\leqq 1.9$	0″.017	0″.014	0″.016	+0.01	8.14	9.00	8.57
301....	$\geqq 2.0$	0.046	0.044	0.045	+4.03	13.89	14.07	13.98
K and M Stars								
268....	$\leqq 1.9$	0.013	0.014	0.014	−0.13	10.31	9.91	10.11
184....	$\geqq 2.0$	0.043	0.056	0.050	+4.62	16.17	14.15	15.16

These results indicate that the K and M type stars have a velocity of from 1.0 to 1.5 km higher than the F and G stars of the same absolute magnitude.

CONCLUSIONS AND SUMMARY

1. Radial velocity is a function of absolute magnitude for the 1300 stars used in this investigation. The increase in velocity is about 1.5 km for a decrease in brightness of one magnitude.

a) This effect cannot be ascribed to distance from the sun, since stars at the same distance show the effect strongly.

b) It cannot be due to the law of frequency-distribution of the velocities, since the component velocities calculated under no assumption of the nature of the frequency-law show the effect equally with the radial velocity..

c) It cannot be due to the effect of stream-motion.

2. A modified form of the formula connecting mean parallax with proper motion has been derived and applied to the stars under investigation. This formula gives mean errors considerably below those of the formulae commonly in use.

3. A method has been devised for calculating the geometrical means of the parallaxes of groups of stars, and these values have

been compared with those used in the investigation. The agreement is good for the groups of large parallax; for those of smaller parallax the observed values are somewhat too large.

4. The radial velocity and the τ-component velocity increase with decrease in apparent brightness.

5. The giant and dwarf division among the K and M stars is indicated clearly in the absolute magnitudes (Table V).

6. The use of the geometrical mean of the velocities and the parallaxes possesses some marked advantages for an investigation of this character. In the case of velocities it is unnecessary to exclude abnormally high values, as was done for the arithmetical mean, but only values very near zero. The agreement of the two means in indicating the variation of velocity with absolute magnitude is, therefore, a valuable check on the results. Further, from the geometrical mean of the parallaxes, it is possible to compute directly the mean absolute magnitude by the formula $\overline{M}=\overline{m}+5+5\log\underline{\pi}$, whatever the range in m and π.

7. The stars of types K and M have mean velocities about 1.0 or 1.5 km higher than the F and G stars of the same absolute magnitude. This conclusion is based upon a discussion of the geometrical means of the radial and component velocities.

Mount Wilson Solar Observatory
April 1917

COMPARISON OF VISUAL AND PHOTOGRAPHIC OBSERVATIONS OF ECLIPSING VARIABLES

By HENRY NORRIS RUSSELL, MARY FOWLER, and MARTHA C. BORTON

The present paper contains the first results of a program of co-operation between the Harvard and Princeton observatories in the measurement and discussion of the observations of variable stars contained in the plates of the Harvard Photographic Library.

The photographic magnitudes have been estimated at Harvard by comparison with a sequence of stars in each field for which magnitudes were derived by Miss Leavitt by comparison with the Polar Sequence. For the stars W Delphini, S Cancri, and SW Cygni the estimates were made in 1903, as part of the work done under a grant from the Carnegie Institution; but the magnitudes of the comparison stars have been redetermined in 1916 by Miss Leavitt. For the remaining stars the estimates were made in 1915 and 1916 by Miss Leavitt and Miss Locke.

The discussion of the observations has been carried on at the Princeton University Observatory, this part of the work having been made possible by a gift to the Observatory by Mr. Archibald D. Russell. For the stars here discussed the computations have been made and the published accounts prepared by Miss Fowler in the case of W Delphini, U Sagittae, and S Cancri; by Miss Borton for RW Tauri and SW Cygni; and by both together for W Ursae Majoris. The observations upon which these reductions are based will be published in the *Harvard Annals*, together with the final phase for each and residuals (O−C) from the light-curves adopted below.

The concluding section, containing a general discussion of the work so far completed, has been written by Professor H. N. Russell.

1. W DELPHINI ($20^{h}33^{m}.1, +17^{\circ}56'$ [1900])

When Wendell's later observations, given in *Harvard Annals*, **69**, Part II, which have been published since Professor Russell computed the light-curve based on Wendell's earlier observations, were being plotted, suspicion of a changing period was aroused and all Wendell's observations were studied further. They were divided into seven groups, with the result given in Table I. The fourth column gives the correction (O−C) to Wendell's elements of minimum: 2,412,002.5971+4.8061 E. These results were plotted and it was found that a symmetrical curve, whose equation gives the correction $-0^{d}0377+0.000{,}111\,E-0.000{,}000{,}062\,E^2$, could be drawn through them. After this is applied, the outstanding residuals are those given in the column headed O−C′.

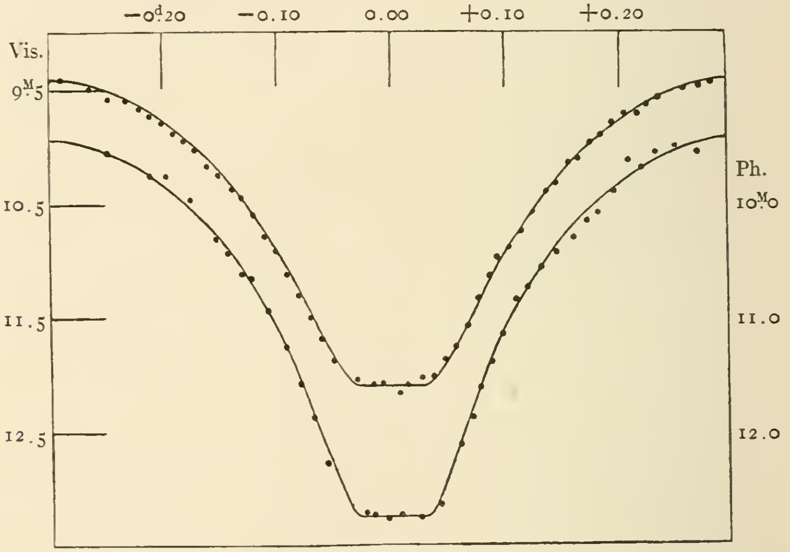
TABLE I

No.	Mean Epoch	No. of Obs.	O–C	O–C′
WENDELL'S OBSERVATIONS				
1.........	+ 322	20	$-0^{d}0125$	$-0^{d}0035$
2.........	+ 381	128	− .004	+ .001
3.........	+ 458	146	.000	.000
4.........	+ 695	15	+ .010	+ .0005
5.........	+ 772	25	+ .010	− .001
6.........	+1222	14	+ .006	.000
7.........	+1374	14	−0.002	0.000
PHOTOGRAPHIC OBSERVATIONS				
1.........	+ 360	113	−0.006	0.000
2.........	+ 482	166	+0.002	0.000
3.........	+ 634	71	+0.009	+0.001
4.........	+ 810	13	+0.012	0.000

Phases for the 401 photographic observations were computed first with Wendell's elements and then corrected to the smooth curve given above. They were then divided into four groups in order of phase, plotted, and the corrections to Wendell's elements obtained, as shown in the lower part of Table I, which fully confirm the correction to Wendell's period during the interval which they cover.

Wendell's elements, thus corrected, which are $2412002.5594+4.806211\,E-0.000{,}000{,}062\,E^2$, are those used in computing the light-curve here discussed. Wendell's elements, as given in *Harvard Annals*, **69**, however, are probably of more value for purposes of prediction, owing to the probability that the effect here appearing as the term in E^2 may ultimately be found to be periodic in nature.

In applying this correction to the visual observations, Wendell's earlier observations on the ascending branch and on the descending branch were treated separately. The mean epoch of 149 observations on the ascending branch is 423.6 and that of 204 observations on the descending branch is 453.3. The corrections for these two epochs are $+0^{d}.0008$ and $+0^{d}.0001$, respectively; but a further allowance must be made for the quadratic term, $+0.000,000,062\ E^2$.



W Delphini

Visual, above; photographic, below

FIG. 1

The mean square difference in epoch between an individual observation and the mean epoch of the group is ±111 for the ascending and ±114 for the descending branch. By applying the value of the quadratic term for these intervals, the complete corrections are found to be $+0^{d}.0016$ and $+0^{d}.0008$, respectively, which fortunately are so nearly equal that it will suffice to apply the mean, $+0^{d}.0012$, to the computed epoch without changing our light-curve or elements. The epoch assumed by Professor Russell in his determination of the light-curve in *Astrophysical Journal*, **36**, 133, 1912, is $0^{d}.0015$ earlier than Wendell's and hence $0^{d}.0027$ earlier than the new elements.

The 390 photographic observations which remained after rejecting eleven which had inadmissably large residuals were combined

TABLE II

NORMAL MAGNITUDES OF W DELPHINI

Phase	Mag.	No. of Obs.	O−C
+0^{d}0062	12.738	5	−0^{M}01
.0235	12.755	2	+ .01
.0417	12.643	10	+ .13
.0597	12.111	10	+ .02
.0702	11.875	11	+ .08
.0776	11.614	11	.00
.0871	11.398	11	+ .01
.0972	11.161	10	+ .02
.1088	10.842	10	− .09
.1183	10.748	9	− .02
.1312	10.560	10	.00
.1433	10.428	10	+ .04
.1583	10.303	9	+ .08
.1707	10.153	7	+ .08
.1807	10.078	9	+ .08
.1953	9.899	10	+ .02
.2067	9.620	9	− .14
.2186	9.680	8	− .03
.2314	9.569	12	− .05
.2495	9.506	12	− .05
+ .2684	9.552	9	+ .08
− .2484	9.544	9	− .02
.2105	9.740	10	.00
.1959	9.752	9	− .07
.1758	9.964	10	− .06
.1534	10.302	11	+ .04
.1428	10.427	12	+ .06
.1313	10.621	11	+ .11
.1225	10.650	11	− .01
.1079	10.945	12	.00
.0925	11.271	11	+ .04
.0799	11.580	12	+ .04
.0689	11.875	11	+ .01
.0506	12.278	12	+ .11
.0243	12.720	6	− .03
.0174	12.738	5	− .01
− .0046	12.774	5	+0.02
0.302	9.46	6	+0.03
0.349	9.31	5	− .12
2.67	9.30	3	− .13
3.32	9.45	6	+ .02
3.44	9.37	5	− .06
3.67	9.35	5	− .08
4.459	9.48	7	+0.05

into thirty-seven normals, which are given in Table II. Constant minimum light determined from twenty-three observations was found to be 12.743, and constant maximum light from thirty-nine observations to be 9.377, or 9.405 if two very discordant observations were rejected. The visual light-curve, given by Professor Russell,[1] was then readjusted to the photographic relation between the relative brightness of the two stars by multiplying the visual loss of intensity from maximum light $(1-l)$ by 1.0417. This curve was then applied to the photographic normals and was found to be too high throughout the whole upper part of the curve. As maximum light was determined from so few observations in comparison with the large number of observations on the upper part of the curve, all of which had positive residuals, it seemed best to take the latter into consideration, and maximum light was therefore assumed to be 9.43, and minimum light 12.75. This shows a range of light of $3^{M}.32$ or a loss of light of 0.9530, while the visual depth is only $2^{M}.70$, the range being from 9.40 to 12.10, or a loss of light of 0.9168 of the whole.

The computed curve being readjusted to this new relation by changing the factor from 1.0417 to 1.0395, the light-curve given in Table III was adopted as satisfactory as regards the range of variation and form of the curve. A least-square solution for the magnitude at maximum and the epoch of minimum was then made,

TABLE III

t	Visual Mag.	Visual Δm	Visual $1-l$	Photo. $1-l$	Photo. Δm	Photo. Mag.
$0^{d}030$	12.10	2.70	0.9168	0.9530	3.32	12.75
.040	12.01	2.61	.9096	.9455	3.16	12.59
.050	11.86	2.46	.8962	.9316	2.91	12.34
.060	11.68	2.28	.8775	.9122	2.64	12.07
.070	11.50	2.10	.8555	.8893	2.39	11.82
.090	11.09	1.69	.7891	.8203	1.86	11.29
.110	10.77	1.37	.7169	.7452	1.48	10.91
.140	10.33	0.93	.5754	.5981	0.99	10.42
.160	10.10	0.70	.4752	.4940	0.74	10.17
.180	9.94	0.54	.3919	.4074	0.57	10.00
.200	9.78	0.38	.2953	.3070	0.40	9.83
.220	9.64	0.24	.1983	.2061	0.25	9.68
.240	9.54	0.14	.1210	.1253	0.15	9.58
.260	9.48	0.08	.0710	.0738	0.08	9.51
0.300	9.40	0.00	0.0000	0.0000	0.00	9.43

In this table Δm denotes the difference in stellar magnitude between the brightness of the star at any given moment and at full light, and $1-l$ the loss of light at this moment, in terms of the whole light of the system as a unit.

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[1] *Astrophysical Journal*, **36**, 134, 1912.

the conditional equations being of the form $x+sy=n$, where s is the slope of curve at the phase of a given normal, n the residual O−C for this normal from the provisional curve, x a correction to the assumed magnitude at maximum, and y a correction to the assumed epoch of mid-eclipse. In the present case, x and n were expressed in hundredths of a magnitude, and y in thousandths of a day, so that $s=1$ corresponded to a rate of variation of a magnitude in $0^{\mathrm{d}}.10$.

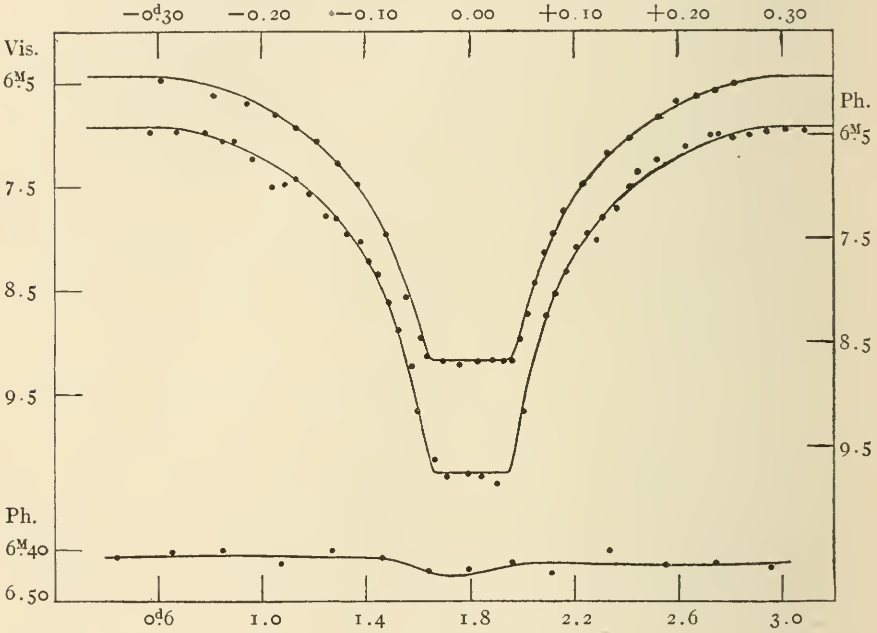
The values derived from thirty-two conditional equations were $x=+0^{\mathrm{M}}.0169\pm0^{\mathrm{M}}.0077$ and $y=-0^{\mathrm{d}}.00016\pm0.00046$, that is, the epoch of minimum derived from the photographs is earlier by 0.23±0.66 minutes than that given by the formula previously adopted.

The probable error of one normal magnitude was $\pm0^{\mathrm{M}}.043$. As the average number of observations combined into a normal was 9.5, this corresponds to a probable error of $\pm0^{\mathrm{M}}.132$ for a single photographic observation.

A least-square solution was not made for the visual observations on account of the uncertainty concerning the nature of the variable term in the expression for the period; but the value found above—according to which the visual minimum comes $0^{\mathrm{d}}.0027$, or 3.9 minutes, earlier than indicated by the formula—must be very near the truth. The photographic minimum therefore comes $3^{\mathrm{M}}.7$ later than the visual—a difference decidedly too great to be explained by any uncertainty in the variable correction aforesaid.

2. U SAGITTAE ($19^{\mathrm{h}}14^{\mathrm{m}}.4, +19°26'$)

Five hundred and eighty photographic observations of U Sagittae were available, twenty of which were eventually rejected because marked uncertain by the observers, and nine on account of large and unexplained discordances. These observations were made twice on the majority of plates, first by Miss Locke and then by Miss Leavitt, at Cambridge. The instruments used were the 1-inch Cooke Anastigmat in Cambridge, the 1-inch Cooke in Arequipa, and the 0.5-inch Ross-Zeiss in Cambridge. The exposures were usually 60 minutes. Some plates with the 1-inch Cooke were taken with several successive exposures, usually of ten minutes each, but sometimes of 30 minutes. Owing to these long exposures, the brightness recorded on the photograph, when the star is varying rapidly, will not be exactly that corresponding to the moment of mid-exposure. The necessary correction can easily be determined from a provisional light-curve, if it is assumed that the photographic action is proportional to the intensity of the star's light, and is simply the difference between the mean of the intensity during the exposure and the intensity at the middle of exposure. This correction is greatest when half the exposure falls in the constant phase at minimum and half in the rapid increase or decrease, and may amount to $0^{\mathrm{M}}21$ for a one-hour exposure or $0^{\mathrm{M}}05$ for a half-hour exposure, but is usually insensible. The long exposures in this case nearly all occur during constant maximum light, so that corrections for long exposure were applied in only 20 cases, and many of these are for half-hour exposures and therefore are small.



U Sagittae

Upper curve, visual; two lower curves, photographic

FIG. 2

Phases were first computed with Graff's[1] elements: minimum = 2,417,130.4133+3.380,604 E. Wendell's 92 early and 36 later observations were reduced with these elements. On plotting the photographic times of minimum, it was quite evident that the period needed revision, and, taking into consideration Wendell's, Graff's, Maggini's,[2] and the Harvard photographic observations, we find that the best period seemed to be $3^{d}.380621$, with no definite indication of a variable period.

Forty-five normals were then formed from the 202 observations during eclipse, with phases corrected to this new period and magnitudes corrected for length of exposure. Wendell's visual observations were also collected into 32 normal groups. Tables IV and V give these normals, with a correction of $+0^{d}.007$ already applied to the photographic phases, and $+0^{d}.010$ to the visual. This correction is explained below. Intensity-curves were drawn, and the

TABLE IV

NORMAL MAGNITUDES OF U SAGITTAE—HARVARD PHOTOGRAPHS

Phase	Mag.	No. of Obs.	O−C	Phase	Mag.	No. of Obs.	O−C
$+0^{d}.0109$..	9.793	7	$+0^{M}.04$	$-0^{d}.2830$..	6.470	2	$+0^{M}.01$
.0190..	9.860	3	+ .11	.2550..	6.480	3	− .04
.0510..	9.158	4	.00	.2378..	6.563	3	.00
.0728..	8.248	5	− .05	.2270..	6.562	5	− .04
.0817..	8.033	3	.00	.2087..	6.730	3	+ .10
.0927..	7.809	7	.00	.1900..	7.000	2	+ .20
.1020..	7.590	4	− .08	.1777..	6.980	3	+ .11
.1128..	7.460	5	− .06	.1677..	6.928	4	− .02
.1215..	7.530	4	+ .13	.1540..	7.060	5	+ .04
.1312..	7.290	4	.00	.1385..	7.268	6	+ .08
.1407..	7.200	3	+ .05	.1277..	7.300	9	− .01
.1535..	7.005	2	− .05	.1170..	7.467	6	.00
.1643..	6.847	3	− .10	.1054..	7.526	5	− .09
.1795..	6.740	2	− .10	.0960..	7.712	6	− .03
.2067..	6.617	3	− .06	.0883..	7.839	10	− .05
.2320..	6.500	3	− .08	.0781..	8.108	8	− .01
.2390..	6.500	2	− .05	.0690..	8.380	11	− .04
.2518..	6.512	4	.00	.0566..	8.748	5	− .12
.2698..	6.500	4	+ .04	.0502..	9.160	6	− .01
.2805..	6.470	4	.00	.0348..	9.610	5	− .14
.3137..	6.447	3	+ .03	.0230..	9.792	4	+ .04
+ .3212..	6.485	4	− .06	−0.0017..	9.756	7	+0.01
−0.3087..	6.480	3	+0.06				

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[1] *Mitteilungen der Hamburger Sternwarte*, Nr. 11 (1907).

[2] *Astronomische Nachrichten*, **200**, 54, 1914.

best possible adjustment was made between the visual and photographic curves.

The visual duration of total eclipse appeared to be a little longer than the photographic, but as this seems to be physically incredible, a compromise was made, such geometric elements of the eclipse being used that the two light-curves were represented as well as could be done with elements identical in the two cases (except for the assumed brightness of the components). This solution assumes the stars to be uniformly luminous disks, and the eclipse central. The solution was made in the usual manner, Table IV being used (*Astrophysical Journal*, **36**, 252, 1912). From this solution we find the elements given in Table XVIII (p. 340).

TABLE V

NORMAL MAGNITUDES OF U SAGITTAE—WENDELL

Phase	Mag.	No. of Obs.	O−C	Phase	Mag.	No. of Obs.	O−C
$+0^{\rm d}.0059$..	9.192	5	$+0^{\rm M}.01$	$+0^{\rm d}.2353$..	6.573	3	$+0^{\rm M}.03$
.0208..	9.173	6	− .01	.2548..	6.500	4	.00
.0330..	9.180	3	.00	− .2975..	6.480	2	+ .05
.0400..	9.180	2	+ .04	.2463..	6.610	3	+ .09
.0488..	8.970	4	+ .03	.2147..	6.697	3	+ .09
.0550..	8.730	3	.00	.1863..	6.803	3	+ .04
.0617..	8.433	3	− .05	.1670..	6.920	4	.00
.0710..	8.140	2	− .02	.1460..	7.057	3	− .03
.0800..	7.953	3	.00	.1270..	7.270	4	− .01
.0902..	7.733	3	− .02	.1075..	7.480	2	− .01
.1095..	7.487	4	+ .01	.0800..	7.958	4	+ .01
.1333..	7.168	3	− .05	.0605..	8.560	5	+ .05
.1533..	7.040	3	+ .01	.0480..	8.958	4	+ .03
.1798..	6.825	4	.00	.0410..	9.140	4	+ .01
.1987..	6.677	3	− .03	.0255..	9.190	4	+ .01
+0.2182..	6.620	4	+0.02	−0.0108..	9.212	4	+0.03

The difference between the visual and photographic relative brightness of the two components was adjusted as for W Delphini; in this case the visual $(1-l)$ was multiplied by the constant factor, 1.0356. The visual range of light is from maximum 6.43 to minimum 9.18, giving a depth of $2^{\rm M}.75$ or a loss of light of 0.9206. The photographic range is from maximum 6.42 to minimum 9.75, giving a depth of $3^{\rm M}.33$ or a loss of light of 0.9534.

The light-curve thus derived was applied both to the visual and to the photographic normal observations. It was found necessary to add $0^{d}.010$ to the visual phases, or, in other words, the epoch of minimum assumed was $0^{d}.010$ too late. In the same way the assumed epoch of minimum for the photographic observations was found to be $0^{d}.007$ too late. Least-square solutions were made to find more exactly the correction as described in the case of W Delphini. For the visual curve $x = +0^{M}.0111 \pm 0^{M}.0047$ and $y = -0^{d}.0006 \pm 0^{d}.00026$; that is, the minimum comes $0^{d}.0006$ earlier than the epoch, as already corrected. The solution of the photographic observations gives $x = +0^{M}.002 \pm 0^{M}.008$ and $y = +0^{d}.00044 \pm 0^{d}.00040$, or the photographic minimum is $0^{d}.00044$ later than the adopted time. Combining the two corrections each, for visual and photographic, we find that the visual minimum comes $0^{d}.0106$ early and the photographic $0^{d}.0066$ early, hence the photographic minimum comes $0^{d}.0040 \pm 0^{d}.00047$, or 5.8 ± 0.7 minutes later than the visual minimum. The probable error of one visual observation, as deduced from this solution, was $\pm 0^{M}.024$, and that of a photographic normal of unit weight depending on the average of 3.4 observations was $\pm 0^{M}.044$, corresponding to $\pm 0^{M}.091$ for a single plate. There seems to be no doubt that the difference, given above, between the visual and photographic minima, whatever its origin, is real.

The 349 observations falling outside the limits of the principal eclipse were collected into 14 normals, as shown in Table VI. These indicate that the light between eclipses is very nearly constant. The effects of ellipsoidal form of the stars and of the illumination of the side of the secondary which faces the primary are quite

TABLE VI

Photographic Magnitudes of U Sagittae, Outside Minimum

Obs.	Phase	Mag.	Comp.	O−C	Obs.	Phase	Mag.	Comp.	O−C
26.....	$0^{d}.440$	6.416	6.413	$+0^{M}.003$	22....	$1^{d}.790$	6.439	6.451	$-0^{M}.013$
27.....	0.647	6.405	6.411	− .006	23....	1.956	6.426	6.432	− .006
32.....	0.843	6.403	6.410	− .007	26....	2.114	6.448	6.427	+ .021
22.....	1.074	6.428	6.411	+ .017	17....	2.336	6.402	6.429	− .027
27.....	1.265	6.403	6.413	− .010	33....	2.548	6.430	6.430	.000
20.....	1.460	6.418	6.416	+ .002	26....	2.740	6.426	6.429	− .002
18.....	1.635	6.441	6.442	−0.001	30....	2.953	6.435	6.423	+0.008

insensible—which is not surprising, as the components are relatively widely separated.

The observations during the secondary minimum, which, according to Miss Fowler's spectrographic elements[1] should come at phase $1\overset{d}{.}744$, lie perceptibly below the mean of the others and are in entire accord with the theoretical depth of $0\overset{M}{.}031$ for uniform star-disks.

There appears, however, to be definite evidence that the star is brighter in the interval between the primary and secondary minima than in the other half of the period, as was found by Dugan[2] from his visual observations of RV Ophiuchi. The explanation suggested by Dugan—that the principal component is a little brighter on the preceding than on the following face—appears to be necessary here also, since the periastron, as determined from the spectrographic observations, falls in the half of the period when the star is fainter than in the other.

The expression for the magnitude which has finally been adopted is $6\overset{M}{.}420-0\overset{M}{.}010 \sin \theta$ plus the effects of eclipse, θ being the longitude in the orbit, measured from the principal eclipse. The computed magnitudes and residuals are given in Table VI. The probable error of one normal place comes out $\pm 0\overset{M}{.}0086$, which, since the average number of observations combined into a normal is 25, corresponds to a probable error of $\pm 0\overset{M}{.}043$ for a single photograph. This remarkably small value is confirmed by a study of the residuals of the individual observations, as will be shown later.

3. S CANCRI ($8^{h}38\overset{m}{.}2$, $+19^{\circ}23\overset{\prime}{.}6$)

S Cancri was handled in much the same way as the stars just discussed. Phases for the 365 observations, of which 20 were afterward rejected for large and unexplained discordances, were computed by the light-elements given by Shapley in *Contributions from Princeton University Observatory*, No. 3. These are 2403210.6053 +9.4845485 E, which are the same as given by Wendell,[3] except that the epoch of minimum here adopted is $0\overset{d}{.}02$ later. This period was adopted as definitive, and, as the exposures were all short, no correction to the magnitudes was necessary.

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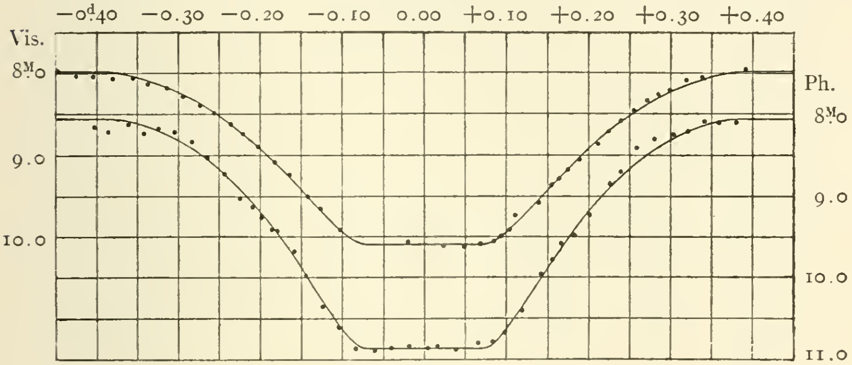
[1] *Publications of the Allegheny Observatory*, **3**, 11, 1912.

[2] *Astrophysical Journal*, **43**, 142, 1916.

[3] *Harvard Annals*, **69**.

Maximum constant light from 35 observations is 8.06 and constant minimum light from 37 observations, 10.86, giving a range of $2^{\mathrm{M}}80$, or a loss of light of 0.924 of the maximum intensity. In the visual curve as found by Shapley we have a range from 7.98 to 10.10, or $2^{\mathrm{M}}12$, meaning a loss of light of 0.858.

The observations during eclipse were grouped into 43 normals and are given in Table VII. It was found that Shapley's curve, if merely altered to correspond to the photographic difference of magnitude of the two stars, did not satisfactorily represent the observations, but that a good agreement could be obtained by supposing that, while the disks appeared uniformly bright visually, they were darkened toward the limb for the photographically active wavelengths, so that the limb was half as bright as the center. The transformation of the curve was made with great ease and rapidity by means of Table VI, *Astrophysical Journal*, 36, 403, 1912, in which values of $X(k, \alpha)$ are given, and the correction applied to the loss of light is, in this case, $\frac{1}{2}(1-l)X(k, \alpha)$. In the present case k is 0.370. The light-curve, thus altered, proved a perfectly satisfactory fit to the photographic observations, so no new solution was necessary. It was necessary, however, to make the time of



S Cancri

Visual, above; photographic, below

FIG. 3

minimum for the photograph $0\overset{\mathrm{d}}{.}010$ later. A least-square solution, similar to those made for W Delphini and U Sagittae, was made with the following result: $x=-0\overset{\mathrm{M}}{.}012\pm0\overset{\mathrm{M}}{.}0008$, $y=-0\overset{\mathrm{d}}{.}0015\pm0\overset{\mathrm{d}}{.}0008$. The two corrections being combined, the photographic minimum was found to be $0\overset{\mathrm{d}}{.}0085\pm0\overset{\mathrm{d}}{.}0008$, i.e., 12.2 minutes later than the assumed minimum. A similar solution was also made for the visual curve as given by Shapley, with the result: $x=-0\overset{\mathrm{M}}{.}002\pm0\overset{\mathrm{M}}{.}003$, and $y=+0\overset{\mathrm{d}}{.}00004\pm0\overset{\mathrm{d}}{.}00041$, meaning that the visual minimum comes later than the assumed minimum by 0.1 ± 0.6 minutes. The photographic minimum therefore comes 12.1 ± 1.3 minutes later than the visual. The probable error of one visual observation of unit weight is $\pm0\overset{\mathrm{M}}{.}021$; that of a photographic normal, depending, on the average, on 8.1 observations, is $\pm0\overset{\mathrm{M}}{.}045$, and the corresponding probable error for a single photograph is $\pm0\overset{\mathrm{M}}{.}128$.

TABLE VII

NORMAL MAGNITUDES OF S CANCRI—HARVARD PHOTOGRAPHS

Phase	Mag.	No. of Obs.	O−C	Phase	Mag.	No. of Obs.	O−C
$+0\overset{\mathrm{d}}{.}0036$..	10.87	5	$+0\overset{\mathrm{M}}{.}01$	$-0\overset{\mathrm{d}}{.}4031$..	8.16	5	$+0\overset{\mathrm{M}}{.}10$
.0163..	10.84	3	− .02	.3862..	8.21	6	+ .15
.0382..	10.89	6	+ .03	.3620..	8.12	4	+ .03
.0654..	10.81	5	− .05	.3418..	8.24	6	+ .09
.0837..	10.80	4	− .03	.3245..	8.17	8	− .02
.0979..	10.68	8	+ .02	.3049..	8.22	9	− .05
.1194..	10.41	7	+ .06	.2844..	8.33	8	− .10
.1409..	9.97	7	− .05	.2666..	8.53	7	− .03
.1550..	9.79	3	− .01	.2431..	8.73	5	.00
.1658..	9.58	8	− .02	.2250..	9.03	10	+ .12
.1825..	9.49	8	+ .08	.2093..	9.13	8	+ .04
.2021..	9.23	9	+ .05	.1996..	9.26	7	+ .08
.2256..	8.86	5	− .04	.1867..	9.41	8	+ .05
.2397..	8.71	6	− .08	.1781..	9.42	11	− .03
.2597..	8.42	11	− .18	.1588..	9.68	9	− .06
.2808..	8.31	12	− .12	.1450..	9.98	6	+ .03
.3036..	8.25	10	− .05	.1242..	10.37	3	+ .09
.3216..	8.21	13	.00	.1014..	10.62	7	+ .01
.3415..	8.09	13	− .05	.0841..	10.87	5	+ .06
.3593..	8.11	11	.00	.0612..	10.90	5	+ .04
+0.3803..	8.11	7	+0.05	.0402..	10.86	6	.00
				−0.0179..	10.84	7	−0.02
0.419...	8.06	6	0.00	4.52....	8.05	5	−0.01
0.447...	8.12	6	+0.06	4.58...	8.06	5	0.00
3.87....	8.15	2	+0.09	9.061...	8.04	5	−0.02
4.44....	8.03	6	−0.03				

4. W URSAE MAJORIS ($9^{h}36^{m}.7$, $+56°25'$)

Miss Leavitt remarks: "On account of the brightness of this variable and the distance of the comparison stars used, the only plates on which it can usually be well observed are those taken with the 0.5-inch Ross-Zeiss and Voigtländer lenses." As exposures with the 0.5-inch telescopes are usually an hour in length, as in the case of U Sagittae, the recorded magnitudes do not represent the true brightness of the variable at the middle of the exposure. The mean length of exposure of all the observations was found to be 65.27 minutes. The visual light-curve given in *Astrophysical Journal*, **36**, 141, 1912, was smoothed out to allow for this mean exposure by interpolating the magnitudes to 5-minute intervals and taking means of these over intervals of 65 minutes, including as middle the specified time. It was then found that the photographic observations indicated a slightly greater range than this modified visual curve, the difference being $0^{M}.064$. The changes in the correction for the length of exposure on account of this fact amounted at most to only $0^{M}.004$, but have been applied. The corrections given in Table VIII were thus determined and applied eventually to the light-curve.

TABLE VIII

Phase	Correction	Phase	Correction
$0^{d}.0834$......	$-0^{M}.02$	$0^{d}.0313$......	$-0^{M}.04$
.0735......	$-$ .02	.0208......	$-$ .03
.0625......	$-$ .02	.0104......	$+$ .07
.0520......	$-$ .01	0.0000......	$+$ .13
0.0417......	$-$ .01		

Phases for the 367 observations were first computed with Baldwin's elements,[1] which are 2,416,129.1926+0.33364044 E. All the observations were divided into four groups, and times of minima were determined for each separately. As there seemed to be a systematic divergence from the assumed period, a careful investigation of all available observations was made, the results of which appear in Table IX. These corrections to Baldwin's time of minimum were found, when plotted (with the exception of all of Müller and Kempf's observations), to lie nearly within their probable errors upon a straight line whose equation gives the correction of $+0^d.0092-0^d.00000142$ E to Baldwin's elements as adopted in computing preliminary phases. This correction gives as final elements 2,416,129.2018+0.33363902 E. Müller and Kempf's observations were found to lie on a line parallel to the line going through all the other observations, but at a distance of about $-0^d.006$. The epochs of minimum assigned to Müller and Kempf have all been corrected by $+2^m$ to allow for the difference between the form of the light-curves assumed by these observers and in this paper (see *Astrophysical Journal*, **36**, 140, 1912). No explanation has been found for this remarkable systematic difference between their results and those of other observers, but it is evident that their observations, taken alone, are satisfied by the finally assumed period, though the actual epoch would have to be taken about 8 minutes earlier.

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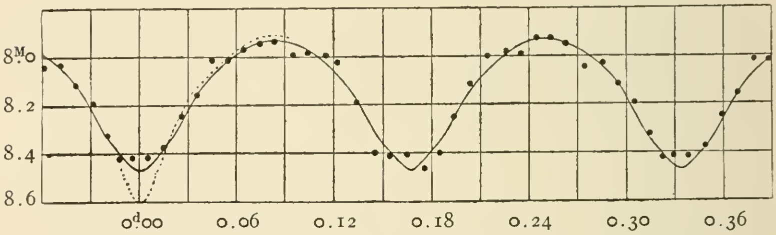
[1] *Monthly Notices*, **69**, 86, 1908—doubling the period there given, as there are two minima in each orbital revolution.



W Ursae Majoris—Photographic

FIG. 4

Phases were then corrected to this period, and normals were formed for each of the four groups separately. The observed normals were plotted, corrected for the influence of the length of exposure, and the resulting true photographic light-curve compared with the visual curve.

It was found that the portion of the variation due to eclipse was sensibly identical in the two cases, but that the variation outside eclipse, arising from the ellipticity of the stars, was greater photographically than visually. This can be accounted for by assuming that the star-disks appear to be darkened toward the limb to a greater degree in the former case than in the latter; for, if an ellipsoidal star whose eccentricity of the meridian section is ϵ revolves so that its equatorial plane is inclined to an angle ι, the variations of light due to its differing presentation are given by the equation

TABLE IX

MINIMA OF W URSAE MAJORIS

Authority	Method	Mean Epoch	Correction to Baldwin	O−C
Harvard.............	Photographic.........	560	+0.0079±0.0015	−0ᵈ0005
Parkhurst and Jordan*	Photographic.........	3,200	+ .004	− .001
Harvard.............	Photographic.........	3,690	+ .0033±0.0012	− .0007
Tikhoff†............	Photographic (orange).	4,500	+ .002	− .001
Tikhoff.............	Photographic (ultra-violet)............	4,500	+ .003	.000
Baldwin‡............	Photometric..........	5,475	+ .0020± .0004	+ .0004
Horn§...............	Photographic.........	7,900	− .004	− .003
Harvard.............	Photographic.........	8,000	− .0012± .0008	+ .0010
Lazzarino‖	Photometric..........	9,286	− .003	.000
Harvard.............	Photographic.........	12,300	− .0086± .0011	− .0003
Müller and Kempf¶..	Photometric..........	0	+ .0034± .0007	− .0058
Müller and Kempf....	Photometric..........	111	+ .004	− .005
Müller and Kempf....	Photometric..........	300	+ .002	− .007
Müller and Kempf**..	Photometric..........	1,184	+ .001	− .006
Müller and Kempf....	Photometric..........	2,200	− .000	− .006
Müller and Kempf....	Photomertic..........	3,864	−0.003	−0.007

* *Astrophysical Journal*, **23**, 82, 1906.

† *Pulkova Mittheilungen*, **2**, 21, 1908.

‡ *Monthly Notices*, **69**, 78–90, 1908.

§ *Rivista di Astronomia*, July, 1910.

‖ *Astronomische Nachrichten*, **190**, 95, 1901.

¶ *Astrophysical Journal*, **17**, 231, 1903.

** *Monthly Notices*, **69**, 86, 1908.

$$L^2 = L_0^2(1 - \epsilon^2 \sin^2 \iota \ \cos^2 \theta)$$

if it appears as a uniform disk, but by

$$L - L_0[1 - (\tfrac{4}{5}\epsilon^2 + \tfrac{16}{175}\epsilon^4) \sin^2 \iota \ \cos^2 \theta]$$

if the disk appears darkened toward the limb, and the range is greater in the second case (see *Astrophysical Journal*, **36**, 400, 1912).

In the present case it was found that the photographic light-curve outside eclipse could be excellently represented by the geometric elements derived from the visual observations, if the computed light was taken halfway between the values obtained on the assumption of uniform disks and complete darkening—that is, if photographically the disks appear 50 per cent as bright at the limb as at the center. The changes in the variation due to eclipse resulting from this hypothesis were computed by Table VI, *Astrophysical Journal*, **36**, 142, 1912, as described in the case of S Cancri—the correction nowhere exceeding $0^{\mathrm{M}}.011$.[1]

Solutions were made from each of the four groups of photographic observations, for magnitudes at maximum, and the correction to Baldwin's elements for the time of minimum, using the mean light-curve. The resulting times of minima are given in Table IX. The corrections to the assumed magnitude at maximum (7.912) came out -0.030 ± 0.010, $+0.012 \pm 0.004$, $+0.005 \pm 0.005$, and $+0.020 \pm 0.007$, respectively, for the four series, while the probable errors of a single observation came out $\pm 0^{\mathrm{M}}.082$, $\pm 0^{\mathrm{M}}.090$, $\pm 0^{\mathrm{M}}.053$, and $\pm 0^{\mathrm{M}}.068$. The small residuals found for the magnitude at maximum indicate that the star is not subject to sensible secular variations of brightness, and that the gradation and color-sensitiveness of the plates have been remarkably uniform from year to year.

In forming the final normals two observations with residuals exceeding $0^{\mathrm{M}}.40$ were rejected, and the remaining 364 combined into 34 groups. The resulting normal magnitudes and phases are given in Table X, together with the number of observations combined into each normal, and the residuals from the final curve, which is derived from the theoretical curve for eclipses of stars partially darkened toward the limb, described above, by applying the corrections for the length of exposure given in Table VIII, with reversed sign, and hence represents the variation as it should be observed for exposure of 65 minutes' duration.

If these residuals are given weights proportional to the number of observations, the probable error of a single observation is found to be $\pm 0^{\mathrm{M}}.066$, from the first powers of the residuals, or $\pm 0^{\mathrm{M}}.070$ from their squares—a very satisfactory showing.

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[1] What the difference in the visual and photographic curves indicates is really that the *degree* of darkening toward the limb is greater in the latter case by 50 per cent of its maximum possible amount. Light-curves sensibly identical with those here given could be obtained, with slightly different geometrical elements, by the assumption that the loss of light at the limb was x per cent visually and $(x+50)$ per cent photographically, where x may have any value from 0 to 50.

TABLE X

W URSAE MAJORIS—FINAL NORMALS

Phase	Obs.	Mag.	O−C
$0^{\mathrm{d}}.0049$	15	8.418	$-0^{\mathrm{M}}.037$
.0148	17	8.376	− .002
.0253	11	8.246	− .032
.0352	10	8.157	.000
.0448	15	8.017	− .057
.0543	15	8.015	.000
.0638	11	7.968	− .001
.0744	13	7.942	+ .003
.0832	11	7.936	+ .004
.0948	14	7.993	+ .048
.1039	17	7.985	+ .011
.1150	9	7.997	− .033
.1222	6	8.022	− .056
.1338	5	8.190	+ .012
.1446	7	8.400	+ .088
.1532	5	8.414	+ .031
.1647	6	8.408	− .057
.1753	7	8.464	+ .033
.1844	5	8.400	+ .050
.1935	2	8.250	.000
.2038	6	8.117	− .024
.2137	3	8.000	− .058
.2254	5	7.980	− .014
.2351	17	7.993	+ .041
.2443	20	7.925	− .013
.2527	15	7.923	− .009
.2624	7	7.950	+ .005
.2743	7	8.043	+ .053
.2851	16	8.031	− .019
.2945	21	8.117	.000
.3048	13	8.192	− .040
.3141	11	8.323	− .014
.3214	10	8.420	+ .021
0.3289	12	8.417	−0.041

5. RW TAURI ($3^{\mathrm{h}}57^{\mathrm{m}}45^{\mathrm{s}}$, +27°51′[1900])

Material for the light-curve was obtained from the visual observations of Wendell,[1] and of Shapley,[2] and from the Harvard photographic observations, as previously described. Wendell's observations gave a well-defined curve; and, as Shapley did not have these at his disposal when determining his elements, a new light-curve and corresponding set of elements were computed by combining Shapley's visual observations published in the volume just mentioned—taking the magnitude of his comparison star as 8.88, Wendell's observations—using Shapley's period, slightly lengthened to $2^d.768,903$ and his epoch 2,417,198.4094, which is $0^d.129$ earlier than that of Wendell.

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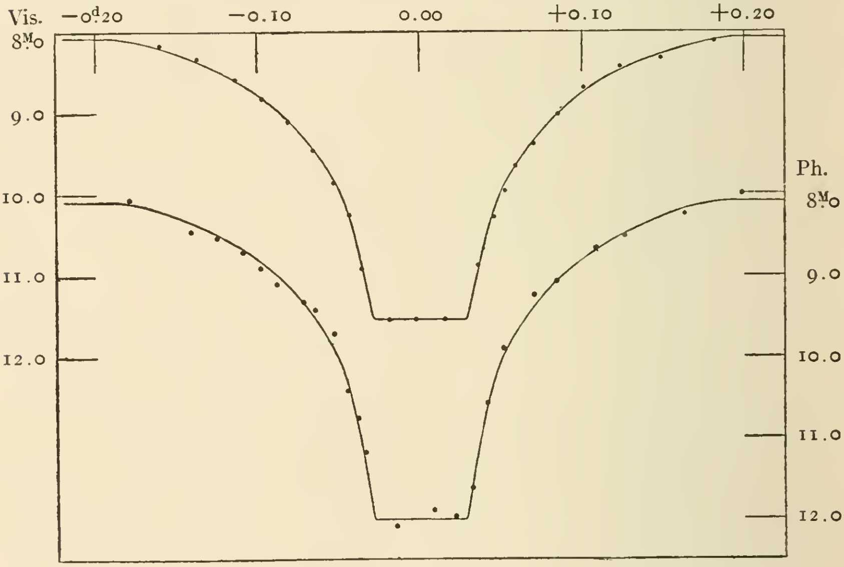
[1] *Harvard Annals*, **69**, 147, 1914.

[2] *Contributions from the Princeton Observatory*, No. 3, p. 136, 1915.

Two of Wendell's observations were rejected, as it was evident some error had been made. From 38 observations by Wendell and 7 by Shapley the maximum was determined as 8.07; and from 14 by Wendell and 6 by Shapley the constant light at minimum was found to be 11.54, making the range $3^M.47$. In solving for the elements on the assumption of uniformly luminous star-disks it was found that the assumption of a central total eclipse led to the elements given in Table XVIII, and an excellent representation of the observations, as shown in Table XI.



RW Tauri

Visual, above; photographic, below

FIG. 5

The period derived from the visual observations represents the photographic data from 1899 to 1915 in a very satisfactory manner;

but a few observations in earlier years show large discrepancies. The observed minima came earlier than those computed by $0^{\mathrm{d}}.07$ in 1891 (J.D. 1782), and by about $0^{\mathrm{d}}.10$ in 1888 (J.D. 0954) and $0^{\mathrm{d}}.14$ in 1886 (J.D. 9913), the last two photographs being taken during totality, so that the exact correction is uncertain.

TABLE XI

RW TAURI—VISUAL NORMALS

Phase	Mag.	Obs.	O−C	Phase	Mag.	Obs.	O−C
$-0^{\mathrm{d}}.1601$..	8.162	8	$-0^{\mathrm{M}}.038$	$+0^{\mathrm{d}}.0152$..	11.538	6	$-0^{\mathrm{M}}.002$
.1370..	8.334	7	− .026	.0354..	10.864	5	− .066
.1131..	8.589	7	− .001	.0452..	10.290	5	+ .110
.0971..	8.811	7	+ .011	.0523..	9.971	6	+ .106
.0811..	9.100	7	+ .010	.0584..	9.657	7	.000
.0666..	9.456	7	− .014	.0700..	9.380	8	+ .040
.0530..	9.862	6	+ .022	.0850..	9.013	7	+ .013
.0434..	10.267	6	− .048	.1012..	8.682	5	− .058
.0362..	10.912	5	+ .012	.1233..	8.433	7	− .042
.0194..	11.541	8	+ .001	.1488..	8.330	6	+ .055
0.0023..	11.540	6	0.000	0.1818..	8.110	5	+0.010

These observations can be represented by the assumption that the period previous to 1895 was about 10 seconds shorter than subsequent to that year; but the data are too scanty to determine the true nature of the change in period, although its reality seems to be unquestionable. These early observations were omitted from the further discussion.

The remaining observations were plotted, and it was found that many of those made while the star was changing rapidly gave values of its light which were obviously half a magnitude or more too bright. This was soon recognized as the effect of the length of exposure. The star at certain times varied by two or even three magnitudes during the longer exposures, and the corrections on this account are of unusual magnitude.

It being assumed, as usual, that the photographic action was proportional to the intensity of the light, it was found that, outside the constant phase of minimum, the curve giving the intensity as a function of the time was nearly linear, so that the intensity at mid-exposure was substantially equal to the mean value during the whole time of exposure, and no correction was necessary. But when part of the exposure fell during the constant phase, the results are very different. From a provisional curve based on the plates with short exposure, the intensity at $9^{M}.50$ being taken as 1000, it was found that the actual intensity was 100 from phase $0^{d}.000$ to $0^{d}.028$ and beyond this point was represented closely enough, though not exactly, by the formula

$$I=100+25.6(t-31) \qquad (1)$$

where t is measured in thousandths of a day from mid-eclipse.

Hence, for an exposure extending from t_1 to t_2, where t_1 is less and t_2 greater than 31, the mean intensity is easily found to be

$$M=100+\frac{12.8(t_2-31)}{t_2-t_1} \qquad (2)$$

For observations on the descending branch t_1+31 takes the place of t_2-31. If I_0 is the value of I at the middle of the exposure, as given by formula (1), I_0 being 100 if $\frac{1}{2}(t_2+t_1)$ is numerically less than 31, then the difference in stellar magnitude corresponding to the ratio M/I_0 is evidently the correction desired. This should always be *added* to the observed magnitude to obtain the magnitude which would have been found from a short exposure.

The computation of these corrections is summarized in Table XII, which needs no further explanation except to say that the first column contains the serial number of the observation concerned in the detailed list (to be published in the *Harvard Annals*) to facilitate its identification there.

The observed magnitudes, so corrected, were combined into normals; 7 observations which made the star more than half a magnitude brighter or fainter than was indicated by the mean curve were rejected.

It was found that a very satisfactory representation of the photographic observations could be obtained by using the geometrical elements derived from the visual solution, but by taking the photographic magnitudes as 8.09 at maximum (derived from 31 observations outside the eclipse) and 12.00 at minimum (from 12 observations) and displacing the curve in time so that the middle of eclipse came $0^{d}.0025$ earlier photographically than visually.

These normals and the residuals (O−C) from the curve are given in Table XIII. The representation is quite as satisfactory

TABLE XII

No.	Phase $\frac{1}{2}(t_1+t_2)$	t_1 or t_2	M	I_0	Correction	Obs. Mag.	Corr. Mag.
8........	−0.029	−52	223	100	$0\overset{\mathrm{M}}{.}87$	10.45	11.32
16........	+ .046	+95	642	484	0.31	10.58	10.89
19........	− .018	−40	124	100	0.23	11.65	11.88
30........	+ .032	+69	350	126	1.11	10.55	11.66
37........	− .034	−49	238	177	0.32	10.95	11.27
48........	+ .010	+33	101	100	0.01	11.85	11.86
50........	+ .014	+47	150	100	0.44	11.55	11.99
51........	− .020	−43	140	100	0.37	10.95	11.32
53........	+ .029	+57	255	100	1.04	11.57	12.61
62........	+ .028	+52	218	100	0.85	10.90	11.75
64........	− .038	−59	339	280	0.20	10.40	10.60
70........	+ .043	+71	466	408	0.14	10.11	10.25
71........	− .050	−72	590	587	0.01	9.95	9.96
121........	+ .024	+45	160	100	0.51	11.65	12.16
125........	+ .041	+62	393	356	0.11	10.50	10.61
128........	− .051	−73	613	612	0.00	9.75	9.75
136........	+ .042	+61	404	382	0.06	10.45	10.51
106........	−0.035	−42	209	202	0.04	11.05	11.09

TABLE XIII

RW TAURI—PHOTOGRAPHIC NORMALS

Phase	Obs.	Mag.	O−C	Phase	Obs.	Mag.	O−C
$-0\overset{\mathrm{d}}{.}179$...	4	8.07	−0.05	$-0\overset{\mathrm{d}}{.}015$...	3	12.09	+0.09
.141...	4	8.46	+ .10	+ .008...	3	11.89	− .11
.125...	4	8.54	+ .04	.022...	3	11.97	− .03
.109...	5	8.71	+ .03	.032...	2	11.61	+ .02
.096...	6	8.90	+ .04	.041...	4	10.56	− .03
.088...	6	9.10	+ .10	.051...	2	9.90	− .10
.072...	5	9.32	− .01	.070...	3	9.24	− .15
.065...	3	9.42	− .11	.089...	4	8.97	− .01
.053...	6	9.71	− .21	.108...	5	8.67	− .02
.045...	2	10.42	+ .12	.126...	3	8.52	+ .03
.038...	2	10.75	− .03	.163...	3	8.25	+ .05
−0.034...	2	11.18	−0.12	+0.199...	1	8.00	−0.09
0.56....	6	8.08	−0.01	1.80....	5	8.01	−0.08
1.05....	5	8.09	.00	2.18....	5	8.15	+ .06
1.36....	4	8.04	−0.05	2.46....	6	8.13	+0.04

as could possibly be expected under the circumstances, the probable error of a single observation, as derived from the weighted sum of the square of the residuals, being $0\overset{\mathrm{M}}{.}111$. The 31 observations outside eclipse yield a smaller probable error, $\pm 0^{\mathrm{M}}.076$, so that the "brute mean" probable error for all the 116 observations of this star is $\pm 0^{\mathrm{M}}.099$.

In addition to these plates, 13 others showed no image of the star, proving it to be fainter than specified limiting magnitudes. In all cases the computed magnitude was well below the limit.

The photographic range of this star, $3^{\mathrm{M}}.91$, is the greatest range so far observed in any eclipsing variable.

6. SW CYGNI ($20^{h}03^{m}50^{s}$, $+46^{\circ}0'.5$ [1900])

The material for the study of this star consists of 192 visual observations by Wendell,[1] and of 286 estimates from Harvard photographs.

The elements derived from the visual observations were 2,411,343.630+4.572908 E, the period being the same as that adopted by Graff and Shapley,[2] but the epoch $0^{d}.029$ later than Shapley's.

The visual observations (with the exception of five which showed large discordances) were plotted with Shapley's theoretical "uniform" curve, which proved to be an excellent fit, as is shown by the residuals given in Table XIV. The maximum is 9.06 and minimum 11.72. A least-square solution gave a correction to the assumed epoch of minimum of $-0^{d}.0040\pm 0^{d}.0004$, which has been applied to the phases given in Table XIV.

The photographic material consists of 220 observations down to magnitude 12.5, from estimates made in 1903, reduced with magnitudes of the comparison stars furnished by Miss Leavitt in 1916, and of 66 estimates of the fainter magnitudes made by Miss Leavitt in 1916. Only two observations of the former group had to be rejected for large discordances, exceeding one magnitude. The remainder were combined into 37 normals, as indicated in Table XV. The residuals there given are derived from a curve computed from the same geometrical elements as Shapley's visual curve, but with the magnitude at maximum 9.29 and at minimum 12.85. The latter value was well determined by 79 observations during the total eclipse. As there were only six observations outside eclipse, the magnitude at this phase was determined mainly from the observations on the upper parts of the curve, where the eclipse is partial and small.

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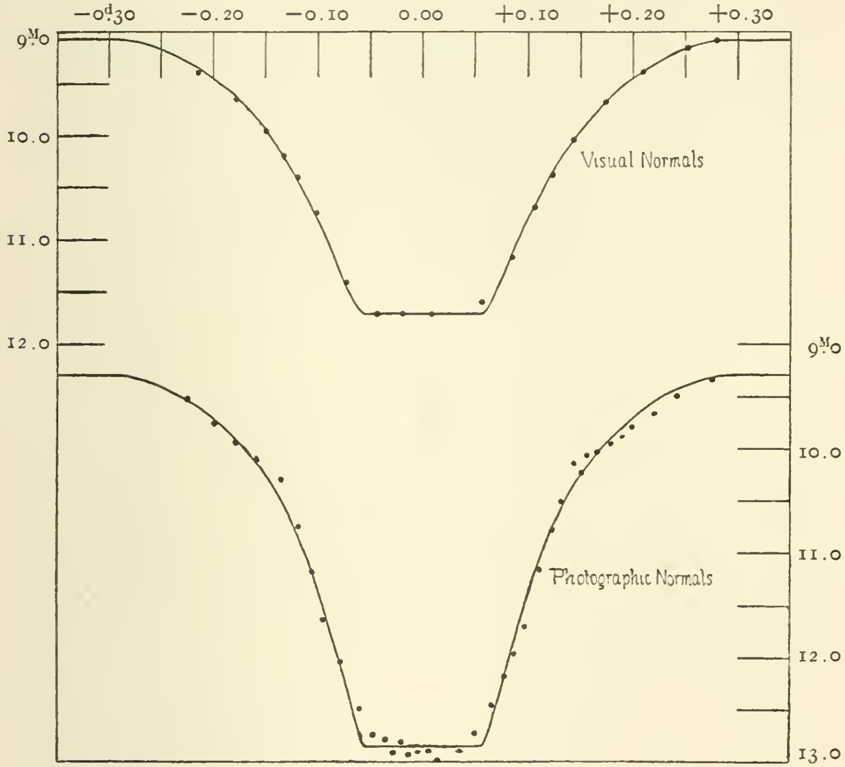
[1] *Harvard Annals*, **69**, 83, 161, 1892–1912.

[2] *Astrophysical Journal*, **36**, 269, 1912.



SW Cygni

Fig. 6

The representation of the observations by the curve is less satisfactory than for the other stars, the residuals being large and positive about $0^{d}.10$ on each side of minimum, and large and negative about $0^{d}.14$ on either side of the middle. Though a freehand curve would represent the observations much better than the curve here given, such a curve would have to cross the observed curve three times during the ascent or descent; and, since only three parameters are available in fixing the shape of an eclipse curve, this means that the freehand curve would be of a shape impossible to represent on the eclipse hypothesis.

TABLE XIV

SW CYGNI—VISUAL NORMALS

Phase	Mag.	Obs.	O−C	Phase	Mag.	Obs.	O−C
$-0^{d}.215$...	9.38	10	$+0^{M}.08$	$+0^{d}.009$...	11.72	10	$-0^{M}.01$
.178...	9.65	11	+ .02	.056...	11.60	10	− .13
.150...	9.95	10	+ .05	.086...	11.17	10	+ .05
.133...	10.18	10	.00	.106...	10.69	11	+ .02
.120...	10.40	10	+ .01	.123...	10.37	11	+ .04
.102...	10.74	9	− .02	.143...	10.05	10	+ .01
.073...	11.41	10	− .03	.174...	9.67	10	− .01
.044...	11.72	10	− .01	.210...	9.38	10	.00
−0.019...	11.71	10	−0.02	.253...	9.15	10	.00
				+0.280...	9.09	5	0.00

TABLE XV

SW CYGNI—PHOTOGRAPHIC NORMALS

Phase	Mag.	Obs.	O−C	Phase	Mag.	Obs.	O−C
$-0^{d}.224$...	9.52	7	$-0^{M}.01$	$+0^{d}.049$...	12.73	9	$-0^{M}.12$
.198...	9.75	8	+ .03	.065...	12.41	8	− .18
.179...	9.94	8	+ .04	.077...	12.17	7	− .01
.158...	10.10	7	− .03	.087...	11.96	7	+ .10
.134...	10.29	7	− .21	.096...	11.70	8	+ .22
.119...	10.74	9	− .08	.109...	11.16	7	+ .04
.106...	11.17	7	.00	.122...	10.78	6	+ .04
.096...	11.63	8	+ .15	.131...	10.51	7	− .06
.078...	12.04	9	− .10	.143...	10.14	7	− .22
.062...	12.49	9	− .25	.150...	10.23	7	− .01
.048...	12.74	9	− .11	.157...	10.07	7	− .09
.036...	12.78	8	− .07	.166...	10.03	8	− .01
.028...	12.91	8	+ .06	.178...	9.95	8	+ .05
.021...	12.81	8	− .04	.190...	9.89	7	+ .10
.013...	12.93	8	+ .08	.199...	9.78	7	+ .07
− .005...	12.91	8	+ .06	.221...	9.66	7	+ .12
+0.003...	12.90	7	+ .05	.243...	9.48	7	+ .06
.014...	12.99	7	+ .14	+0.276...	9.32	5	.00
+0.034...	12.90	7	+0.05	3.732...	9.14	6	$-0^{M}.15$

The principal discordances, already mentioned, are such as would arise if the assumed magnitudes of the comparison stars were somewhat erroneous. Whether this be the true explanation or not, these discordances, since they are symmetrical on each side of the minimum, will have no sensible influence on the computed time of mid-eclipse.

A least-square solution gave, as correction to the assumed epoch of minimum (1343.630), the value $+0^{\mathrm{d}}.0002 \pm 0^{\mathrm{d}}.0008$. The probable error of one normal place, depending, on the average, upon 7.5 observations, comes out $\pm 0^{\mathrm{M}}.072$, which makes one probable error of a single observation $\pm 0^{\mathrm{M}}.196$. This is much greater than in any of the cases previously discussed—mainly on account of the systematic deviations already mentioned, but partly because the individual observations are in any case mutually discordant. The residuals from a freehand curve lead to a probable error of $\pm 0^{\mathrm{M}}.130$ for a single observation.

From the visual and photographic solutions it appears that the photographic minimum is later than the visual by $0^{\mathrm{d}}.0042 \pm 0^{\mathrm{d}}.0009$—which, in view of the number and quality of the observations concerned, appears to be unquestionably real.

DISCUSSION OF THE RESULTS

A. *Errors of the photographic observations.*—At this stage of a work which it is hoped may prove of considerable extent, it is well to consider with some care the degree of precision which the observations possess. The individual residuals for all the observations of each star have therefore been arranged in order of their numerical magnitude, and the numbers between given limits compared with the theoretical law of distribution corresponding to the probable error computed in the ordinary fashion. The principal results of this inquiry are presented in condensed form in Table XVI.

For each star there are first given the numbers of positive and negative residuals between the limits specified at the left (residuals of a limiting value, such as $0^{\mathrm{M}}.05$, being equally divided between the preceding and following groups). Following these are two columns, the whole number of residuals, regardless of sign, between the assigned limits, and the number which might be expected on the Gaussian theory, with the probable errors given below. The observations of U Sagittae during and outside the principal eclipse are listed separately, as they are of different degrees of precision.

The lowest line of this part of the table gives the whole number of positive and negative residuals. The table so far deals only with those observations which were finally admitted into the discussion.

TABLE XVI

Residual Errors of the Photographic Observations

Limits	S Cancri				W Delphini				SW Cygni			
	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory
$0^{\mathrm{M}}.00$ to $0^{\mathrm{M}}.05$	40	52	92	85	53	47	100	98	25	33	58	56
.05 to .10	38	50	88	79	48	49	97	88	31	23	54	53
.10 to .15	30	21	57	64	38	31	69	72	22	19	41	46
.15 to .20	22	24	46	49	30	20	50	54	19	21	40	39
.20 to .30	24	22	46	50	27	23	50	57	26	24	50	52
.30 to .40	9	8	17	14	9	5	14	17	13	16	29	25
.40 to .50	2	3	5	4	2	3	5	3	6	4	10	9
Over 0.50	....	....	....	....	....	5	5	1	....	2	2	4
Total	165	180	345	(345)	207	183	390	(390)	142	142	284	(284)
Rejected	3	17	20	....	8	3	11	....	0	2	2	....
Limit	$0^{\mathrm{M}}.40$				$0^{\mathrm{M}}.55$				$0^{\mathrm{M}}.60$			
Average	0.56				1.07				1.09			
Probable error:												
First powers	$\pm 0^{\mathrm{M}}.105$				$\pm 0^{\mathrm{M}}.107$				$\pm 0^{\mathrm{M}}.136$			
Squares	± 0.107				± 0.111				± 0.136			
Normals	± 0.128				± 0.132				± 0.196			

LIMITS	RW TAURI				W URSAE MAJORIS				U SAGITTAE							
									During Eclipse				Outside Eclipse			
	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory
$0^{\mathrm{M}}.00$ to $0^{\mathrm{M}}.05$	16	18	34	32	65	87	152	139	35	39	74	70	112	96	208	209
.05 to .10	14	20	34	28	47	52	99	109	34	30	64	58	72	28	100	108
.10 to .15	7	14	21	22	33	38	71	67	18	16	34	38	6	20	26	28
.15 to .20	4	4	8	15	11	14	25	32	5	8	13	21	2	7	9	4
.20 to .30	9	3	12	15	7	7	14	16	4	7	11	13	1	5	6	0
.30 to .40	2	3	5	3	3	......	3	1	3	1	4	2	......	......	......	......
.40 to .50	......	2	2	1	......	......	......	......	1	1	1	0	......	......	......	......
Over 0.50	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
Total	52	64	116	(116)	166	198	364	(364)	100	102	202	(202)	193	156	349	(349)
Rejected	2	5	7	......	0	2	2	......	4	4	8	......	1	0	1	......
Limit	$0^{\mathrm{M}}.50$				$0^{\mathrm{M}}.40$				$0^{\mathrm{M}}.50$				$0^{\mathrm{M}}.30$			
Average	0.89				0.52				1.76				1.16			
Probable error:																
First powers	$\pm 0^{\mathrm{M}}.094$				$\pm 0^{\mathrm{M}}.068$				$\pm 0^{\mathrm{M}}.075$				$\pm 0^{\mathrm{M}}.040$			
Squares	±0.098				±0.068				±0.079				±0.045			
Normals	±0.099				±0.070				±0.091				±0.043			

The next three lines give particulars concerning the observations rejected on account of inadmissibly large discordances: the first giving the number of those with positive and negative residuals and the whole number rejected; the second, the limit of rejection[1] (observations with residuals numerically exceeding this being rejected); and the third, the average value, regardless of sign, of the rejected residuals. It is clear that these very large residuals cannot have arisen from accidental errors of the same character as those of the bulk of the observations. Some of them may be due to defects of the plates not obvious on inspection; but it is probable that others arise from mistakes of some sort. In extensive routine work like the present, where hundreds of observations of the same star are available, the best course to pursue with regard to such discordant observations is clearly to reject them summarily. To spend an exceptional amount of work upon these doubtful observations—on the chance that in some cases the mistake, whatever it is, may be rectified—would demand so great an expenditure of time and money, in proportion to the increase of accuracy which might conceivably be attained in the final results, that this policy would be utterly inconsistent with the principles of "scientific management."

The percentage of observations so rejected varies from 0.6 for W Ursae Majoris to 5.5 for S Cancri and to 5.7 for RW Tauri. For the second of these stars the limit of rejection was set relatively low in proportion to the probable error later determined. For the third, four out of the seven rejected plates are long exposures made while the star was very faint and varying rapidly, under which conditions changes in the transparency of the atmosphere during the exposure may introduce large errors.

In the foregoing list of rejections no account is taken of plates recorded as unsatisfactory by the measurer, or of those on which the star was invisible, being near minimum.

The last three lines in Table XVI give for each star the probable error of a single photographic observation, as determined (1) from the numerical average of the first powers of the individual residuals,

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[1] For S Cancri the assigned limit is only approximate, as no rigorous rule of rejection was followed in this, the first case discussed.

(2) from the squares of these residuals, and (3) from the squares of the residuals for the normal points previously given. The values found in the first two ways are always very nearly equal. Those found from the normals are often perceptibly greater, but the difference is serious only in the case of SW Cygni, where the existence of systematic deviations from the computed curve has already been remarked. In this case the substitution of a smooth and symmetrical freehand curve reduced the computed probable error of a normal point from $\pm 0^{\mathrm{M}}.072$ to $\pm 0^{\mathrm{M}}.048$. If, as is probable, the square of the probable error of an observation, derived from the individual residuals, would be diminished by the same amount, it would become $\pm 0^{\mathrm{M}}.126$, as against $\pm 0^{\mathrm{M}}.130$ derived from the discordance between the normal points and this same curve.

Upon examination of Table XVI it is noticeable that the comparison of the observed with the theoretical distribution of the residuals shows usually an excess of small and large values, with a deficiency of those of intermediate size. This indicates, as is well known, that the observations, even though the seriously discordant ones have already been rejected, are not all of the same precision—the best observations furnishing the excess of small residuals, and the worst the superfluity of large ones.

The estimates made recently at Harvard are much more precise than those made in 1903, the average probable error (as derived from the individual residuals, weighting the various series in proportion to the number of observations) being $\pm 0^{\mathrm{M}}.068$ for the former and $\pm 0^{\mathrm{M}}.117$ for the latter. The recent observations, which are probably a good indication of what may be expected in future work, show that the Harvard photographs, though taken with no particular reference to the variables here studied, or, indeed, to photometric work at all, give results of a very considerable degree of precision.

When the great number of photographs available is considered, it is evident that the fund of information contained in the Harvard Photographic Library is of very great value and that the discussion of as much of this material as possible is urgently to be desired.

B. *Periods and epochs of eclipse.*—The periods and epochs of mid-eclipse which were finally adopted for the stars here discussed are given in Table XVII. The upper part of this table gives the elements for prediction of the visual minima in the ordinary form. The irregularity in the period of W Delphini, here represented by a quadratic term, is very likely to turn out ultimately to be periodic, and predictions by the formula here given should therefore be made with caution.

TABLE XVII

VISUAL MINIMA

Star	J. D.
W Delphini	2412002.5594+4.806211 E−0.000000062 E^2
U Sagittae	2417130.4027+3.380621 E
S Cancri	2413210.6053+9.4845485 E
W Ursae Majoris	2416129.2022+0.33363902 E
RW Tauri	2417198.4094+2.768903 E
SW Cygni	2411343.6260+4.572908 E

Star	Phot.-Vis.	P.E.	Duration of Increase	Distance Light-Years	ΔV Meters/Second
W Delphini	+0^{d}0026		0^{d}264	2900	+0.7
U Sagittae	+ .0040	±0^{d}0005	.262	960	+3.4
S Cancri	+ .0085	±0.0009	.315	1370	+5.1
W Ursae Majoris	− .0004	±0.0007	.034	165	−2.0
RW Tauri	− .0025		.175	2200	−0.9
SW Cygni	+0.0042	±0.0009	0.230	2400	+1.4

The lower half of the table contains data relating to the difference in time between the moments of mid-eclipse, as determined from the visual and photographic observations.[1] The second column gives this difference, which is positive if the photographic minimum comes late; and the third gives the probable error of this quantity (taking into account the errors of both the visual and photographic observations) as derived in the cases where least-square relations were made. For the two cases in which no such solutions were made the probable errors are not given, but the determinations are of about the same accuracy as the others.

In five of the six cases these differences are unquestionably real, and light-curves with the same epoch of mid-eclipse cannot give a satisfactory representation of both the visual and photographic observations. Similar differences in the times of minimum, as observed with different wave-lengths, have often been reported before; but the evidence of their reality is in the present case far more conclusive than previously, depending as it does upon hundreds of observations of each star, at practically the same epochs for the visual and photographic work (the former always falling within the interval covered by the latter), which have been reduced with the aid of light-curves whose form is not arbitrary, but is determined by a well-tested theory.

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[1] Baldwin's visual observations in the case of W Ursae Majoris. See p. 320.

The origin of these differences however remains decidedly obscure. The most obvious suggestions are (*a*) that the velocity of the visual and photographic rays in interstellar space is slightly different, as suggested by Nordmann, and (*b*) that the distribution of brightness over the surface of the star-disks is uneven, and different for the different wave-lengths, as suggested by Lebedew.[1]

The fact that the observed differences are not always of the same sign is strongly against the first of these explanations. Moreover, the distances of the various stars can now be approximately estimated, in the manner described by Russell and Shapley.[2] These distances are given in the fifth column of Table XIX, the data for RW Tauri and U Sagittae being recalculated to correspond to the new elements of the present paper. The difference in velocity of the visual and photographic rays which would be required to explain the observed discrepancy for each star is given in *meters* per second in the last column of Table XVII. The various results are utterly discordant and do not furnish the slightest evidence in favor of the hypothesis. It is, however, decidedly interesting to note that the observations of even a single star prove the identity of the velocity of light of the different wave-lengths within a few meters per second, that is, on the average, to within a hundred-millionth part of the whole.

Probably no other direct observational test is of comparable delicacy; but since a difference of velocity of even one meter per second would demand that the medium occupying interstellar space should have a dispersive power of about one fifteen-hundredth that of ordinary air at the earth's surface, there is good indirect reason to suppose that, if any such influence really exists, its effects must be far too small to measure, even in this way.

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[1] *Astrophysical Journal*, **29**, 101, 1908.

[2] *Ibid.*, **40**, 417–434, 1914.

The second explanation appears more probable, especially as the amount of shift of the minimum is always but a small fraction of the whole time occupied by the increase or decrease of light during the eclipse (which is given in the fourth column). But in the case of S Cancri the shift is one thirty-seventh of the whole time of increase, which seems a rather large amount to be accounted for by a *difference* in the manner of distribution of yellow and violet light over the star-disk. Again, in the case of U Sagittae, when the visual light of the brighter star is half eclipsed (visual magnitude 7.10), the photographic magnitude is 7.08 during the decreasing, and 7.16 during the increasing, phase. At these times the visible portions of the bright component are practically the two opposite halves of the disk (since the eclipse is central), and it is evident that these two regions must differ in color-index by at least $0\overset{M}{.}08$ to explain the observations in this way—the preceding half of the disk being the redder. But, when these opposite faces of the star are turned toward us a quarter-period before and after eclipse, the difference of magnitude is only $0\overset{M}{.}02$, and the preceding face appears to be photographically brighter than the other.

Wendell has four visual observations in the interval between primary and secondary minimum, with a mean magnitude of 6.415, and nine in the other half of the period, with a mean of 6.429. It appears, therefore, very doubtful that differences in color-index between the opposite sides of the star, if present at all, can be great enough to account for the observed retardation of the photographic minimum.

A third explanation deserves mention. It has been shown by Pickering that, when several successive exposures are made on one plate, the plate usually appears to be more sensitive for the first exposure than for the others. Hence, when a star of varying brightness is photographed, the effective mean instant of exposure, this variation of sensitiveness being taken into account, will be earlier than the middle of the exposure. A simple calculation shows that, if the sensitiveness during the first half of the exposure were 10 per cent greater than the average and that in the second half 10 per cent less, the effective mean time of exposure for a star varying in brightness at a uniform rate would be earlier than the middle of exposure by one-twentieth of the whole duration of exposure. If no allowance is made for this, the photographic variation will appear to come late, the shift being $0^{d}002$ for exposures of an hour. This may have something to do with the tendency of the observed photographic minima to come late, but can hardly explain the case of S Cancri, in which the shift is 12 minutes, and the average length of exposure between 20 and 30 minutes.

For the present it appears necessary, therefore, to leave the origin of these small shifts as a problem for future solution.

C. *Astrophysical results.*—1. The present investigation furnishes the most comprehensive test which has yet been made of the theory of eclipsing variables. Two independent, extensive, and precise series of observations were available for each star, and these have in all cases been satisfactorily represented by exactly the same geometrical elements—size and shape of the components and inclination of the orbit—only the brightness of the components and, in some cases, the degree of darkening toward the limb being different for the visual and photographic observations. The representation of the latter by the elements derived from the visual observations is excellent for all the stars but SW Cygni, and fair in this case, in which the discrepancies can reasonably be attributed to errors in the early and less precise photographic estimates. Table XVIII gives the elements of the six systems. The geometrical elements are first given, those of U Sagittae and RW Tauri being derived from the computations described above and the others being taken from Shapley's list.[1] These are in all cases the "uniform elements" derived on the assumption that the visual brightness of the star-disks is everywhere the same. For S Cancri and W Ursae Majoris the photographic brightness appears to be only half as great at the edge of the disks as at the center. The stars are spherical (or at least spheroidal) except in the case of W Ursae Majoris, where they are ellipsoidal, the two shorter radii being 0.745 and 0.681 times the greatest radius given in the table. The radii, as tabulated, are expressed as fractions of the radius of the relative orbit.

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[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 82–85.

The visual and photographic ranges of variation are next given, the former being taken in most instances from Shapley's work. The latter is invariably the greater, showing that the stars are redder at minimum than at maximum—and usually considerably so.

TABLE XVIII

ELEMENTS OF ECLIPSING BINARIES

	W Delphini	U Sagittae	S Cancri	RW Tauri	SW Cygni	W Ursae Majoris
Radius of bright star	0.135	0.230	0.075	0.190	0.131	0.366
Radius of faint star	0.256	0.295	0.203	0.254	0.266	0.366
Inclination of orbit	$83^\circ 27'$	$90^\circ 0'$	$83^\circ 10'$	$90^\circ 0'$	$83^\circ 42'$	$77^\circ 32'$
Visual range	$2^{\mathrm{M}}.70$	$2^{\mathrm{M}}.75$	$2^{\mathrm{M}}.12$	$3^{\mathrm{M}}.47$	$2^{\mathrm{M}}.66$	$0^{\mathrm{M}}.60$
Photographic range	3.32	3.33	2.80	3.91	3.56	0.68
Bright component:						
Visual magnitude	9.49	6.51	8.14	8.11	9.16	(8.66)
Photographic magnitude	9.48	6.47	8.14	8.12	9.33	8.68
Color-index	−0.01	−0.04	0.00	+0.01	+0.17	(+0.02)
Spectrum	A	B8	A	B5	A	G
Density	0.18	0.073	0.25	0.20	0.21	2.45
Faint component:						
Visual magnitude	12.10	9.18	10.10	11.54	11.72	(8.66)
Photographic magnitude	12.75	9.75	10.86	12.00	12.85	8.68
Color-index	+0.65	+0.57	+0.76	+0.46	+1.13	(+0.02)
Density	0.008	0.011	0.005	0.024	0.009	2.45

Following this appear data concerning each component separately—the visual and photographic magnitudes, the color-index, the spectral class (which is known only for the brighter components), and the density—in computing which the relative masses of the components have been estimated on the basis of their visual brightness.[1]

These visual magnitudes have been determined by comparison with stars which appear in some photometric catalogues (usually only one star in each case), and the photographic magnitudes by comparison with the Polar Sequence. The zero-points to which the magnitudes of the different stars are referred (especially the visual magnitudes) may be subject to small errors, and too much weight should not be laid upon the absolute amounts of the resulting color-indices. For W Ursae Majoris, in particular, the visual magnitude is given on the Potsdam scale and requires a large correction—

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[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 121–123.

probably about $-0^{\mathrm{M}}.3$—to reduce it to the Harvard scale, so that the color-index given in the table is not comparable with the others.

The *scale* of magnitudes, however, should in all cases be well determined, and the relative color-index of the two components of each system should be trustworthy.

2. All these stars except W Ursae Majoris have been selected from a well-defined class—those with total eclipses, large range, and components markedly unequal in brightness, the smaller star being much the brighter and denser of the two.

The brighter components of these five systems are very similar, their average spectrum being B 9, their average color-index $+0^{\mathrm{M}}.03$—in very good agreement with the spectral class—and their average density 0.18 that of the sun. They appear to be entirely typical of the first-type stars in general. The fainter components are also very similar *inter se*, but are of much lower density, averaging only 0.011 that of the sun, and are all far redder than their primaries, their average color-index being $+0^{\mathrm{M}}.72$. If the relation between color-index and spectrum is the same for these point-images of faint stars as for the extra-focal images of brighter ones upon which the ordinary system is based, this would correspond to the spectral class G 0; but there is evidence that in this case the color-equation is smaller than the standard value, so that the given difference of color-index would indicate an even greater difference of spectral type.

In any case it is clear that these large faint companions are *second-type stars*. Direct observations of their spectra during the constant phase at minimum should be possible in some cases at least and are much to be desired.[1]

Similar differences in color-index, always in the same sense, have previously been reported by Seares[2] and Shapley[3] in the case of RR Draconis, by Shapley for Y Piscium[2] and TW Andromedae,[4] and by Dugan for RZ Cassiopeiae.[5] The additional results here given show clearly that the typical eclipsing binary of large range consists of a small, bright, dense component of the first spectral type (Class A or near it) and a large, faint component of much lower density and of the second type (Class G or redder).

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[1] In the very similar system U Cephei, the spectrum of the companion is K (Miss Cannon, *American Astronomical Society*, December meeting, 1916).

[2] *Astrophysical Journal*, **36**, 368, 1912.

[3] *Ibid.*, **37**, 160, 1913.

[4] *Publications of the Astronomical Society of the Pacific*, **26**, 258, 1914.

[5] *Astrophysical Journal*, **44**, 120, 1916.

It can hardly be doubted that, in such pairs, the component of low density represents the earlier stage of evolution, and that the terms "early" and "late," as applied to the spectral classes near the beginning and end of the series of the Draper classification, have no further significance than as describing the position of the stars in this sequence, and not at all in the sequence of evolution.

3. It is of much interest to compare the differences of color-index between the components with the differences of surface brightness (expressed likewise in stellar magnitudes), which may readily be computed from the data of Table XVIII. If the stars radiated according to Wien's law and were observed with monochromatic light, these two quantities should be in a fixed ratio to one another, no matter what the temperatures of the stars compared might be—namely, the ratio of the difference of the wave-lengths used in the visual and photographic observations to the photographic wave-length.

For bodies radiating according to Planck's law, this ratio would fall off slowly at very high temperatures. More serious deviations may arise from the departure of the stars from the laws of "black-body" radiation and from the fact that a considerable range of wave-lengths is effective in both the visual and the photographic observations. Nevertheless it might be expected that the difference in color-index would prove to be approximately proportional, at least, to that in surface brightness.

Table XIX contains all the data at present available which bear upon this question. The first column gives the name of the star; the second, the difference in color-index (ΔI) between the components, except for the last two stars, where the tabulated quantity is the difference in color-index between the light of the brighter component and that of the system at minimum, which is a mixture of the light of the two components (the eclipses being large, but partial). For RR Draconis and TW Andromedae two sets of values are given—one derived by comparison of the photo-visual and photographic results of Seares[1] and Shapley,[2] respectively; and the other, from the same photographic observations and the visual measures of Shapley[3] and Van Biesbroeck.[4]

The third and fourth columns give the difference in surface brightness (ΔJ) between the components, expressed in stellar magnitudes and derived from the known depths of eclipse, and the ratios of the radii of the components. The solutions assuming the star-disks to be of uniform brightness, or darkened toward the limb, usually give different values of the ratio of the radii, and hence of the relative surface brightness, and both sets of values are therefore tabulated.

TABLE XIX

STAR	DIFFERENCE OF COLOR-INDEX ΔI	DIFFERENCE OF SURFACE BRIGHTNESS ΔJ		$\Delta I/\Delta J$	
		Uniform	Darkened	Uniform	Darkened
RR Draconis:					
Visual	$+0^{\rm M}.84$	$+4^{\rm M}.9$	$+4^{\rm M}.1$	0.17	0.20
Photo-visual	.57	5.2	4.4	.11	.13
RW Tauri	.45	4.1	3.9	.11	.12
SW Cygni	.96	4.1	3.5	.23	.27
S Cancri	.76	4.1	3.4	.19	.22
W Delphini	.66	4.0	3.4	.16	.19
U Sagittae	.61	3.2	3.1	.19	.20
TW Andromedae:					
Visual	.33	3.0	2.4	.11	.14
Photo-visual	.51	2.9	2.2	0.18	0.23
RX Herculis	− .02	+0.14	+0.14	....	....
W Ursae Majoris	.00	0.00	0.00	....	....
Y Piscium	+ .37	>4.0	>3.5	....	....
RZ Cassiopeiae	+0.29	+3.6	+4.8	....	....

The elements of RZ Cassiopeiae are from Dugan's discussion,[5] the tabular quantities being the difference in surface brightness between those sides of the components which are visible at primary minimum. The geometrical elements of TW Andromedae are by Stewart.[6] The darkened elements of RW Tauri ($r_b=0.192$, $r_f=0.239$, $i=90^\circ.0$) and U Sagittae ($r_b=0.226$, $r_f=0.282$, $i=90^\circ.0$)

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[1] *Astrophysical Journal*, 36, 368, 1912.

[2] *Publications of the Astronomical Society of the Pacific*, 26, 156, 1914.

[3] *Astrophysical Journal*, 37, 154, 1913.

[4] *Annales de l'Observatoire Royal de Belgique*, 13, 11, 1914.

[5] *Astrophysical Journal*, 44, 117, 1916.

[6] *Ibid.*, 42, 315, 1915.

have been calculated from the new visual normals given above. The elements of the other stars may be found in Table XVIII or in Shapley's list[1] except in the case of RX Herculis, where the tabular data depend on Shapley's statement[2] that the visual surface brightness of the star eclipsed at principal minimum is 1.14 times that of the other, and Baker's value[3] 1.12 for the same ratio, as derived from his photographic observations.

The fifth and sixth columns give the ratios $\Delta I/\Delta J$, derived from the "uniform" and "darkened" solutions, for those cases in which the differences are large enough to permit a reliable determination. The results for different systems differ considerably, but not much more than those obtained by different observers for the same system. The mean values of $\Delta I/\Delta J$, giving each star equal weight, are 0.17 ± 0.010 for the uniform elements and 0.19 ± 0.012 for the "darkened"—the probable errors being derived in the usual way from the discordances between the individual values and the mean.

If the mean effective visual wave-length—to be exact, the "Crova wave-length"—is estimated as $0.52\,\mu$ for the visual and $0.42\,\mu$ for the photographic observations, the theoretical value of the ratio $\Delta I/\Delta J$ comes out 0.24; but neither of these wave-lengths has yet been determined even roughly for such methods of observation as are used in stellar photometry.

All that can be said at present is that, if, as seems probable, the color-indices given in this paper need to be increased considerably to reduce them to the standard scale, the observed ratio is in good accordance with theory.

For the two systems—RX Herculis and W Ursae Majoris—in which the components are nearly equal in surface brightness, they are also sensibly of the same color, as might be expected. For Y Piscium, where it is probable that the light at minimum is a mixture of that of the fainter component and of the uneclipsed portion of the brighter, the color-index is relatively small, as might again be anticipated.

[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 82–91, 1915.

[2] *Astrophysical Journal*, **40**, 399, 1914.

[3] *Laws Observatory Bulletin*, **2**, 170, 1916.

The case of RZ Cassiopeiae is more interesting. According to Dugan's "uniform" elements, which are determined with great accuracy by a long series of observations covering the whole curve, the fainter component conceals 0.806 of the area of the brighter one at mid-eclipse, and the radius of the brighter star is 0.768 that of the fainter.

The depth of the principal minimum (measured from the adjacent portion of the light-curve, to eliminate the effects of ellipticity and mutual radiation) is $1^{\mathrm{M}}.54$ visually and $1^{\mathrm{M}}.77$ photographically, so that the ratio of the brightness at mid-eclipse to that outside eclipse is 0.242 in the first case and 0.196 in the second. If b and f denote the light of the two stars, we then have the equations:

$$
\begin{aligned}
f+0.194b &= 0.242(b+f) \qquad \text{(visual)} \\
&= 0.196(b+f) \qquad \text{(photographic)}
\end{aligned}
$$

whence $f/b=0.063$ visually and 0.002 photographically.

The first of these two results is in accordance with Dugan's data, it being remembered that the fainter side of the larger star is visible at this time; but the second appears to lead to the absurd conclusion that the companion gives out no violet light at all (within the errors of observation). A clue to the riddle may be found in the assumption that the brighter component is darkened toward the limb for the photographic wave-lengths, so that the light of the remaining uneclipsed portion is redder than that of the whole disk, leaving less to be accounted for by the redness of the companion. If the loss of brightness at the limb is x times the central brightness, the amount of light lost when the fraction α_0 of the apparent area of the disk is concealed by a star of $1/k$ times the radius of the one eclipsed is[1]

$$\alpha_0(1-xX[k_1\alpha_0])$$

From the table of the function X it appears that in the present case

$$\alpha_0=0.806,\ k=0.768,\ X=+0.071$$

and the fraction of the light of the brighter star which is lost by eclipse is $0.806+0.057x$. The equation for the photographic minimum then becomes

$$f+(0.194-0.057x)b=0.196(b+f)$$

whence

$$f/b=+0.002+0.071x.$$

---

[1] *Astrophysical Journal*, **36**, 402–403, 1912.

To the visual ratio $f/b = 0.063$ corresponds a difference of surface brightness of $3\overset{\mathrm{M}}{.}6$, which would lead us to anticipate a color-index of about $+0\overset{\mathrm{M}}{.}6$ for the light of the fainter star relative to the brighter one and hence a photographic ratio $f/b = 0.036$. This value may be exactly reproduced by setting $x = 0.48$ in the equation derived above. It appears, therefore, that, if the brighter component of RZ Cassiopeiae is redder at the limb than at the center to the degree represented by the loss of about half the photographic light at the limb, the large faint comparison is of normal color-index in comparison with those of the other stars studied here. It is probable that this assumption is not far from the actual facts.

SUMMARY

1. Two thousand one hundred and one observations of the photographic brightness of six eclipsing variable stars have been made at Harvard and discussed at Princeton, and the resulting light-curves compared with those derived from visual observations.

2. In all cases it has been found possible to obtain a satisfactory representation of both the visual and the photographic observations by means of identical geometrical elements of the eclipsing system. The relative brightness of the two stars and in some cases the degree of darkening toward the limb are different for the visual and photographic rays.

3. The probable error of an estimate of magnitude depending on one Harvard plate is $\pm 0\overset{\mathrm{M}}{.}068$, for those plates investigated in 1914–16. (For the older estimates in 1903 it is considerably higher.) The Harvard Photographic Library therefore contains material for very precise light-curves of all regular variable stars brighter than the twelfth magnitude.

4. The epochs of the middle of eclipse differ for the visual and photographic observations by amounts which, though small (twelve minutes at most), are much greater than can be explained by the accidental errors of observation. No explanation of these differences has been found.

5. In three cases—S Cancri, W Ursae Majoris, and RZ Cassiopeiae—there is definite evidence that the disks of the brighter components appear redder at the edge than at the center, as is the case

with the sun. In other cases there is little evidence of this effect, but it would be premature to say definitely that it was absent.

6. When the two components of an eclipsing system are nearly equal in surface brightness, they are of very nearly the same color, but when they differ considerably in surface brightness, the star of lower surface intensity is the redder. In many cases the difference in color-index is at least as great as that between the spectral classes A and G. The whiter components are usually smaller and undoubtedly denser than their reddish companions.

7. The difference of color-index is, on the average, about one-sixth of the difference of surface brightness, expressed in stellar magnitudes. It is, however, probable that the ratio will be increased to one-fifth or farther when the color-indices are referred to the standard system. When this ratio is definitely established, the way will be clear to estimate the linear diameters of all stars of known color-index and parallax, and the angular diameters of all stars of known spectral type.

8. The diagrams of the visual and photographic light-curves need no further explanation than to say (*a*) that, in order to avoid overlapping, the photographic curves have been displaced in a vertical direction relative to the visual curves, so that the photographic magnitudes are to be read from the scale at the right and the visual magnitudes at the left; and (*b*) that the phases used in plotting the normal points have in all instances been referred to the finally adopted instants of mid-eclipse, which differ, for the visual and photographic observations of the same star, by the quantities given in Table XVII, so that the differences between the times of photographic and visual minimum are not shown in the figures.

Princeton University Observatory
February 22, 1917

MINOR CONTRIBUTIONS AND NOTES

ON THE AVERAGE PARALLAX OF THE STARS OF THE FOURTH TYPE

In the *Astrophysical Journal*, **32**, 9, 1910, Professor Kapteyn has published a determination of the average parallax of the stars of the fourth type. As this investigation has an important bearing on the question of the connection between the stars of type IV and Miss Maury's collateral series, I should like to point out that no reliable conclusion as to the mean parallax of these stars can be deduced from the data used by Kapteyn.

The material consists of the A.G. positions, some older observations, and the careful determinations made by Dr. Nörlund with the old meridian circle of the Copenhagen Observatory. Of this instrument Schjellerup says in the introduction to his star catalogue that the errors of the Pistor and Martin division amount in one case, at least, to 6″, and that one of the circles through an accident during transportation had become so deformed that the divisions were only indistinctly seen in the microscopes. Consequently the instrument has only been used in cases in which rough positions were better than none. In the present case the reliability of the positions has been further reduced by the method of observation; the bright stars were bisected by the wire, while the fainter ones were placed between two wires 14″ apart, so that the declinations are, strictly speaking, estimates rather than measures.

Although the systematic errors in the Copenhagen positions are partially reduced by the use of more catalogues in the computation of the proper motions, their existence is clearly revealed by a glance at the results. Arranging the secular motions μ_α and μ_δ according to right ascension, we get the means given in Table I.

It thus appears in Table II that the proper motions are positive from 0^h to 8^h and negative from 8^h to 24^h.

If star No. 60 in the group from 6^h to 8^h be rejected, the mean values become $+0^{s}.049$ and $-0''.68$, respectively.

From Tables I and II it appears that the proper motions in right ascension, if real, show a maximum in 7^h, even after the rejection stated, and a minimum in 19^h. This is clearly an effect of systematic errors in the proper motions, as it would require a right ascension of 13^h for the solar apex.

TABLE I

			No. of Stars
0^h to 2^h	$-0^{s}.118$	$-0''.13$	6
2 to 4	+0.083	+0.13	12
4 to 6	+0.012	−0.78	25
6 to 8	+0.149	−0.27	23
8 to 10	−0.049	+0.14	9
10 to 12	−0.070	+0.04	5
12 to 14	−0.047	−1.02	6
14 to 18	+0.172	+0.06	5
18 to 20	−0.028	+0.06	23
20 to 22	+0.020	−1.87	15
22 to 24	−0.060	−0.94	8

TABLE II

			No. of Stars
0^h to 4^h	$+0^{s}.016$	$+0''.04$	18
4 to 6	+0.012	−0.78	25
6 to 8	+0.149	−0.27	33
8 to 18	−0.009	−0.17	25
18 to 20	−0.028	+0.06	23
20 to 24	−0.008	−1.55	23

Arranging the proper motions according to declination, we get the synopsis in Table III.

In the neighborhood of the zenith of Copenhagen there is a strongly marked discontinuity in the proper motions in the right ascension from $+0^{s}.133$ to $-0^{s}.133$, the origin of which must be sought in Copenhagen rather than in the Milky Way. The change of sign has probably some connection with the change of position of the observer, and north of the zenith of Copenhagen the systematic errors of the method used in the observations of declination reappear as a "star-drift" of −2″ common to seven stars! If the declinations of the stars between +65° and +70° be computed from the older observations alone, we find that the Copenhagen declinations in this region require a correction of about 1.″

TABLE III

			No. of Stars
−25° to −15°......	$-0^{s}.037$	$+0''.09$	10
−15 to − 5	+0.037	−0.21	17
− 5 to + 5	−0.029	−0.97	15
+ 5 to +15	0.000	+0.18	11
+15 to +25	−0.055	−0.79	13
+25 to +35	+0.027	−0.28	13
+35 to +45	+0.011	−0.84	21
+45 to +55	+0.133	−0.75	14
+55 to +65	+0.010	−0.07	10
+65 to +75	−0.133	−2.17	7

The imaginary character of the proper motions is further clearly revealed by their rapid increase with the magnitude of the stars. Dividing the stars in groups of twenty, we obtain the mean values given in Table IV.

TABLE IV

$7^{M}.58$	$2''.24$
8.26	2.86
8.80	2.61
9.01	4.31
9.31	5.86

As the systematic errors in the proper motions are of the same magnitude as the proper motions of the stars of the seventh magnitude, we may safely conclude that the secular motions found for the fainter ones are quite imaginary; hence any attempt at a determination of the mean parallax from these data must be regarded as mere guesswork.

H. E. Laŭ

Hörsholm
March 1917

RECENT CHANGES IN THE VARIABLE NEBULA N.G.C. 2261

This is a bright, comet-shaped nebula situated at R.A. 6^h34^m, Dec. $+8°49'$ (1900). Plates taken by the writer in the winter of 1915–1916 with the 24-inch reflector of the Yerkes Observatory, when compared under the blink-comparator with a plate taken with the same instrument by Mr. F. C. Jordan in March 1908, showed changes in the outline, and displacement of the structural details of the nebula. An account of the matter is given in this *Journal* for October 1916 (**44**, 190). The changes, though striking, were small, amounting at most to displacements of some 4 or 5 seconds of arc in the interval.

A plate taken by Mr. H. D. Curtis with the Crossley reflector of the Lick Observatory in January 1913 was very kindly lent to us by Director Campbell. This showed that the entire change had occurred between 1908 and 1913.

The nebula apparently remained quiescent until May 1916, when the last plate of that season was taken. In September of last autumn Miss Gushee of the Yerkes Observatory secured a plate of the nebula on which some changes were suspected. These were verified by a series of exposures which she made in the latter part of October.

A Lowell Observatory circular, dated January 29, 1917, stated that a comparison of plates taken, the one in March 1916, the other on January 25, 1917, by Mr. Lampland showed striking differences—apparent displacements which might amount to 15 seconds of arc.

On March 27, 1917, the writer secured a new plate with the Yerkes 24-inch reflector. In the telescope the nucleus was distinctly brighter than in the previous year, at least with respect to the neighboring nebulosity. The developed plate confirmed this impression to a remarkable degree. The general trend of the change seems to be that the nebula has faded very considerably all around its southern perimeter. It is to be regretted that the scale of the reflector is too small to bring out the finer details distinctly. Plate IX has been prepared, showing the nebula as it appeared on February 27, 1916 and on March 27, 1917.

Although the interval is here an entire year, almost the whole change occurred after November 1 and had reached enormous proportions certainly by January 25, when a plate was taken at Lowell Observatory. It remains to be seen whether the change is still in progress.

The details of the change are confused by the small scale of the plate, as the image of the nebula is only half a millimeter in length on the negative. Clearly the former bright band just north of the nucleus has contracted to about a third of its length, and the whole southern portion of the body has contracted about the north-and-south axis—all this, however, without affecting the details in the outer portion, away from the nucleus. It would seem that the former illumination near the nucleus has faded away. The faint streamer running out from the nucleus in the direction south-following has perceptibly brightened. The recent plates have certain points in common with a plate taken by the late Dr. Isaac Roberts at Starfield on January 27, 1901, copies of which Mme Roberts had the kindness to prepare and send at the request of Director Frost.

The nucleus has long been known as the variable star R Monocerotis, for which a range between magnitudes 9.5 and 13.0 has been reported. The period is irregular. The photographs showed no appreciable variation until this spring, when the nucleus brightened by half a magnitude.

Director Slipher of the Lowell Observatory reports a continuous spectrum crossed by bright lines or bands, not those typical of the gaseous nebulae.

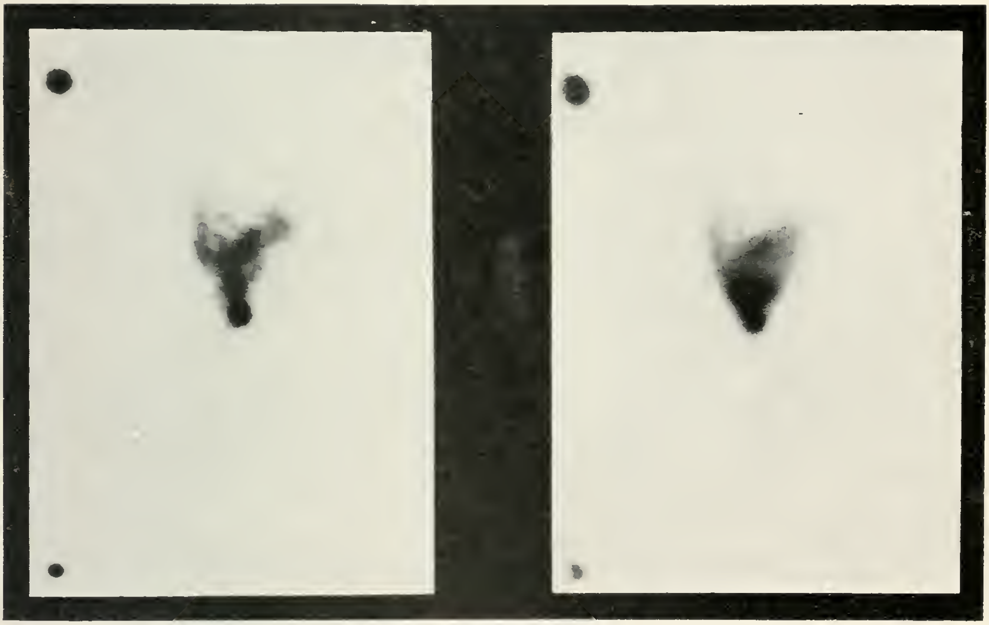
This nebula is of the same class as N.G.C. 6729, the variability of which was studied photographically by Knox-Shaw at Helwan. N.G.C. 2261, however, is larger and brighter, and hence the more easily studied in detail. Hind's variable nebula by T Tauri (N.G.C. 1555) is probably a third member of this class.

No attempt is here made to explain the phenomenon, save to remark that, unless we are witnessing a phenomenon of illumination, the nebula must be very near. Parallax and proper motion

PLATE IX

Scale: 1 mm = 3″.6

North



1917 March 27

1916 February 27

Taken with Yerkes 24-inch Reflector
Enlarged 24 times

N.G.C. 2261

are related to linear velocity across the line of sight by the formula $\pi = \frac{5\mu}{V}$. In this case μ could not be less than 20–30 seconds of arc, hence a velocity of, say, 500 km per second would point to a parallax of 0.″2 at the least, and the matter could be settled by a direct measure of the parallax. If it is a question of illumination, then, as was pointed out by Slipher and Lampland, the nebula would not be more than ten or twelve thousand light-years distant at the most.

EDWIN P. HUBBLE

UNIVERSITY OF CHICAGO
April 10, 1917

REVIEWS

X-Rays. By G. W. C. KAYE. Second Edition. London and New York: Longmans, Green, & Co., 1917. Pp. xxi+285. $3.00 net.

"In this edition the writer, so far as his military duties would permit, has subjected the book to a thorough revision and incorporated original work of note published up to the middle of 1916." Although the first edition was far from outgrown, the rapid growth of this fascinating and highly important subject warrants a revision at this time. The additions which have been made are characterized by the same conciseness, yet comprehensiveness, found in the first edition. The substance of many scattered and often lengthy papers is clearly stated in remarkably few words.

Thirty pages in all have been added, twenty of which comprise a wholly new chapter on "X-Ray Equipment and Technique," which gives a brief account of the installation and procedure for taking X-ray photographs. From the standpoint of the physicist the changes of greatest interest are: (1) a careful revision of the chapter on "The Diffraction of X-Rays by Crystals," the recent work of the Braggs, De Broglie, Siegbahn, and others being incorporated, and the discoveries of Moseley being dealt with more at length; (2) a detailed description of the Coolidge tube; (3) a brief summary of the recent work of Barkla on the scattering of X-rays, on the gaseous absorption of X-rays, and on the velocity of the corpuscles ejected by X-rays; (4) a brief discussion of the quantum-theory relations which appear in X-ray phenomena.

Other changes and additions are: (1) the addition of sections on "The Ionics of an X-Ray Tube," "The Snook Hydrogen Tube," "Metal X-Ray Bulbs," "Energy Measurements," "Present Day Radiology and Radiotherapy," and "Recommendations for the Protection of X-Ray Workers"; (2) twenty-three new cuts have been added, one left out, and four changed; (3) three new tables have been added.

An undue prominence is given to the paper of Rutherford, Barnes, and Richardson (*Philosophical Magazine*, **30**, 339, 1915), the fundamental assumption of which (namely, that wave-lengths may be deduced from the absorption coefficients) has been shown to be incorrect, at any rate for high frequencies, by several observers (see Duane and Hunt, *Physical Review*, **6**, 166, 1915; Hull, *ibid.*, **7**, 156, 1916; Allen and Alexander, *ibid.*, **9**, 198, 1917). As a result an entirely wrong impression is given in several places. On pp. 129 and 46 the statement is made that the shortest wave-length emitted by a Coolidge tube is 0.171×10^{-8} cm with a penetration of about one-half that of the gamma rays from radium C, and that this radiation appears at about 140,000 volts. Dr. Hull, of the General Electric Laboratory, has observed a radiation with wave-length 0.08×10^{-8} cm at a voltage of about 150,000, and he states that there is no indication of the existence of a limit to the hardness of the X-rays which may be generated by any particular anticathode (see *General Electric Review*, **19**, 314, 1916; also *Proceedings of the National Academy*, **2**, 265, 1916). On p. 230 credit is given to Rutherford for pointing out that absorption results are not a reliable guide for deducing wave-lengths. This was first pointed out by Duane and Hunt (*Physical Review*, **6**, 166, 1915) and has been verified by Hull and Rice (*Physical Review*, **7**, 326, 1916). As has been said, Rutherford worked on the opposite assumption. Siegbahn's discovery of an M series for the heavy metals (gold to thorium) softer than the K or L series (*Comptes Rendus*, **162**, 787,1 916) seems to have been overlooked (see p. 122). Moreover, I believe that the work of Webster (*Physical Review*, **6**, 599, 1916) shows clearly that the quantum relation $Ve = h\nu$, holds as well for the characteristic radiations as for the independent (the statement on p. 245 to the contrary), except that in this case the frequency to be used is that of the shortest (γ) line of the series.

The paper upon which the book is printed is not as good as that of the old edition, and as a consequence some of the cuts are a little blurred and not as clear as in the first edition. This was probably occasioned by war conditions and was unavoidable.

E. C. WATSON

RYERSON PHYSICAL LABORATORY

Annuaire pour l'an 1917 publié par le Bureau des Longitudes. Paris: Gauthier Villars et Cie, 1916. Pp. 661. Fr. 1.50.

The *Annuaire* for 1917 is devoted principally to astronomical tables and tables of measure and of meteorology. According to the plan adopted in 1904, it should contain also tables on geography and statistics. These have been omitted, together with those giving physical and chemical information.

Besides the usual ephemerides and lists of double stars, parallaxes, etc., there are tables for finding the value of gravity in different places, and for the calculation of altitude from barometric observations.

Revisions occur in the chapter on constellations, which contains six excellent charts with "catch-figures" outlined in red, and notes on the study of constellations, and in the chapter dealing with stellar spectra, where particular attention is given to recent developments. We find additions in the part dealing with the Julian and Gregorian calendars, and in the section on the rising and setting of the sun and moon.

Among the notices is one of particular interest to the physicist—"La Détermination du mètre en longueurs d'ondes lumineuses" by Maurice Hamy.

E. W. W.

Annuaire astronomique et météorologique pour 1917. Par Camille Flammarion. Paris: Ernest Flammarion. Pp. 436. 140 figures, charts, and diagrams. Fr. 2.

This well-known publication has this year the distinction of being the fifty-third issue by the same editor, M. Flammarion. In addition to the usual astronomical and meteorological tables, notices on the recent progress of astronomy, and a summary of celestial phenomena for the year, it contains also guides for the study of the constellations, descriptions of the sun, moon, planets, and stars, and methods of observation. Of particular interest this year are the references to the war situation in Europe and records of celestial observations in letters received from the front.

The book is well adapted to furnish inspiration for the amateur observer and general reader as well as to meet some needs of the more experienced astronomer.

V. M. G.

Astronomischer Jahresbericht. Bearbeitet im Kgl. Astronomischen Rechen-Institut. Berlin: Georg Reimer, 1914–1916. Bd. XV (Literatur des Jahres 1913), pp. 545, Marks 18; Bd. XVI, pp. 391, M. 14; Bd. XVII, pp. 299. M. ?

We are glad that this valuable annual of the literature of astronomy has been maintained even during war times, through the editorial efforts of Professor P. V. Neugebauer and the chief of the Rechen-Institut, Professor Fritz Cohn. The first volume of the three here considered has its preface dated in June 1914. The subsequent shrinkage in size and in number of reviews is natural and is chiefly due, in the opinion of the editors, to the diminished output of scientific papers in war-ridden Europe.

Many papers not available to the editors were reviewed by colleagues at the observatory at Copenhagen. The shortening of the notices is not wholly a disadvantage. They are impersonal and generally adequate. The necessity of co-operation in the various phases of astronomy justifies the hope of a resumption of such relations between all the nations after the war.

F.

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